A Deterministic Inventory Model for Deteriorating Items with Price Dependent Demand and Time Varying Holding Cost under Trade Credit

Mukesh Kumar, Anand Chauhan, Rajat Kumar

Abstract— **In this proposed research, we developed a deterministic inventory model for price dependent demand with time varying holding cost and trade credit under deteriorating environment, supplier offers a credit limit to the customer during whom there is no interest charged, but upon the expiry of the prescribed time limit, the supplier will charge some interest. However, the customer has the reserve capital to make the payments at the beginning, but decides to take the benefit of the credit limit. This study has two main purposes, first the mathematical model of an inventory system are establish under the above conditions. Second this study demonstrate that the optimal solution not only exists but also feasible. Computational analysis illustrates the solution procedure and the impact of the related parameter on decision and profits.**

Keywords — **Deterioration, price dependent Demand, Trade credit, time varying holding cost.**

I. INTRODUCTION

In the EOQ model, we assumed that the supplier must be paid for the items as soon as the items are received. However, in practice, this may not true. In today's business transactions, it is more and more to see that a supplier will allow a certain fixed period for setting the amount owed to him for the items supplied. Usually there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged. Recently Haley and Higgins (1973), Kingsman (1983), Chapman et al. (1985), Bregman (1993) examined the effect of the trade credit on the optimal inventory policy. Furthermore, Goyal (1985) explored a single item economic order quantity model under conditions of permissible delay in payments. Chung (1998) studied the same model as Goyal (1985) and developed an alternative approach to find a theorem to determine the EOQ under conditions of permissible delay in payments and Aggarwal and Jaggi (1995) extended
Goyal's model to the case of deterioration, Jamal et al. (1997) generalized Aggarwal and Jaggi (1997) to the case of allowable shortage, Kumar, M et al. (2008) developed an EOQ model for time varying demand rate under trade credits, Kumar, M et al. (2009) presented an inventory model for power demand rate incremental holding cost under permissible delay in payments and Kumar et al. developed an inventory model for quadratic demand rate, inflation with permissible delay in payments. Chen and Kang (2010). Proposed an integrated inventory models considering permissible delay in payment and variant pricing strategy, M. Liang et.al. (2011) developed an optimal order quantity under advance sales and permissible delays in payments, C.K. Jaggi (2011) developed a pricing and replenishment policies for imperfect quality deteriorating items under inflation and permissible delay in payments. Deterioration is applicable to many inventories in practice like blood, fashion goods, agricultural products and medicine, highly volatile liquids such as gasoline; alcohol and turpentine undergo physical depletion over time through the process of evaporation. Electronic goods, radioactive substances, photographic film, grain, etc. deteriorate through a gradual loss of potential or utility with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the necessity to use this factory into consideration.

Shah and Jaiswal (1977) presented an inventory model for items deteriorating at a constant rate, Covert and philip (1973), Deb and Chaudhuri (1986), Kumar, M et al. (2009) developed an inventory model with time dependent deterioration rate. Some of the recent work in this field has been done by Chung and Ting (1993), Hariga (1996), Giri and Chadhuri (1997), Jalan and Chadhuri (1999).

In the classical inventory models, the demand rate is assumed to be a constant. In reality, demand for physical goals may be time dependent, stock dependent and price dependent. Selling price plays an important role in field of inventory system. Burwell (1997) developed an economic lot size model for price dependent demand under quantity and freight discounts, Mondal et al. (2003) presented an inventory system of ameliorating items for price dependent demand rate, You (2005) developed an inventory model with price and time dependent demand, Teng et al. (2005) developed an inventory model with price dependent demand rate.

