Digital Linear and Nonlinear Controllers for Buck Converter

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Abstract—Both linear PID controllers and fuzzy controllers are designed and implemented for a buck converter. Comparison between the two controllers is made in the aspect of design, implementation and experimental results. Design of fuzzy controllers is based on heuristic knowledge of the converter and tuned using trial and error, while the design of linear PID and PI controllers is based on the frequency response of the buck converter. Implementation of linear controllers is quite straightforward, while implementation of fuzzy controllers has its unique issues. A comparison of experimental results indicates that the performance of the fuzzy controller is superior to that of the linear PID and PI controllers. The fuzzy controller is able to achieve faster transient response, has more stable steady-state response, and is more robust under different operating points.

Index Terms—DC-DC Converter, Buck Converter, PID controller, Fuzzy logic controller

I. INTRODUCTION

Proportional-integral-derivative (PID) control and fuzzy control are two different control approaches for DC-DC converters. Fuzzy control is a kind of nonlinear control, while PID control is a traditional linear control method used prevalently in industrial applications. Linear PID and PI controllers are usually designed for DC-DC converters using standard frequency response techniques based on the small signal model of the converter. A Bode plot is used in the design to obtain the desired loop gain, crossover frequency and phase margin. The stability of the system is guaranteed by an adequate phase margin. However, linear PID and PI controllers can only be designed for one nominal operating point. A boost converter’s small signal model changes when the operating point varies. Both the poles and a right-half plane zero, as well as the magnitude of the frequency response, are all dependent on the duty cycle. Therefore, it is difficult for the PID controller to respond well to changes in operating point. Design of fuzzy controllers is based on expert knowledge of the plant instead of a precise mathematical model. Fuzzy controllers can be designed to adapt to the nonlinear property of buck converters under varying operating points. Linear PID control and fuzzy control are compared in the aspect of design and implementation issues. Experimental results for a buck converter using the two different control methods are evaluated and compared.

In this paper, PID control and fuzzy control are compared in the aspects of design, implementation, and performance. Two sections of the paper are devoted to explaining the design methods. Section III describes the linear system design methods used with the buck converter. Fuzzy controller design is detailed in Section IV. The authors believe the material in these two sections will help readers understand the differences between the linear and fuzzy design approaches. These design descriptions are preceded by Section II, where the models that support linear system design are developed. Implementation and experimental results for a buck converter using the two different control methods are reported and compared in Sections V and VI.

II. SMALL-SIGNAL MODELS

Linear controllers for dc–dc converters are often designed based on mathematical models. To achieve a certain performance objective, an accurate model is essential. A number of ac equivalent circuit modeling techniques have appeared in the literature, including circuit averaging, averaged switch model-ing, the current injected approach, and the state-space averaging method [18]. Among these methods, the state-space averaged modeling is most widely used to model dc–dc converters. In this section, the basic models are reviewed, and experimental circuit parameters are presented.

TABLE I

Circuit parameters of the prototype buck converter

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter capacitance, C</td>
<td>1000</td>
<td>µF</td>
</tr>
<tr>
<td>Filter inductance, L</td>
<td>150</td>
<td>µH</td>
</tr>
<tr>
<td>Load resistance, R</td>
<td>10</td>
<td>Ω</td>
</tr>
<tr>
<td>ESR of capacitor, R_C</td>
<td>30</td>
<td>m Ω</td>
</tr>
<tr>
<td>ESR of inductor, R_L</td>
<td>10</td>
<td>m Ω</td>
</tr>
</tbody>
</table>

