Application of Truncated Pyramid Model in Determination of Escape Velocity of Particles of Different Diameters in Varying Conditions

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Abstract— ***The behavior of the particles in a riverbank is actually a function of inter-particle distances, radii of the particles and the volume of water entrapped between them. It is also different for pure and saline water. In the present paper a deterministic method has been suggested for calculation of the cohesive force between the adjacent particles. It has been shown that the cohesive force is actually a function of all those parameters. The escape velocity is an important parameter in determination of other relevant parameters like volumetric rate of bank erosion and entrainment rate. The results indicate that the escape velocity of a particle on a riverbank increases no further after a certain value of the volume of the water bridge between the pairs of particles is attained. The results fall well in line with those obtained by previous researchers.***

Index Terms— ***Cohesive force, escape velocity, inter-particle distance, liquid (water) bridge.***

I. INTRODUCTION

A river is a dynamic system. Every river system, how much stable it is, has some eroding banks. Riverbank erosion is a natural process that involves a complex analysis. Obviously, stable river systems erode much slowly compared to unstable systems. Besides this, the amount of erosion in a stable river system is lower than that in an unstable system. The initiation of the riverbank erosion involves a number of factors. Among these, the force of cohesion between the particles on the riverbank plays an important role. This force, in turn, depends on the radii of particles, the inter-particle distance, the volume of the water bridge between the particles, the salinity of water and many other factors. A strong force means that a particle would require a greater velocity to escape from the riverbank. The escape velocity has direct relation with other parameters like the volumetric rate of bank erosion and the entrainment rate of particles. A good estimate of the escape velocity would be a great help in determining the rate of erosion of a riverbank.

II. LITERATURE SURVEY

There are many literary materials available on the sediment transport and riverbank erosion. Unfortunately, most of them being case studies are not found suitable to the subsequent researchers for application in their problems. Cornelis et al. (2004) parameterized the threshold shear velocity to initiate deflation of dry and wet particles. They made the balance of moments acting on particles at the instant of particle motion. Their model included a term for the aerodynamic forces, including the drag force, the lift force and the aerodynamic-moment force, and a term for the inter-particle forces. They incorporated the effect of gravitation in both terms [1]. Duan (2005), in her analytical model, showed that the escape velocity of the sediment particle can be derived from force analysis considering dynamic equilibrium, and the predominant forces acting on the particle are lift force, submerged weight of the particle and cohesive force between the particles. She expressed the cohesive force as a function of a number of parameters related to bank material. The cohesive force plays a quite significant role in bank erosion phenomenon. It has a telling effect on the escape velocity and the impending acceleration of the particles on a riverbank [2]. Julian and Torres (2006) made a study to assess the hydraulic erosion of cohesive riverbanks on a 600-m reach of an urban ephemeral stream with active bank erosion. They examined bank erosion by separating estimated bank shear stress into four properties: magnitude, duration, event peak, and variability. They combined the results of their study with results from previous bank erosion studies to produce a conceptual model for estimating bank erosion rates based on their silt-clay content [3].

Zhang and Li (2006) simulated the mechanical behavior of cohesive soil by Distinct Element Method (DEM). To simulate and analyze the behavior of cohesive soil accurately, they established the DEM mechanical model of cohesive soil with parallel bonds between particles by considering the capillary forces and the dynamic viscous forces induced by the presence of water between soil particles [4]. Soulie et al. (2006) expressed capillary force as an explicit function of local geometric configuration and relevant physical parameters. They suggested a relation between the geometric parameters of the particles of unequal radii and the force acting between those particles of radius R_1 and R_2 having inter-particle distance (between periphery) D. The equation proposed by them is

$$F_c = \pi\sigma\sqrt{R_1 R_2}\left[c + \exp\left(a\frac{D}{R} + b\right)\right] \qquad (1)$$

Where the coefficients a, b and c are functions of the volume $\forall$ of the liquid bridge, the surface tension σ, the contact angle ϕ and R, i.e., the greater value among R_1 and R_2.