In this paper, we develop an economic order quantity inventory model for deteriorating items,

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where deterioration rate and holding cost are linear and shortages are allowed and are fully backlogged. Demand rate is a function of selling price with permission delay in payments

II. ASSUMPTIONS AND NOTATIONS

A. The fundamental assumptions for the developing the model is as follows

- The deterioration rate is time varying. $\theta(t) = \theta t$ Inventory deterioration rate.
- Shortages are allowed and are fully backlogged.
- The demand rate is a function of selling price. $f(p) = (a - p) > 0$.
- The holding cost is linear with time dependent, $h(t) = (h + \alpha t)$, where $\alpha > 0,\ h > 0$ is the inventory holding cost per unit time.
- Replenishment is instantaneous.
- Lead time is zero.
- Delay in payment is allowed.
- During time T_1, inventory is depleted due to deterioration and demand of the item. At time T_1 the inventory becomes zero and shortages start occurring.

B. In addition the notations are as follows:

i). $\theta(t) = \theta t$ is Inventory deterioration rate.
ii). *a is* parameter used in demand function which hold the condition a > p.
iii). p is the selling price per unit item.
iv). C_1 is the inventory shortage cost per unit time.
v). C_2 is the unit cost of an item.
vi). A is the ordering cost of an order.
vii). T is the length of the cycle.
viii). q is the order quantity per cycle.
ix). T_1 is the length of the period with positive stock of the item.
x). I_e is the interest earned per Rs./unit time.
xi). I_p is the interest paid per Rs. / unit time, $I_p > I_e$
xii). M is the permissible delay in settling the account.les.

III. MATHEMATICAL FORMULATION AND SOLUTION

If The inventory model with above described assumption and notation is depicted in fig 1. The variation of inventory level Q (t) with respect to time t due to combine effect of demand and deterioration. At time T_1 inventory level goes to zero and shortage occurs. During the period (0, T) can be described by differential equation (1) and (2) with boundary condition

$Q(T_1)=0$

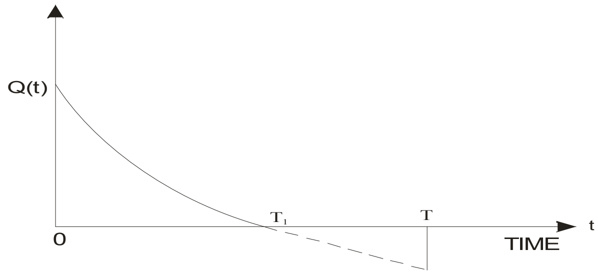

Fig.1

$$\frac{dQ(t)}{dt} + \theta t \,.\, Q(t) = -(a-p), \quad 0 \le t \le T_1 \qquad (1)$$

$$\frac{dQ(t)}{dt} = -(a-p), \quad T_1 \le t \le T \qquad (2)$$

The solutions of above differential equation are affected from the relation between T_1 and M, through the price dependent demand rate.

$$Q(t) = (a-p)\left[(T_1 - t) + Q\left(\frac{T_1^3}{6} - \frac{T^3}{3} - \frac{T_1 t^2}{2}\right) + \theta^2\left(\frac{T_1^5}{40} - \frac{t^5}{15} - \frac{t^2 t_1^3}{12} + \frac{t_1 t^4}{8}\right)\right], \quad 0 \le t \le T_1 \qquad (3)$$

And

$$Q(t) = -(a-p)(t - T_1) = (a-p)(T_1 - t), \quad T_1 \le t \le T \qquad (4)$$

Stock loss due to deterioration

$$D = (a-p)\int_0^{T_1} e^{\frac{\theta t^2}{2}}\,dt - (a-p)\int_0^{T_1} dt$$

$$= (a-p)\left[\frac{\theta T_1^3}{6} + \frac{\theta^2 T_1^5}{40}\right] \qquad (5)$$

Order quantity

$$q = C_2\left[D + \int_0^T (a-p)\,dt\right]$$

$$= C_2(a-p)\left[\frac{\theta T_1^3}{6}\ \frac{\theta^2 T_1^5}{40}\right] + (a-p)\,T \qquad (6)$$