A. Buck converter’s small-signal model

A buck converter’s small-signal control-to-output transfer function, derived by the standard state-space averaging technique, is given by (1). In this transfer function, Vo is the output voltage, D is the duty cycle, C is the output capacitance, L is the inductance, and R is the load resistance.
Parameters $R_C$ and $R_L$ are the equivalent series resistance (ESR) of $C$ and $L$, respectively,
\[
\frac{v'_0(s)}{d'(s)} = \frac{V_0}{D} \frac{1 + sR_C C}{1 + a_1 s + a_2 s^2} \tag{1}
\]
Where
\[
a_1 = R_C C + \left( \frac{R_L}{R_C} \right) C + \frac{L}{R + R_L}
\]
\[
a_2 = \frac{R + R_C L C}{R + R_L}
\]
The buck converter transfer function is a second-order low-pass filter, with a left-half-plane zero introduced by the ESR of the filter capacitance [3]. The cutoff frequency of the low-pass filter is $\omega_c = 1/\sqrt{LC}$. The magnitude of the transfer function depends on the duty cycle $D$. Variations of $D$ do not change the shape of the frequency response, but only shift the magnitude plot upward or downward. The prototype buck converter’s nominal operating point is as follows: $V_{in} = 20V$, $V_o = 12V$, and $D = 0.6$. Values of the circuit parameters of the prototype buck converter are listed in Table I. The transfer function at the nominal operating point is given by
\[
\frac{v'_0(s)}{d'(s)} = \frac{6 \times 10^{-4} s + 20}{1.503 \times 10^{-7} + 5.4975 \times 10^{-5} + 1} \tag{2}
\]
The model has complex conjugate poles at $615.9 \pm j 2481.5$, which causes a $180^\circ$ phase delay at the approximate frequency of 2500 rad/s (400 Hz). The model also has a zero at 33 333 rad/s (5.3 kHz). Frequency response data for all of the experimental circuits was measured using an analog network analyzer (AP Instruments Model 102B). The measured frequency response of the buck converter is shown in Fig. 1. The measured frequency response compares favorably with the theoretical model, particularly below the 20-kHz switching frequency, so a linear controller can be designed based on (2). Above 20 kHz, switching frequency noise introduces greater uncertainty in the experimental data, particularly in the phase plot.

III. LINEAR PID CONTROL FOR A BUCK CONVERTER

A PID controller was designed for the buck converter to improve the loop gain, crossover frequency, and phase margin. One zero was placed an octave below the cutoff frequency (approximately 260 rad/s) and the other one at $4.6 \times 10^3$ rad/s. The transfer function of the PID controller is given by
\[
G_C(s) = 0.5786 + \frac{142.4}{s} + 1.19 \times 10^{-4} s \tag{3}
\]

The Bode plot for the compensated system is shown in Fig. 3. As shown in this plot, the gain at low frequency is high, the phase margin is $107^\circ$ at a gain crossover frequency approximately 3 kHz. A PI controller was also designed for the buck converter to reduce steady-state oscillation. One pole was placed at the origin, and one zero was placed at 800 rad/s. The dc gain of the controller was adjusted to obtain sufficient phase margin.

![Fig. 2. Comparison of the frequency response obtained using the analog analyzer and from the generated transfer function](image)

![Fig. 3. Bode plot of PID controller compensated buck converter](image)
Fig 4: Bodeplot of PI controller compensated buck converter

IV. DESIGN OF FUZZY CONTROL FOR DC–DC CONVERTERS

Fuzzy systems can be considered a type of nonlinear function interpolator [17]. The design of fuzzy controllers does not require an exact mathematical model. Instead, they are designed based on general knowledge of the plant. Fuzzy controllers can be designed to adapt to varying operating points. A fuzzy controller contains four main components: 1) the fuzzification interface that converts its input into information that the inference mechanism can use to activate and apply rules; 2) the rule base that contains the expert’s linguistic description of how to achieve good control; 3) the inference mechanism that evaluates which control rules are relevant in the current situation; and 4) the defuzzification interface that converts the conclusion from the inference mechanism into the control input to the plant[4]. There are two inputs for the fuzzy controller for the buck and boost converters.

