Group Delay Variations in Wideband Transmission Lines: Analysis and Improvement

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Abstract—Although poorly studied in the literature, Group Delay Variations (GDV) versus frequency is an essential factor which causes distortion and degradation in wideband satellite signals especially when using phase modulation and high data rates. In this paper, transmission line is analyzed as a dispersive medium and some kinds of coaxial cables such as RG58U, RG59U, RG213 and ECOFLEX15 are compared as GDV parameter point of view. Then the effect of reflections from discontinuities and impedance mismatches at transmission lines, on GDV quantity, is investigated by suggesting a novel network model of transmission line with discontinuity or impedance mismatch, and extracting a new formula for GDV. Graphical data are presented based upon the formula developed, and the simulation results are also given by AWR software which confirms the theory and formula. At last, based on the developed formula, some calculations will be carried out both to predict the values of GDV parameter and to compensate it. In this paper the frequency range of 100-1000 MHz is selected. The main reason of this selection is due to the practical application of coaxial cables for transmitting wideband satellite signals in remote sensing ground stations from down-converter to modem at IF frequencies such as: 140, 375, 720 MHz, etc. In addition, the introduced model and formula are generalizable to upper frequency bands.

Index Terms—group delay variations, transmission lines, coaxial cables, dispersion, discontinuity, mismatch.

I. INTRODUCTION

In wideband communication systems-e.g. satellite communication systems- which are used for transmitting wideband signal/data, any distortion causes Signal to Noise Ratio (SNR) or Bit Error Rate (BER) degradation [1-4]. Distortions are grouped to linear and nonlinear types.

Nonlinear distortion happens in systems such as mixers, High Power Amplifiers (HPAs), etc., which have a nonlinear characteristic. Figure 1(a) shows such systems. This phenomenon is explained by AM-to-AM, AM-to-PM, Intermodulation Distortion (IMD), etc., and can be compensated by several linearization techniques or decreased by using elements with weak nonlinear characteristic [5-7].

Linear distortion happens in linear systems in which the magnitude of frequency response is not constant and the phase of frequency response is nonlinear. Figure 1(b) shows such systems. The magnitude and phase of frequency response in these systems are defined as:

\[ A(\omega) = |H(j\omega)| \quad \text{and} \quad \theta(\omega) = \angle H(j\omega) \]  

From this section, input the body of your manuscript according to the constitution that you had. For detailed information for authors, please refer to [1].

\[ T_{pd} = -\frac{d\theta}{d\omega} = -\frac{1}{2\pi} \frac{d\theta}{df} \]  

Figure 1. block diagram of (a) linear and (b) nonlinear systems

Phase distortion is measured using Group Delay (GD) parameter [11-15]. It is expressed in units of time (\(T_{pd} : \text{nanosecond}\)) [8, 9]. The GD is related to the phase shift variation with frequency and for a linear system, at an angular frequency \(\omega_0 = 2\pi f_0\), is defined as follows:

In equation (2), \(\theta\) is the phase of frequency response defined in equation (1) or phase of \(S\)-parameters (e.g. \(\angle S_{21}\)).

In distortionless systems, the phase characteristics must have a linear slope so that the ratio \(\frac{\theta(f)}{f}\) is constant for all frequencies and this represents a constant GD [9]. However, any deviations from linear phase over the frequency range will cause Group Delay Variations (GDVs); therefore, for a linear system, linear distortion over a frequency bandwidth is caused by GDV over that bandwidth not GD [2, 8, 9]. The past researches show that GDV over a frequency range can be caused degradations in wideband satellite signals with phase modulation and high data rates. This degradation can be neglected when the value of GDV over a frequency bandwidth is sub one nanosecond [2].

This paper deals with GDV versus frequency caused by transmission lines, e.g. coaxial cables that are used in the RF block of satellite receiver. Investigations on GDV caused by transmission lines have been very scarce [8-10]. The effects due to discontinuities of transmission lines on the GD are explained in [8] and GD caused by Impedance mismatch is also studied in [9, 10, and 11].