Holding cost

HC=

$$\int_0^{T_1}(h + \alpha t)\,e^{-\theta\frac{t^2}{2}}\left\{\int_t^{T_1}\left(1 + \frac{\theta u^2}{2} + \frac{\theta^2 u^4}{8}\right)du\right\}dt$$

$$= (a-p)\,h\,.\left[\frac{T_1^2}{2} + \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90}\right]$$

$$+ \alpha(a-p)\left[\frac{T_1^3}{6} + \frac{\theta T_1^5}{40} + \frac{\theta^2 T_1^7}{336}\right] \qquad (7)$$

Shortage cost

$$SC = -C_1 \int_{T_1}^{T} [-(a-p)(t-T_1)]\, dt$$

$$= C_1 \frac{(a-p)}{2} (T-T_1)^2 \qquad (8)$$

The total profit of the system consist of the following elements

Net stock loss due to deterioration

Net Annual holding cost HC

Annual shortage cost SC

Interest Charged I_P

Interest Earned I_E

Annual ordering cost A.

Unit cost of an item order quantity per cycle q

Total profit per unit time is

$$P(T, T_1, p) = p(a-p) - \frac{1}{T}[A + SC + HC + q + IP_1 - IE_1]$$

Now, there are two possibilities regarding the period M of permissible delay in payments.

Case I: $M \le T_1$ (Payment at or before total depletion of inventory i.e. the inventory not being sold after the due date and evaluate the interest payable IP_1 and interest earned IE_1 per cycle)

Case II: $M > T_1$ (Payment at or after depletion i.e. the interest payable per cycle is zero because the supplier can be paid in full at time M , So only evaluate the interest earned per cycle Which is earned during the positive inventory period plus the interest earned from the cash invested during time period (T_1, M) after the inventory is exhausted at time T_1).

A. Case I: $M \le T_1$ (Payment at or before total depletion of inventory)

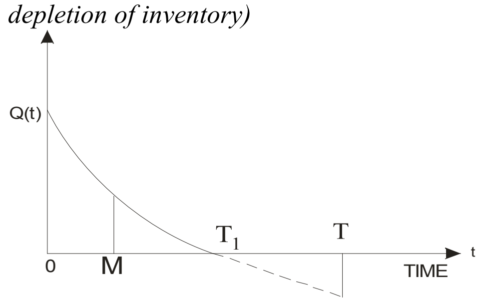

Fig.2 M<T$_1$

In this case, the credit time expires on or before the inventory depleted completely to zero. The interest payable per cycle for the inventory not being sold after the due date M is Interest payable in the time horizon When $M < t \le T_1$

$$IP_1 = C_2 I_p \int_{M}^{T_1} Q(t)\, dt$$

$$= C_2(a-p)I_p \left[\left(\frac{T_1^2}{2} - \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90}\right)\right.$$

$$\left. -\left(MT_1 - \frac{M^2}{2}\right) - \theta\left(\frac{MT_1^3}{6} - \frac{M^4}{12} - \frac{M^3 T_1}{6}\right) - \theta^2\left(\frac{MT_1^2}{40} - \frac{M^6}{90} - \frac{M^3 T_1^3}{36} + \frac{M^5 T_1}{40}\right)\right] \qquad (9)$$

In addition, the interest earned per cycle IE_1 is the interest earned during the positive inventory level, and is given by Interest earned in the time horizon When $T_1 < t \le 0$

$$IE_1 = C_2 I_e \int_0^{T_1} (a-p)t\, dt$$

$$= C_2 I_e (a-p) \frac{T_1^2}{2} \qquad (10)$$

Total profit per unit time is

$$P(T, T_1, p) = p(a-p) - \frac{1}{T}[A + SC + HC + q + IP_1 - IE_1]$$

$$= p(a-p) - \frac{1}{T}\left[A + \frac{C_1(a-p)}{2}(T-T_1)^2 \right.$$

$$+(a-p)h\left\{\frac{T_1^2}{2} + \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90}\right\} + \alpha(a-p)\left\{\frac{T_1^3}{6} + \frac{\theta T_1^5}{40} + \frac{\theta^2 T_1^7}{336}\right\}$$

$$+C_2(a-p)\left\{T + \frac{\theta T_1^3}{6} + \frac{\theta^2 T_1^5}{40}\right\}$$

$$+C_2(a-p)I_p\left\{\left(\frac{T_1^2}{2} - \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90}\right) - \left(MT_1 - \frac{M^2}{2}\right) - \theta\left(\frac{MT_1^3}{6} - \frac{M^4}{12} - \frac{M^3 T_1}{6}\right)\right.$$

$$\left.\left. -\theta^2\left(\frac{MT_1^5}{40} - \frac{M^6}{90} - \frac{M^3 T_1^3}{36} + \frac{M^5 T_1}{40}\right)\right\} - \frac{C_2 I_e (a-p) T_1^2}{2}\right]$$