![Fig 5. Block diagram of Fuzzy controlled Buck converter](image)

The first input is the error in the output voltage given by (12), where ADC[k] is the converted digital value of the k th sample of the output voltage and Ref is the digital value corresponding to the desired output voltage. The second input is the difference between successive errors an is given by

\[ e[k] = \text{Ref} - \text{ADC}[k] \]  
\[ ce[k] = e[k] - e[k-1] \]  

The two inputs are multiplied by the scaling factors g0 and g1 respectively, and then fed into the fuzzy controller. The output of the fuzzy controller is the change in duty cycle \( \Delta d \) [k], which is scaled by a linear gain h. The scaling factors g0, g1, and h can be tuned to obtain a satisfactory response.

A. Two Methods for Computing the Commanded Duty Cycle

There are two methods to calculate the new duty cycle from the fuzzy controller’s output \( \Delta d[k] \). A block diagram model of the first method is shown in Fig. 5(a). In this method, the fuzzy controller output \( \Delta d[k] \) is scaled by the output gain h, and then added to the previous sampling period’s duty cycle d[k – 1]

\[ d[k] = d[k-1] + h\Delta d[k] \]  

(7)

The first method (7) represents a discrete time integration of the fuzzy controller output. Integrating the fuzzy controller’s output increases the system type and reduces steady-state error. The second method of computing the new duty cycle is shown in Fig. 5(b). The fuzzy controller’s output is scaled by h, and then added to the output of a parallel integrator

\[ d[k] = K_i\Delta I[k] + h\Delta d[k] \]  

(8)

Here, \( I[k] \) is the output of the discrete time integration of the error \( e[k] \), and \( K_i \) is the gain of the integrator. The integrator is used to eliminate steady-state error. In the first structure of the fuzzy controller, an integrator is in series with the fuzzy logic controller, while in the second structure, the integrator is in parallel with the fuzzy logic controller. A disadvantage of the first structure is that the output gain h has to be tuned to very small values to avoid voltage oscillations in steady state. On the other hand, a very small output gain h tends to slow down the transient response time because more sampling periods are needed to arrive at the desired duty cycle. In the second structure, the change of duty cycle is not accumulated every sampling period, so the output gain h can be increased to reduce transient response time. The second structure is a combination of linear and nonlinear controllers. In the literature, the first method in Fig. 6(b) is more prevalent than the second method in Fig. 6(a). In this paper, only the second method is applied to the buck converter to obtain satisfactory response. The second structure is applied during startup transient to obtain fast transient response, and the first structure is applied during steady state in order to obtain stable steady-state response and to reduce steady-state error. For both methods, the duty cycled [k] was limited to be between 10% and 90% for the buck converter.

B. Fuzzification

The first step in the design of a fuzzy logic controller is to define membership functions for the inputs. Seven fuzzy
levels or sets are chosen and defined by the following library of fuzzy-set values for the error \( e \) and change in error \( ce \):

![Membership functions for \( e \) and \( ce \).](image)

They are as follows:

- NB: negative big;
- NM: negative medium;
- NS: negative small;
- ZE: zero equal;
- PS: positive small;
- PM: positive medium;
- PB: positive big.

The number of fuzzy levels is not fixed and depends on the input resolution needed in an application. The larger the number of fuzzy levels, the higher is the input resolution. The fuzzy controller utilizes triangular membership functions on the controller input. The triangular membership function is chosen due to its simplicity. For a given crisp input, fuzzifier finds the degree of membership in every linguistic variable. Since there are only two overlapping memberships in this specific case, all linguistic variables except two will have zero membership.

### C. Rule Base

Fuzzy control rules are obtained from the analysis of the system behavior. In their formulation it must be considered that using different control laws depending on the operating conditions can greatly improve the converter performances in terms of dynamic response and robustness. The control rules that associate the fuzzy output to the fuzzy inputs are derived from general knowledge of the system behavior. However, some of the control actions in the rule table are also developed using “trial and error” and from an “intuitive” feel of the process being controlled.