Total GDV is caused by two factors. The first factor is the dispersive nature of transmission line which is analyzed here. Low-loss coaxial cables (RG58U, RG59U, RG213 and ECOFLEX15) are also compared with respect to GDV they cause. The second and essential factor is reflection from...
discontinuities and impedance mismatches at transmission lines whose effects have been investigated for one discontinuity between two transmission lines or two impedance mismatches separated by a transmission line. A novel model of this factor is presented and a new formula is extracted. The simulation results confirm the formula and theory. At last, on the basis of formula, the prediction methods of GDV values are presented and practical solutions are introduced for compensation of the GDV.

II. GDV CAUSED BY DISPERSIVE NATURE OF TRANSMISSION LINES

Here we use a uniform transmission line of length \( l \) without any discontinuities inside it and without any impedance mismatch at two ends. A simple network model is shown in Figure 2. \( S_{21} \) can be obtained for the 2-ports:

\[
S_{21} = \frac{V_2^-}{V_1^+} = e^{-j\gamma l} = e^{-j(\alpha + j\beta)l} = e^{-\alpha l} e^{-j\beta l}
\]

(3)

Where \( \gamma \) is the propagation constant defined by \( \gamma = \alpha + j\beta \). \( \alpha \) is the attenuation constant in nepers or dB per meter, \( \beta \) is the phase constant in radians per meter, \( \beta = \frac{2\pi}{\lambda} \); and \( \lambda \) is the wavelength.

Knowing the phase of S-parameter (\( \angle S_{21} = -\beta l \)), the GD is obtained by equation (2):

\[
T_{gd} = \frac{d}{d\omega} \beta
\]

(4)

In equation (4), \( \beta \) is the imaginary term of \( \gamma \). This equation shows that GDV in a transmission line, without any discontinuities and impedance mismatches, is caused by nonlinear variations of phase constant (\( \beta \)) with frequency [20, 23]. If \( \beta \) is not a linear function of frequency, then the various frequency components of a wideband signal will travel with different phase velocities, and so arrive at the receiver end of the transmission line at slightly different times [23]. This will lead to dispersion and a distortion of the signal and generally has an undesirable effect.

The piece of transmission line can be modeled as a lumped-element circuit, as shown in Figure 3. Where \( R \), \( L \), \( G \), \( C \) are per unit length quantities defined as follows: [21]

\( R \) = series resistance per unit length, for both conductors, in \( \Omega/m \).

\( L \) = series inductance per unit length, for both conductors, in \( H/m \).

\( G \) = shunt conductance per unit length, in \( S/m \).

\( C \) = shunt capacitance per unit length, in \( F/m \).

The general expression for the complex propagation constant is [20, 21]:

\[
\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

(5)

The series resistance \( R \) represents the resistance due to the finite conductivity of the conductors, and the shunt conductance \( G \) is due to dielectric loss in the material between the conductors. \( R \) and \( G \), therefore, represent loss. In the lossless or ideal line, setting \( R = G = 0 \) in (5), gives that:

\[
\alpha = 0 \quad , \quad \beta = \omega \sqrt{LC}
\]

(6)

In this case, \( \beta \) varies linearly with frequency and, therefore, according to (4) GD is constant with frequency.

In practice, all transmission lines have loss due to finite conductivity and/or lossy dielectric, but these losses are usually small (low-loss lines). If \( R \ll \omega L \) and \( G \ll \omega C \), the following approximations are obtained by retaining up to second-order terms in the binomial expansions [20].

\[
\alpha \approx \frac{1}{2} \left[ \frac{R}{L} + \frac{G}{C} \right]
\]

(7)

\[
\beta \approx \omega \sqrt{LC} \left[ 1 - \frac{RG}{4\omega^2 LC} + \frac{G^2}{8\omega^2 C^2} + \frac{R^2}{8\omega^2 L^2} \right]
\]

(8)

In this case, \( \beta \) varies nonlinearly with frequency and GDV will exist over a frequency range. In fact, these variations of GD are due to inherent nature of transmission line as a dispersive medium [17-19].