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Manuscript Received September 16, 2011.
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$$a = -1.1\left(\frac{\forall}{R^3}\right)^{-0.53} \qquad (2a)$$

$$b = \left(-0.148\ln\left(\frac{\forall}{R^3}\right) - 0.96\right)\phi^2 - 0.0082\ln\left(\frac{\forall}{R^3}\right) + 0.48 \qquad (2b)$$

$$c = 0.0018\ln\left(\frac{\forall}{R^3}\right) + 0.078 \qquad (2c)$$

These equations can be used to express the capillary cohesion between two soil particles assuming their shape to be spherical [5]. Mu and Su (2007) analyzed the effects of liquid volume and separation distance on static liquid bridge by assuming perfect-wet condition. They also studied the relation between rupture energy and the volume of the liquid bridge [6]. Grof et al. (2008) made an analysis of the strength of capillary bridges in randomly packed granular media by means of computer simulations. They followed a simulation method based on the tracking of moving interfaces. They used the method for determination of the equilibrium shape of capillary bridges in a granular medium under a range of liquid saturations and solid-phase geometry. They also calculated the net force acting on each grain due to the capillary bridges, as well as the aggregate force acting between two wet granular media during their separation in the normal directions [7]. Kudrolli (2008) studied the mechanical properties of granular matter as affected by the addition of liquid. He showed that after a certain limit the strength of the cohesive force became independent of the volume of liquid between the particles. He explained that this occurs because the increase in liquid bridge size is balanced by the decrease in its curvature [8].

Recently, Mukherjee and Mazumdar (2010) proposed a new analytical model named the Truncated Pyramid Model that ably considered the micro-level variations taking place in a cohesive riverbank. They suggested equations for determination of the escape velocity of a particle considering dynamic equilibrium. The model considered more degrees of freedom, as compared to its previous research, experienced by a particle on a riverbank under the influence of the particles in its vicinity [9].

III. TRUNCATED PYRAMID MODEL AND GENERAL EQUATIONS FOR IMPENDING ACCELERATION AND ESCAPE VELOCITY

An analytical model called the Truncated Pyramid Model (Mukherjee and Mazumdar, 2010) has been used in the present paper to calculate the escape velocity of a particle on a riverbank. As depicted in the model, each soil particle, spherical in shape and materially homogeneous, rests on a pair of particles in a pyramidal structure. Suffices i and j indicate the spatial location of a particle in two-dimensional frame. Here i is the row number and j is the column number. For example, particle 13 is the third particle in the first row and it rests on particles 23 and 24. Radius of a particle at a location ij is denoted as R_{ij}. Radius of the next particle in the same row is $R_{i+1,j}$ and so on. Inter-particle distance between these two particles is designated as ${}_{ij}D_{i+1,j}$.

Impending acceleration of a particle in dynamic equilibrium, can be found out in x and y direction. The general equations of impending acceleration as given by Mukherjee and Mazumdar (2010) can be written for x and y direction as follows [10]:

$$\ddot{x}_{ij} = \left(3\sigma/4R_{ij}^3\rho_s\right)\left[F_{1x} + F_{2x} + F_3 - F_{4x} - F_{5x} - F_6\right] \qquad (3)$$

where

F_{1x} = Part of x-component of force between particles ij and $i+1,j+1$

$$= \sqrt{R_{ij}R_{i+1,j+1}}\left[c_{i+1,j+1} + \exp\left(a_{i+1,j+1}\left\{{}_{ij}D_{i+1,j+1}/R_{i+1,j+1}\right\} + b_{i+1,j+1}\right)\right]\left[1 - \left\{2R_{ij}R_{i+1,j}/\left(R_{ij}+R_{i+1,j+1}\right)\left(R_{i+1,j}+R_{i+1,j+1}\right)\right\}\right] \qquad (4a)$$

F_{2x} = Part of x-component of force between particles i-1,j and ij

$$= \sqrt{R_{i-1,j}R_{ij}}\left[c_{ij} + \exp\left(a_{ij}\left\{{}_{i-1,j}D_{ij}/R_{ij}\right\} + b_{ij}\right)\right]\left[1 - \left\{2R_{i-1,j}R_{i,j+1}/\left(R_{i-1,j}+R_{ij}\right)\left(R_{ij}+R_{i,j+1}\right)\right\}\right] \qquad (4b)$$

F_3 = Part of force between particles ij and $i,j+1$

$$= \sqrt{R_{ij}R_{i,j+1}}\left[c_{i,j+1} + \exp\left(a_{i,j+1}\left\{{}_{ij}D_{i,j+1}/R_{i,j+1}\right\} + b_{i,j+1}\right)\right] \qquad (4c)$$