The control rules for the dc–dc converter in Table I resulted from an understanding of converter behavior. A typical rule can be written as follows. If \( e \) is NB and \( ce \) is PS then output is ZE. Where are the labels of linguistic variables of error \( e \), change of error \( (ce) \) and output respectively. \( e, ce \) and output represent degree of membership. To obtain the control decision, the max-min inference method is used. It is based on the minimum function to describe the AND operator present in each control rule and the max sum function to describe the OR operator. Control rules are given below.

The derivation of the fuzzy control rules is heuristic in nature and based on the following criteria [5]:

1) When the output of the converter is far from the set point, the change of duty cycle must be large so as to bring the output to the set point quickly.

2) When the output of the converter is approaching the set point, a small change of duty cycle is necessary.

### TABLE II

<table>
<thead>
<tr>
<th>Rule</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZE</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) When the set point is</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>ZE</td>
<td>PB</td>
</tr>
<tr>
<td>reached and the output is</td>
<td>NB</td>
<td>NB</td>
<td>NB</td>
<td>NM</td>
<td>NS</td>
<td>ZE</td>
<td>PB</td>
</tr>
<tr>
<td>still constant, the duty</td>
<td>NB</td>
<td>NM</td>
<td>ZE</td>
<td>PS</td>
<td>PM</td>
<td>PM</td>
<td>PB</td>
</tr>
<tr>
<td>cycle must be changed a bit</td>
<td>NB</td>
<td>NM</td>
<td>ZE</td>
<td>PS</td>
<td>PM</td>
<td>PM</td>
<td>PB</td>
</tr>
<tr>
<td>to prevent output from</td>
<td>NS</td>
<td>ZE</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
<tr>
<td>overshoot</td>
<td>ZE</td>
<td>PS</td>
<td>PM</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
<td>PB</td>
</tr>
</tbody>
</table>

3) When the output of the converter is near the set point and is approaching it rapidly, the duty cycle must be kept constant so as to prevent overshoot.

4) When the set point is reached and the output is still changing, the duty cycle must be changed a little bit to prevent the output from moving away.

5) When the set point is reached and the output is steady, the duty cycle remains unchanged.

6) When the output is above the set point, the sign of the change of duty cycle must be negative, and vice versa.

### D. Inference Mechanism

The results of the inference mechanism include the weight factor \( w_i \) and the change in duty cycle \( c_i \) of the individual rule. The weight factor \( w_i \) is obtained by Mamdani’s min fuzzy implication of \( \mu_e(e[k]) \) and \( \mu_{ce}(ce[k]) \), where \( w_i = \min\{\mu_e(e[k]), \mu_{ce}(ce[k])\} \) and \( \mu_e(e[k]), \mu_{ce}(ce[k]) \) are the membership degrees [21]. Control \( c_i \) is taken from the rule base. The change in duty cycle inferred by the \( i \)th rule \( z_i = w_i \times c_i \) is given by

\[
\min\{\mu_e(e[k]), \mu_{ce}(ce[k])\} \times c_i
\]

### E. Defuzzification

The center of average method is used to obtain the fuzzy controller’s output, which is given in (17), where \( N \) is the number of rules that are active [4]

\[
\Delta d[k] = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}
\]

### F. Comparison of the Design of PID/PI Controllers to the Design of Fuzzy Controllers

The analysis and design procedure for the linear PID and PI controller are quite different from that for the fuzzy controller in several aspects: design conditions, inputs to the design process, and analysis of design. 1) Design Conditions: Linear controller design begins with a design model of known structure, as described in Section II. For a chosen operating condition, a design model can be
derived. On the other hand, design of the fuzzy controller is not based upon a precise mathematical model; fuzzy rules are based upon general knowledge of the converters’ dynamic behavior under various operating conditions. 3) Analysis of Design: There are more control design and analysis tools available for PID controllers, and the response is highly predictable for linear plants. Bandwidth, loop gain, and phase margin are the main factors to consider when designing a linear controller based on frequency response techniques. However, because a human’s heuristic knowledge is used in the design of fuzzy controllers, there are fewer tools for the design and analysis of fuzzy controllers, and the analysis tends to be more complex. In the absence of expert understanding, extensive tuning is required for the fuzzy controller design process. Computer simulations can provide some guidance and reduce the amount of time needed for tuning.