For example, group delay (\( T_{gd} \)) of a transmission line that has the following per unit length parameters:

\( R = 0.1 \) ohm/m , \( L = 200 \) nH/m , \( C = 80 \) pF/m and length \( l = 100 \) m, versus frequency and with variations of \( G \) is represented in Figure 4. This figure shows that GDV is nearly equal \( 0.012, 0.005 \), and \( 0.01 \) s/m, respectively, over the frequency range of 0.1-1GHz. This points that the increase of the transmission line loss results in increase of GDV. Therefore, the variations of \( \beta \) from a linear function and GDV caused by the low-loss transmission lines are very small.

 Granted, as we have argued above, the deviation of \( \beta \) from a linear function may be quite small, but the effect can be significant if the line is very long. There is an exception, however, for a lossy line that has a linear phase constant as a function of frequency. Such a line is called a distortionless line [23], and is characterized by line parameters that satisfy the following relation:

\[
\frac{R}{L} = \frac{G}{C}
\]

(9)

From (5) the exact complex propagation constant, under
condition specified by (9), reduces to
\[ \gamma = j\omega \sqrt{LC} \left(1 + \frac{R}{j\omega L}\right) = \alpha + j\beta \]
\[ \alpha = R\frac{C}{L} = G\frac{L}{C}, \quad \beta = \omega \sqrt{LC} \quad (10) \]

Which shows that \( \beta \) is a linear function of frequency. Therefore, if the equation (9) is satisfied then GD will be constant and GDV will be zero. Since the line parameters (R, L, G and C particularly R) are usually weak functions of frequency, equation (8) may not be satisfied over a frequency range, exactly. To obtain a transmission line with parameters that satisfy (8) it is often required to increase L by adding series loading coils spaced periodically along the line [20].

For example, GD of a transmission line that has the following per unit length parameters: \( G = 0.001 \) s/m, \( L = 200 \) nH/m, \( C = 80 \) pF/m and length \( l = 100m \), versus frequency, and with various \( R \)'s is represented in Figure 5. This figure shows that GDV is nearly equal 0.01, 0 and 0.06 nanoseconds for \( R \)'s: 0.1, 2.5 and 8 ohm/m, respectively, over the frequency range of 0.1~1GHz. Although, for \( R=2.5 \) ohm/m, the losses of the line is more than that of \( R=0.1 \) ohm/m, but since \( R=2.5 \) ohm/m satisfies equation (9), GD is constant with frequency, and this line is a distortionless line.

\[
R = \frac{\mu}{\pi} \left( \frac{1}{D} + \frac{1}{d} \right), \quad L = \frac{\mu}{2\pi} \ln \frac{D}{d}
\]
\[
C = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln \frac{D}{d}}, \quad G = \frac{2\pi \varepsilon_r \varepsilon_0}{\ln \frac{D}{d}} \tan \delta
\]

Where:
- \( d \): diameter of the inner conductor
- \( D \): diameter of the outer conductor
- \( \varepsilon_r \): relative dielectric constant of the medium between conductors
- \( \varepsilon_0 \): permittivity of air and which is equal to \( 8.85 \times 10^{-12} \) F/m
- \( \mu \): permeability of the medium between conductors
- \( \tan \delta \): loss tangent of dielectric
- \( R_s \): surface resistivity of conductor

In table 1, four types of coaxial cables that used in practical applications have been brought. GDV is calculated for 100m length of the line over the frequency range of 100~1000 MHz by consideration the characteristics of lines.