F_{4x} = Part of x-component of force between particles ij and $i+1,j$

$$= \sqrt{R_{ij}R_{i+1,j}}\left[c_{i+1,j} + \exp\left(a_{i+1,j}\left\{{}_{ij}D_{i+1,j}/R_{i+1,j}\right\} + b_{i+1,j}\right)\right]\left[1 - \left\{2R_{ij}R_{i+1,j+1}/\left(R_{ij}+R_{i+1,j}\right)\left(R_{i+1,j}+R_{i+1,j+1}\right)\right\}\right] \qquad (4d)$$

F_{5x} = Part of x-component of force between particles i-1,j-1 and ij

$$= \sqrt{R_{i-1,j-1}R_{ij}}\left[c_{ij} + \exp\left(a_{ij}\left\{{}_{i-1,j-1}D_{ij}/R_{ij}\right\} + b_{ij}\right)\right]\left[1 - \left\{2R_{i-1,j-1}R_{i,j-1}/\left(R_{i-1,j-1}+R_{ij}\right)\left(R_{i,j-1}+R_{ij}\right)\right\}\right] \qquad (4e)$$

F_6 = Part of force between particles ij and i,j-1

$$= \sqrt{R_{i,j-1}R_{ij}}\left[c_{ij} + \exp\left(a_{ij}\left\{{}_{i,j-1}D_{ij}/R_{ij}\right\} + b_{ij}\right)\right] \qquad (4f)$$

$$\ddot{y}_{ij} = \left(1 - \left\{\rho/\rho_s\right\}\right)g + \left(3\sigma/4R_{ij}^3\rho_s\right)\left[F_{1y} + F_{2y} - F_{3y} - F_{4y}\right] \qquad (5)$$

where

F_{1y} = Part of y-component of force between particles ij and $i+1,j$

$$= \sqrt{R_{i+1,j}R_{ij}}\left[c_{i+1,j} + \exp\left(a_{i+1,j}\left\{{}_{ij}D_{i+1,j}/R_{i+1,j}\right\} + b_{i+1,j}\right)\right]\left[\left\{2\sqrt{R_{ij}R_{i+1,j}R_{i+1,j+1}\left(R_{ij}+R_{i+1,j}+R_{i+1,j+1}\right)}/\left(R_{ij}+R_{i+1,j}\right)\left(R_{i+1,j}+R_{i+1,j+1}\right)\right\}\right] \qquad (6a)$$

F_{2y} = Part of y-component of force between particles ij and $i+1,j+1$

$$= \sqrt{R_{i+1,j+1}R_{ij}}\left[c_{i+1,j+1} + \exp\left(a_{i+1,j+1}\left\{{}_{ij}D_{i+1,j+1}/R_{i+1,j+1}\right\} + b_{i+1,j+1}\right)\right]\left[\left\{2\sqrt{R_{ij}R_{i+1,j}R_{i+1,j+1}\left(R_{ij}+R_{i+1,j}+R_{i+1,j+1}\right)}/\left(R_{ij}+R_{i+1,j+1}\right)\left(R_{i+1,j}+R_{i+1,j+1}\right)\right\}\right] \qquad (6b)$$

F_{3y} = Part of y-component of force between particles i-1,j-1 and ij

$$= \sqrt{R_{i-1,j-1}R_{ij}}\left[c_{ij}+\exp\left(a_{ij}\left\{{}_{i-1,j-1}D_{ij}/R_{ij}\right\}+b_{ij}\right)\right]$$
$$\left[\left\{2\sqrt{R_{i-1,j-1}R_{i,j-1}R_{ij}\left(R_{i-1,j-1}+R_{i,j-1}+R_{ij}\right)}\Big/\left(R_{i-1,j-1}+R_{ij}\right)\left(R_{i,j-1}+R_{ij}\right)\right\}\right] \quad (6c)$$

F_{4y} = Part of y-component of force between particles i-1,j and ij

$$= \sqrt{R_{i-1,j}R_{ij}}\left[c_{ij}+\exp\left(a_{ij}\left\{{}_{i-1,j}D_{ij}/R_{ij}\right\}+b_{ij}\right)\right]$$
$$\left[\left\{2\sqrt{R_{i-1,j}R_{ij}R_{i,j+1}\left(R_{i-1,j}+R_{ij}+R_{i,j+1}\right)}\Big/\left(R_{i-1,j}+R_{ij}\right)\left(R_{ij}+R_{i,j+1}\right)\right\}\right] \quad (6d)$$

Here

ρ, ρ_s = densities of water and sediment particles, respectively, and g = acceleration due to gravity.