Let $T_1 = \alpha T$; $0 < \alpha < 1$

Hence, we have the profit function

$$P(T, p) = p(a-p) - \frac{1}{T}\left[A + \frac{C_1(a-p)}{2}(1-\alpha)^2 T^2\right.$$

$$+h(a-p)h\left\{\frac{\alpha^2 T^2}{2} + \frac{\theta \alpha^4 T^4}{12} + \frac{\theta^2 \alpha^6 T^6}{90}\right\}$$

$$+\alpha(a-p)\left\{\frac{\alpha^3 T^3}{6} + \frac{\theta \alpha^5 T^5}{40} + \frac{\theta^2 \alpha^7 T^7}{336}\right\}$$

$$+C_2(a-p)\left\{T + \frac{\theta \alpha^3 T^3}{6} + \frac{\theta^2 \alpha^5 T^5}{40}\right\}$$

$$+C_2(a-p)I_p\left\{\left(\frac{\alpha^2T^2}{2}-\frac{\theta\alpha^4T^4}{12}+\frac{\theta^2\alpha^6T^6}{90}\right)\right.$$

$$-\left(M\alpha T-\frac{M^2}{2}\right)-\theta\left(\frac{M\alpha^3T^3}{6}-\frac{M^4}{12}-\frac{M^3\alpha T}{6}\right)$$

$$\left.-\theta^2\left(\frac{M\alpha^5T^5}{40}-\frac{M^6}{90}-\frac{M^3\alpha^3T^3}{36}+\frac{M^5\alpha T}{40}\right)\right\}$$

$$\left.-\frac{C_2I_e(a-p)\alpha^2T^2}{2}\right]$$

Now, our objective is to maximize the profit function $P(T,\,p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0 \quad \text{and} \quad \frac{\partial P(T,p)}{\partial p}=0$$

This implies

$$\left[-\frac{A}{T^2}+\frac{C_1(a-p)(1-\alpha)^2}{2}\right.$$

$$+h(a-p)\left\{\frac{\alpha^2}{2}+\frac{\theta\alpha^4T^2}{4}+\frac{\theta^2\alpha^6T^4}{18}\right\}$$

$$+\alpha(a-p)\left\{\frac{\alpha^3T}{3}+\frac{\theta\alpha^5T^3}{10}+\frac{\theta^2\alpha^7T^5}{56}\right\}$$

$$+C_2(a-p)\left\{\frac{\theta\alpha^3T}{3}+\frac{\theta^2\alpha^5T^3}{10}\right\}$$

$$+C_2(a-p)I_p\left\{\left(\frac{\alpha^2}{2}-\frac{\theta\alpha^4T^2}{4}+\frac{5\theta^2\alpha^6T^4}{4}\right)\right.$$

$$-\left(\frac{M^2}{2T^2}\right)-\theta\left(\frac{M\alpha^3T}{3}+\frac{M^4}{12T^2}\right)$$

$$-\theta^2\left(\frac{M\alpha^5T^3}{10}+\frac{M^6}{90T^2}-\frac{M^3\alpha^3T}{18}\right)$$

$$\left.-\frac{C_2I_e(a-p)\alpha^2}{2}\right]=0 \qquad (11)$$

and

$$(a-2p)-\frac{1}{T}\left[-\frac{C_1(1-\alpha)^2}{2}T^2\right.$$

$$-h\left\{\frac{\alpha^2T^2}{2}+\frac{\theta\alpha^4T^4}{12}+\frac{\theta^2\alpha^6T^6}{90}\right\}$$