<table>
<thead>
<tr>
<th>Type</th>
<th>Conductor Diameter (mm)</th>
<th>Dielectric Diameter (mm)</th>
<th>Velocity Ratio (%)</th>
<th>Capacitance (pF/m)</th>
<th>Characteristic Impedance (ohm)</th>
<th>Dielectric Constant (( \varepsilon_r ))</th>
<th>Attenuation @ 1000MHz (dB/100m)</th>
<th>GDV for 10m length over 100~1000MHz (nanosecond)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RG58U</td>
<td>0.82</td>
<td>2.95</td>
<td>66 %</td>
<td>98</td>
<td>50</td>
<td>2.3</td>
<td>54.6</td>
<td>1.4x10^{-4}</td>
</tr>
<tr>
<td>RG59U</td>
<td>0.82</td>
<td>3.68</td>
<td>83 %</td>
<td>53</td>
<td>75</td>
<td>1.45</td>
<td>26.5</td>
<td>5.6x10^{-7}</td>
</tr>
<tr>
<td>RG213U</td>
<td>2.26</td>
<td>7.24</td>
<td>66 %</td>
<td>100</td>
<td>50</td>
<td>2.25</td>
<td>22.5</td>
<td>2.4x10^{-7}</td>
</tr>
<tr>
<td>ECOFLEX15</td>
<td>4.5</td>
<td>11.3</td>
<td>86 %</td>
<td>77</td>
<td>50</td>
<td>1.35</td>
<td>9.8</td>
<td>7.4x10^{-8}</td>
</tr>
</tbody>
</table>

Figures 4 and 5, also, show that GDV for low-loss transmission lines is quite small (less than 1 nanosecond per 0.1~1GHz frequency band) and can be neglected.
III. GDV CAUSED BY REFLECTED WAVES DUE TO DISCONTINUITY AND IMPEDANCE MISMATCHING

For investigating the effects of reflected waves caused by discontinuity and impedance mismatch on the GD, the previous developed models have presented only discontinuity [8] or only mismatch [9, 10]. Figure 6 proposes the improved model of a transmission line with discontinuity separated by the length \( l \), characteristic impedance \( Z_{\alpha} \) and constant propagation \( \gamma_{\alpha} = \alpha_{\alpha} + j\beta_{\alpha} \). This model depicts the both cases: discontinuity between two transmission lines and two impedance mismatches separated by a length of transmission lines (e.g., a connector can be modeled by this design if the length \( l \) of discontinuity is short).

Figure 6. The proposed model of transmission line with discontinuity separated by length \( l \) of transmission line

It is more convenient to suppose the ends of the transmission line in Figure 6 to be matched to the generator and load. This means that \( \Gamma_1 = \Gamma_2 = 0 \). Therefore all the reflection waves due to discontinuity or mismatching impedance. If we suppose that \( Z_0, \gamma_0 = \alpha_0 + j\beta_0 \) are, respectively, characteristic impedance and propagation constant of the transmission lines that are connected to discontinuity at ends. Also \( l_1, l_2 \) are, respectively, length of transmission lines at the left and right of discontinuity then we will be written:

\[
V_1^+ = (V_1^+ e^{-\gamma_1 l_1}) T_1(e^{\gamma_1 l_1}) T_2(e^{\gamma_2 l_2}) + (V_1^+ e^{-\gamma_1 l_1}) T_1(e^{\gamma_1 l_1}) T_2(e^{\gamma_2 l_2}) \quad (11)
\]

Where:
- \( V_1^+ \): main incident wave transmitted from the generator
- \( V_2^- \): summation of all forward waves at the load
- \( T_1, T_2 \): transmission coefficients at ends of discontinuity
- \( \Gamma_1, \Gamma_2 \): reflection coefficients at ends of discontinuity

Using the geometric series, s-parameter is calculated:

\[
S_{nl} = \frac{V_1^-}{V_1^+} = T_1 T_2 e^{-\gamma_1(l_1 + l_2)} e^{\gamma_1 l_1} (1 + \Gamma_1 T_2 e^{-\gamma_2 l_2} + \Gamma_2^2 e^{-2\gamma_2 l_2} + \ldots)
\]

\[
= T_1 T_2 e^{-\gamma_1(l_1 + l_2)} e^{\gamma_1 l_1} \frac{1}{1 - \Gamma_1 T_2 e^{-\gamma_2 l_2}}
\]

(12)

We can let:

\[
\varphi = -\beta_0 (l_1 + l_2) - \beta_1 l + \theta_{\gamma 1} + \theta_{\gamma 2}
\]

\[
\varphi_B = -2\beta_1 l + \theta_{\gamma 1} + \theta_{\gamma 2}
\]

\[
h = [T_1 \gamma_1^2] e^{-2\alpha_1 l}
\]