The resultant impending acceleration (in m/s^2),

$$f_{ij} = \sqrt{\ddot{x}_{ij}^{\,2} + \ddot{y}_{ij}^{\,2}} \quad (7)$$

The direction of the resultant acceleration is given by

$$\tan^{-1}(\ddot{y}_{ij}/\ddot{x}_{ij})$$

In compliance with the equation used by Duan (2005) the escape velocity (in m/s) of the particle ij would be

$$V_{sij} = \sqrt{R_{ij}\, f_{ij} \times CF} \quad (8)$$

where R_{ij} is expressed in mm, and CF is the conversion factor of value 0.002.

IV. CALCULATION OF THE ESCAPE VELOCITY OF A SEDIMENT PARTICLE

In the present work calculations have been made to study the variations of the escape velocity with gradual increase volume of the water bridge between particles for different inter-particle distances. For the present set of analyses the radii of all particles have been considered to be equal to the mean radius and only the particle sitting at the leftmost position of a row (i.e. j = 1) has been considered. Separate calculations have been made for pure and saline water. In the first case the mean radius of the particles has been considered to be 0.4 mm and in the second case it has been taken as 0.002 mm. This variation in the mean particle radius has been considered to observe the distinctive influence of the particle radii over the end results.

Following properties of pure water have been used in the present analyses:

- Density of water: 1000 kg/m^3;
- Surface tension: 0.073 N/m;
- Contact angle: 0°

Similarly, following properties of saline water have been used in the present analyses:

- Density of water: 1025 kg/m^3;
- Surface tension: 0.0681 N/m [Poorni et al. (2009)] [11];
- Contact angle: 25° [Bakker et al. (2003) suggested the range of the contact angle from 15° to 50° depending on the nature of the solid surface [12], and according to Morgan (1963) contact angle can be considered as 25° for impure water [13].]

These values may vary from case to case but, in this case, the present set of values has been used to study the nature of the variation. The Truncated Pyramid Model has the ability to incorporate similar kind of variations.

V. RESULTS AND DISCUSSIONS

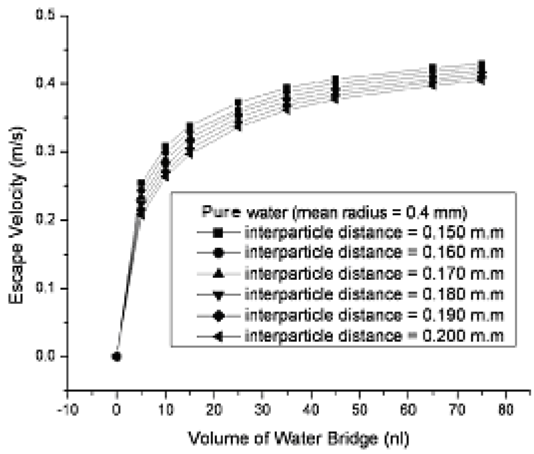

Figure 1: Escape Velocity Vs. Volume of the Water Bridge for Pure Water (Mean Particle Radius = 0.4 mm)

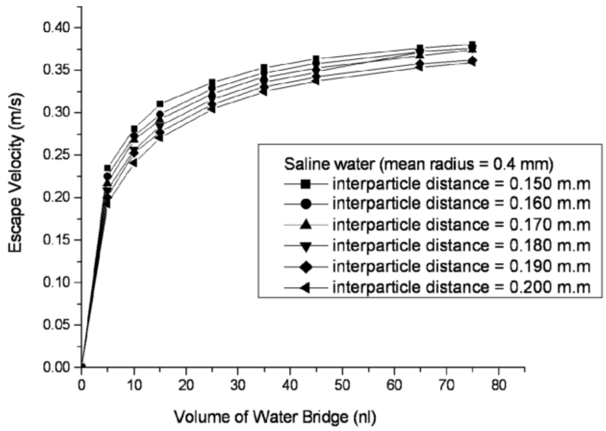

Figure 2: Escape Velocity Vs. Volume of the Water Bridge for saline Water (Mean Particle Radius = 0.4 mm)

Figure 1 shows the variation of the escape velocity of a particle with radius 0.4 mm with gradual increase in the volume of the water bridge for pure water. It is evident that after a sharp initial increase the rate of increase becomes quite low after a value of the volume of water bridge being about 65 nl. Figure 2 shows the same kind of variation of escape velocity for saline water. Here also the rate of increase becomes smaller after a certain volume of the water bridge. But the value of the escape velocity is less than that for pure water.