$$-\alpha\left\{\frac{\alpha^3T^3}{6}+\frac{\theta\alpha^5T^5}{40}+\frac{\theta^2\alpha^7T^7}{336}\right\}$$

$$+C_2\left\{T+\frac{\theta\alpha^3T^3}{6}+\frac{\theta^2\alpha^5T^5}{40}\right\}$$

$$-C_2I_p\left\{\left(\frac{\alpha^2T^2}{2}-\frac{\theta\alpha^4T^4}{12}+\frac{5\theta^2\alpha^6T^6}{4}\right)\right.$$

$$-\left(M\alpha T-\frac{M^2}{2}\right)$$

$$\left.-\theta^2\left(\frac{M\alpha^5T^5}{40}-\frac{M^6}{90}-\frac{M^3\alpha^3T^3}{36}+\frac{M^5\alpha T}{40}\right)\right\}$$

$$\left.+\frac{C_2I_e\alpha^2T^2}{2}\right]=0 \qquad (12)$$

Case II : $M>T_1$ (Payment at or after depletion)

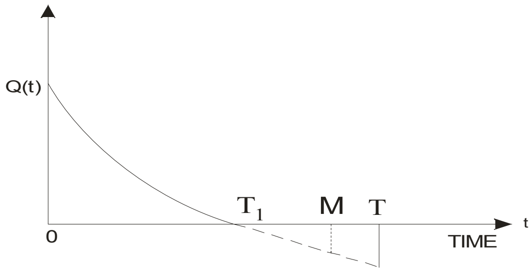

Fig.3 M>T$_1$

In this case, the interest payable per cycle is zero, i.e., $IP_2=0$, when $T_1<M\le T$ because the supplier can be paid in full at time M, the permissible delay. Thus, the interest earned per cycle is the interest earned during the positive inventory period plus the interest earned from the cash invested during time period (T_1,M) after the inventory is exhausted at time T_1, and it is given by

$$IE_2=C_2I_e\int_0^{T_1}f(p).tdt+C_2I_e(M-T_1)\int_0^{T_1}f(p)\,dt$$

$$=C_2I_ef(p)\frac{T_1^{\,2}}{2}+C_2I_e(M-T_1)f(p).T_1$$

$$IE_2=C_2I_e\;f(p)\,T_1\left(M-\frac{T_1}{2}\right) \qquad (13)$$

Total profit per unit time is

$$P(T,T_1,p)=p(a-p)-\frac{1}{T}\left[A+SC+HC+q+IP_2-IE_2\right]$$

$$=p(a-p)-\frac{1}{T}\left[A+\frac{C_1(a-p)}{2}(T-T_1)^2\right.$$

$$+(a-p)h\left\{\frac{T_1^{\,2}}{2}+\frac{\theta T_1^{\,4}}{12}+\frac{\theta^2T_1^{\,6}}{90}\right\}$$

$$+\alpha(a-p)\left\{\frac{T_1^{\,3}}{6}+\frac{\theta T_1^{\,5}}{40}+\frac{\theta^2T_1^{\,7}}{336}\right\}$$

$$\left.-C_2I_e(a-p)T_1\left(M_1-\frac{T_1}{2}\right)\right] \qquad (14)$$

Let $T_1=\alpha T$; $\quad 0<\alpha<1$.

Hence, we have the profit function

$$P(T,\,p) = p(a-p) - \frac{1}{T}\left[A + \frac{C_1(a-p)}{2}(1-\alpha)^2 T^2 \right.$$
$$+h(a-p)\left\{\frac{\alpha^2T^2}{2}+\frac{\theta\alpha^4T^4}{12}+\frac{\theta^2\alpha^6T^6}{90}\right\}$$
$$+\,\alpha(a-p)\left\{\frac{\alpha^3T^3}{6}+\frac{\theta\alpha^5T^5}{40}+\frac{\theta^2\alpha^7T^7}{336}\right\}$$
$$\left. -C_2I_e(a-p)\alpha T\left(M_1-\frac{\alpha T}{2}\right)\right] \qquad (15)$$