Then we can write:

\[
S_{21} = \frac{T_1 T_2 e^{-\gamma_1(l_1 + l_2)} e^{-\gamma_1 l_1} e^{j\varphi_B}}{1 - \beta_1 e^{j\varphi}}
\]

(13)

The characteristic phase \( \theta \) is then:

\[
\theta = \angle S_{21} = \varphi_B + \tan^{-1} \left( \frac{h \sin \varphi_B}{1 - h \cos \varphi_B} \right)
\]

(14)

The GD is obtained by multiplying (14) by \(-1\) and differentiating the result with respect to angular frequency \( \omega \), and is:

\[
V_2^+ = \frac{\partial \varphi_B}{\partial \omega} - 2 l \frac{\partial \beta_1}{\partial \omega} = -2 T_0
\]

(15)

We can see in (15) how a change with frequency of the attenuation or phase constant or the reflection coefficients of the discontinuity, for example, might affect the GD. In practical cases and low-loss transmission lines, we can neglect the variations of \( \theta_{\gamma 1}, \theta_{\gamma 2}, \theta_{\gamma 1}, \theta_{\gamma 2}, h \) with frequency and suppose that \( \beta_0, \beta_1 \) vary linearly with frequency, approximately. Then we can write:

\[
\frac{\partial \varphi_B}{\partial \omega} = - (l_1 + l_2) 2 \frac{\partial \beta_1}{\partial \omega} - 1 \frac{\partial \beta_0}{\partial \omega} = -(T_1 + T_0)
\]

(16)

In which \( T_0 = \frac{d \beta_0}{d \omega} \) and \( T_1 = \frac{d \beta_1}{d \omega} \). \( T_1 \) and \( T_2 \) are propagation delays of discontinuity and transmission lines at the ends of it, respectively, and because of the line is low-loss, these are constant with frequency. Therefore equation (15) is obtained as:

\[
T_{gd} = T_1 + T_0 \left( 1 + \frac{2h \cos \varphi_B - h \cos 2\varphi_B}{1 + h^2 - 2h \cos \varphi_B} \right)
\]

(17)

In practical case, we have \( h \ll 1 \) and can neglect terms of \( h^2 \) in (17). Then the GD is approximated to:

\[
T_{gd} = T_1 + T_0 \left( 1 + \frac{2h \cos \varphi_B}{1 - 2h \cos \varphi_B} \right) = (T_1 + T_0) + \frac{2h \cos \varphi_B - T_0}{1 - 2h \cos \varphi_B}
\]

(18)

In equation (18), first term of the GD \((T_1 + T_0)\) is the sum of the GD’s resulting from all lengths of transmission lines that it is constant with frequency and second term results from reflection waves caused by discontinuity or mismatching impedance. This is called GDV and is equal to:
Equations (18) and (19) show that GDV of reflected waves caused by one discontinuity between two transmission lines or two impedance mismatches separated by a length of transmission lines, are sinusoidal with frequency. Also, more than one discontinuity at length of transmission line or more than two mismatching, causes multiple reflections and results irregular sinusoidal. Equation of GDV of this case is complicated. In formula (19), the magnitude of sinusoidal GDV is affected by $T_0$ and $h$. Also, the ripple frequency of the sinusoidal GDV is increased by $T_0$, because of:

$$
\varphi_m = -2\beta_1 l + \vartheta_{t1} + \vartheta_{t2} \quad \text{and} \quad \beta_1 = \frac{\omega}{V_p} l = \omega T_0 \quad (20)
$$

IV. PREDICTION METHODS OF THE SINUSOIDAL GDV

On the basis of formula (18) and by consideration that the magnitude and the ripple frequency of sinusoidal GDV is affected by $T_0$ and $h$, one can predict the sinusoidal GDV. Parameters such as: the length of the transmission line discontinuity ($l$), the loss and the dielectric constant of discontinuity and the magnitude of reflection coefficients can affect the magnitude and the ripple frequency of the sinusoidal GDV. Here this parameter is studied.