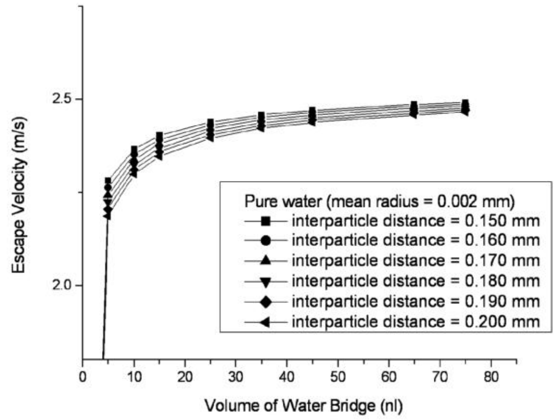

Figure 3: Escape Velocity Vs. Volume of the Water Bridge for Pure Water (Mean Particle Radius = 0.002 mm)

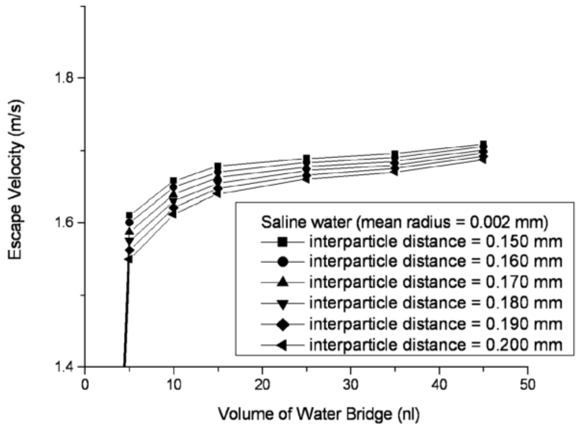

Figure 4: Escape Velocity Vs. Volume of the Water Bridge for saline Water (Mean Particle Radius = 0.002 mm)

Figure 3 and Figure 4 depict the variation of the escape velocity of a particle with radius 0.002 mm with gradual increase in the volume of the water bridge for pure water and saline water, respectively. Both Figure 3 and 4 indicate same kind of variation of the escape velocity but the value of the escape velocity is much higher in this case. This is because smaller particles would be bound with a greater force. Here, the graphs at the lower values of the volume of the water bridge being very close to one another the ordinates are drawn above the zero value.

All the four figures indicate that particles would require a greater velocity to escape for a lower value of the inter-particle distance. For a low value of the inter-particle distance the force between particles is high and, so, the escape velocity required is also high. The escape velocity in both cases is more for pure water than for saline water. The nature of the graphs showing the escape velocity becoming independent of the volume of the water bridge after attaining certain value falls perfectly in line with the findings of Soulie et al. (2006). The results also support the findings and the theory put up by Kudrolli (2008) that after a certain limit the strength of the cohesive force became independent of the volume of liquid (i.e. water) between the particles.

VI. CONCLUSION

A number of points have emerged from the present analyses.

- Firstly, the escape velocity of a particle increases rapidly with the increase in the volume of the water bridge when the volume is small. Ultimately, after reaching a certain value it ceases to increase with the volume of the water bridge.
- Secondly, particles require greater velocity to escape from the riverbank when the inter-particle distance is less.
- Thirdly, the escape velocity is less for saline water than for pure water.
- Fourthly, as the particle size decreases the escape velocity increases. The results obtained using the Truncated Pyramid Model are of the same nature as published in the previous research papers. It has the ability to take into account the variability inherent in a practical system. Therefore, the Truncated Pyramid Model can be used as an analytical tool for determination of the escape velocity, and, hence, other relevant parameters, e.g., entrainment rate and rate of bank erosion, for different real-life situations.

ACKNOWLEDGMENT

The author is indebted to Prof. (Dr.) Asis Mazumdar, Director of School of Water Resources Engineering of Jadavpur University, Kolkata, India, for his kind advice, and Mr. Debasish Ghosh, Mr. Kaustav Saraswati, Mr. Dwaipayan G. Karmakar and Mr. Jayanta Mistri, all from Kalyani Government Engineering College, for their cooperation during the preparation of the manuscript.

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