Our objective is to maximize the profit function $P(T,\,p)$. The necessary conditions for maximizing the profit are

$$\frac{\partial P(T,p)}{\partial T}=0 \;\; \& \;\; \frac{\partial P(T,p)}{\partial p}=0.$$

$$\Rightarrow \left[-\frac{A}{T^2}+\frac{C_1(a-p)(1-\alpha)^2}{2}\right.$$
$$+h(a-p)\left\{\frac{\alpha^2}{2}+\frac{\theta\alpha^4T^2}{4}+\frac{\theta^2\alpha^6T^4}{18}\right\}$$
$$+\alpha(a-p)\left\{\frac{\alpha^3T}{3}+\frac{\theta\alpha^5T^3}{10}+\frac{\theta^2\alpha^7T^5}{56}\right\}$$
$$\left.-C_2(a-p)\alpha(M-\alpha T)\right]=0 \qquad (16)$$

And

$$(a-2p)-\frac{1}{T}\left[-\frac{C_1(1-\alpha)^2}{2}T^2\right.$$
$$-h\left\{\frac{\alpha^2T^2}{2}+\frac{\theta\alpha^4T^4}{12}+\frac{\theta^2\alpha^6T^6}{90}\right\}$$
$$-\alpha\left\{\frac{\alpha^3T^3}{6}+\frac{\theta\alpha^5T^5}{40}+\frac{\theta^2\alpha^7T^7}{336}\right\}$$
$$\left.+C_2I_p\alpha T\left(M-\frac{\alpha T}{2}\right)\right]=0 \qquad (17)$$

The solutions of (11), (12), (16) and (17) will give T^* & p^*. The optimal value $P^*(T,p)$ of the average net profit is determined provided the sufficient conditions for maximizing $P(T,p)$ are

$$\frac{\partial^2 P(T,p)}{\partial T^2}<0,\quad \frac{\partial^2 P(T,p)}{\partial p^2}<0 \quad \text{And}$$

$$\frac{\partial^2 P(T,p)}{\partial T^2}\cdot\frac{\partial^2 P(T,p)}{\partial p^2} - \frac{\partial^2 P(T,p)}{\partial T\partial p}>0 \quad \text{At}$$

$T=T^*$ & $p=p^*$.

IV. Computational Analysis

The goals of the computational analysis in this study are as follows:

1. To consider both the Cases and illustrate the efficiency of the solution approach.
2. To discuss the impact of the related parameter on decision and profit.

4.1 Numerical example

To illustrate the above model described, we applied our procedure to a store in a major cosmetics retailer in mega cities. In which product include sunscreen, lotion, powder, lipstick, baby product; these products was initially promoted by TV / internet advertisements, but the sale of the product decreasing at small rate. In practices, the related parameter can be determined by regression analysis using historical transaction data.

Example 1 The parameters of the product are: A=200, a=100, M=0.055, C_2=20, h=0.4, C_1=1.2, β=0.95, α=0.1, θ=0.01, Ip=0.15, Ie=0.12.

Solution: Based on these input data, the computer outputs are as follows:

Profit= 1514.86, p* = 60.5336,T*= 4.67959.

Example 2 The parameters of the product are: A=200, a=100, M=0.35, C_2=20, h=0.4, C_1=1.2, β=0.95, α=0.1, θ=0.01, Ip=0.15, Ie=0.12

Solution: Based on these input data, the computer outputs are as follows:

Profit= 2420.81, p* =49.9196, T*= 2.29246.