The increase the dielectric constant of discontinuity decreases the velocity of wave propagation and results in increase $T_0$ in formula (19). This causes to increase both magnitude and ripple frequency of the sinusoidal GDV versus frequency. This result is shown in Figure 7. This figure is on the basis of formula (18). The parameters of discontinuity shown in Figure 6, are equal: $l=1m$, $Z_0=75\Omega$, $Z_1=50\Omega$, loss = 0.01$dB/m$ and the relative permittivity of the discontinuity dielectric is varied ($\varepsilon_r = 1, 2.5, 10$).

In Fig. 8, transmission line with discontinuity has been simulated by Microwave-Office with aforementioned parameters and GD of this circuit versus frequency is shown in Fig 9. This figure confirms the theory and formula. Therefore, GDV can be compensated by decrease the dielectric constant. Also the length of transmission line discontinuity has similar effects on sinusoidal GDV.
V. COMPENSATION METHODS

With due attention to effects due to dispersive nature of a transmission line on the GD studied in section 2 of this paper, we can reduce or eliminate GDV over frequency by using low-loss or distortionless lines. A group of transmission lines e.g. coaxial cables with very low loss are introduced in table 1. GDV of these lines are calculated and can be neglected because of their quantities are very low.

By consideration of the section 3, since the GDV versus frequency caused by reflected waves are sinusoidal in nature, it would be very difficult to compensate for them once they occur using GD equalizers. At best, one equalizer could compensate for one cycle of GDV. Also, these equalizers can be quite complex and could introduce additional problems if they are not tuned properly. The best way to minimize GDV versus frequency is to prevent the occurrence of discontinuity and mismatching at transmission lines. If the discontinuity and impedance mismatch happen, then the decrease of the length of discontinuity, dielectric constant and magnitude of reflection coefficients and also increase the transmission line loss can be compensated the GDV. These works are impossible or difficult in practice because for example increase the transmission line loss causes that the transmission line may not be a low-loss line. This causes the dispersion in line. Here attenuators can be desirable compensators instead of increase the line loss, practically. Figure 12 shows the proper placement for attenuators used for GDV compensators.

This compensation method is more wideband than matching impedance networks and is very simpler than the previous methods have been pointed in e.g. [24, 25] as group delay equalizers, but because of the weakness of IF signal power level at the receiver, attenuation is not desirable. Therefore the trade off between attenuation and GD compensation is considerable.

VI. CONCLUSION

In transmission lines, especially coaxial cables, there are some factors which affect group delay. Here a GDV parameter introduced which includes two main components. First component is dispersive nature of transmission lines without any discontinuity in their lengths. The GDV caused by this factor can be vanished by using distortionless lines. Practically all of the transmission lines have some dispersion effects. Meanwhile, GDV can be neglected using low-loss lines. The calculation is carried out for 4 difference types of coaxial cables over a frequency range of 0.1~1GHz. Corresponding GDV are very low and can be neglected.

Second component is characterized by reflected waves due to discontinuity or impedance mismatch in transmission lines. Reflected waves GDV caused by one discontinuity between two transmission lines or two impedance mismatches separated by a length of transmission lines, are sinusoidal with frequency. This investigation is confirmed by formula that extracted in this paper. Also the simulation results confirm the proposed model. The magnitude and the ripple frequency of
the sinusoidal GDV depends on 4 parameters including: the constant dielectric of transmission line ($\varepsilon_r$), the length of discontinuity ($l$), the magnitude of the reflection coefficients ($|F|$) and the transmission line losses ($\alpha$).

Sinusoidal GDV in a transmission line can be reduced and compensated by decreasing the dielectric constant, length of transmission line and the magnitude of reflection coefficients and also by increasing the transmission line loss. Practically, some of improvements are impossible or difficult to be implemented, but sinusoidal GDV can be reduced by impedance matching networks or attenuators placed properly such that they reduce the reflected waves. In spite of the fact that the attenuation is undesirable at the weak IF signal, attenuators are used because of their wide bandwidths and simple structures.

REFERENCES