V. Sensitivity Analysis

To study the effect of change of the parameter on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 20% and 50% and taking one parameter at a time, Keeping the remaining parameter at original value. The results are shown in table 1 and table 2 for permissible delay in payment (trade credit) by using software Mathematica5

A careful study of table 1 and table 2 reveals the following,

$P^*(T,\,p)$ is slightly sensitive to change in the value of parameter **θ, h, a, β** and it is moderately sensitive to change in $\mathbf{C_1}$ and highly sensitive to change in **a**

p is slightly sensitive to change in the value of parameter **θ, h, a, β** and it is moderately sensitive to change in $\mathbf{C_1}$ and highly sensitive to change in **a**

T are insensitive to change in the value of the parameter $\mathbf{C_1}$ and slightly sensitive to change in the value of parameter **a** and it is moderately sensitive to change in **θ, h, a, β.**

A. Table1

Parameter	% change	profit P	p	T
θ	-50	1514.87	60.5336	4.68108
	-20	1514.86	60.5336	4.68019
	20	1514.85	60.5336	4.67899
	50	1514.84	60.5336	4.6781

C_1	-50	1570.07	60.1818	12.9757
	-20	1527.81	60.4509	5.50702
	20	1498.48	60.6384	3.93315
	50	1483.32	60.7358	3.42741
C_2	-50	1911.95	55.6329	3.54042
	-20	1667.39	58.578	4.13295
	20	1370.96	62.4803	5.43873
	50	1172.22	65.3735	7.4679
h	-50	1515.04	60.5324	4.68966
	-20	1514.93	60.5331	4.68361
	20	1514.78	60.5341	4.67558
	50	1514.67	60.5348	4.66959
A	-50	1539.91	60.3746	3.30477
	-20	1523.88	60.4762	4.18341
	20	1506.7	60.5856	5.12876
	50	1495.66	60.6561	5.73813
a	-50	173.31	35.8975	7.82125
	-20	826.326	50.6195	5.42222
	20	2404.8	70.4756	4.17837
	50	4116.49	85.4157	3.65994
β	-50	1514.86	60.5336	4.67959
	-20	1514.86	60.5336	4.67959
	20	1514.86	60.5336	4.67959
	50	1514.86	60.5336	4.67959
α	-50	1477.22	60.7773	3.26819
	-20	1495.86	60.6562	3.84069
	20	1544.8	60.3415	7.14296
	50	1544.8	60.3415	7.14296

B. Table2

Para meter	% change	profit P	p	T
θ	-50	2420.81	49.9196	2.29246
	-20	2420.81	49.9196	2.29246
	20	2420.81	49.9196	2.29246
	50	2420.81	49.9196	2.29246
C_1	-50	2445.87	49.7936	2.67328
	-20	2428.41	49.8813	2.39601
	20	2409.98	49.9741	2.15974
	50	2398.94	50.0299	2.03931
C_2	-50	2389.69	50.3167	2.53991
	-20	2408.13	50.0795	2.38294
	20	2433.76	49.7585	2.21106
	50	2453.65	49.5148	2.10293
h	-50	2420.92	49.919	2.29395
	-20	2420.85	49.9194	2.29306
	20	2420.76	49.9198	2.29186
	50	2420.69	49.9202	2.29096
A	-50	2.29096	49.6629	1.6169
	-20	2439.23	49.8269	2.04855
	20	2404.16	50.0035	2.51336
	50	2381.63	50.1174	2.81321
a	-50	549.372	25.2885	3.26345
	-20	1520.18	40.0235	2.56584
	20	3523.21	59.8433	2.09168
	50	5554.25	74.7592	1.87031
β	-50	2420.81	49.9196	2.29246
	-20	2420.81	49.9196	2.29246
	20	2420.81	49.9196	2.29246
	50	2420.81	49.9196	2.29246
α	-50	2391.46	50.3077	2.56873
	-20	2410.82	50.0659	2.4214
	20	2428.85	49.7831	2.15312
	50	2437.96	49.5933	1.94396

VI. CONCLUSIONS

The main purpose of this study is to formulate a deterministic inventory model for deteriorating items under demand rate is price dependent and holding cost is time varying and when the supplier offer a trade credit period. The supplier offers credit period to the retailer who has the reserve money to make the payments, but decides to avail the benefits of credit limit. Shortages are allowed and are completely backlogged. Finally, numerical example and sensitivity analysis are provide to illustrate and inference the theoretical result.

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