V. IMPLEMENTATION

Fuzzy controllers for power converters have been implemented using microcontrollers, but the computational power of a digital signal processor (DSP) enables higher control sampling frequency. The PID controllers and fuzzy controllers have been implemented and evaluated using a Texas Instruments (TI) TMS320C6713, which is a 32-b fixed point DSP controller with on-board Flash memory. The CPU operates at 225 MHz. The TMS320C6713 supports peripherals used for embedded control applications, such as event manager modules and a dual 12-bit, 16 channel ADC. The conversion period of the A/D is 80 ns. The sampling and switching frequency of the PID controllers and the fuzzy controllers was 150 kHz. Since the clock frequency of the DSP is 150 MHz, in order to obtain 10-b resolution of the PWM signal, the switching frequency is chosen to be 150 kHz, and the sampling frequency is chosen to be the same as the switching frequency.

A. Implementation of PID and PI Controllers

The continuous-time transfer function of the PID and PI controllers designed previously were transformed into the discrete-time domain using the backward Euler integration method [26]. The difference equation used to calculate a new duty cycle for the digital PID controller is given by

\[ u[k] = K_i \sum_{n=0}^{k} e[n] + \frac{K_p}{T} (e[k] - e[k-1]) \]  

(11)

In the difference equation, \( u[k] \) is the controller output for the k th sample, and \( e[k] \) is the error of the k th sample. The error \( e[k] \) is calculated as \( e[k] = \text{Ref} - \text{ADC}[k] \), where \( \text{ADC}[k] \) is the converted digital value of the k th sample of the output voltage, and Ref is the digital value corresponding to the desired output voltage. The second term on the right side of (11) is the sum of the errors, and third term is the difference between the error of the k th sample and the error of the \((k - 1)\) sample. For the PI controller, the derivative gain \( K_p \) is set to zero. The difference equation (11) is a linear combination of feed-back and control signals. A series of scalar multiplication and addition instructions can be used to implement the controller. The TI TMS320C6713 DSP is optimized for implementation of digital filters it has special internal structures to multiply a number by a constant and add the previous product in a single instruction. Therefore, DSP-based implementation of the linear controller in real time is straightforward.

B. Implementation of Fuzzy Controllers

A fuzzy controller is a nonlinear algorithm, which requires frequent use of multiplication and division instructions with high accuracy. There are unique challenges to implement a fuzzy controller on a DSP. When implementing a fuzzy controller in real time, two main issues are the amount of time it takes to compute the output of fuzzy controllers, and the amount of memory used. Between sampling instants, centers in the membership function and their corresponding membership degrees need to be calculated. When there are many inputs to the fuzzy controller, or each input has many membership functions, the efficiency of the implementation of fuzzy controllers becomes even more important, because the number of rules increases exponentially with the increase of the number of inputs.

C. Comparison of Implementation of PID and PI Controllers

With Implementation of Fuzzy Controllers Generally, the implementation of a linear controller is less demanding than the implementation of a fuzzy controller. Most DSPs are optimized for implementation of digital filters. On the contrary, more computation power and memory are required to implement a fuzzy controller than a linear controller. To reduce the computation time and amount of memory used, several techniques have been addressed in this effort. A DSP with fast computation speed and high computation power is more appropriate for the implementation of fuzzy controllers in real time. The computation time for the fuzzy controller was 3.6 μs, while the computation time for the PID controller was 1 μs on the TMS320C6713 DSP.

![Fig 8. Block diagram of TMS320C6713](image)

VI. EXPERIMENTAL RESULTS

Experimental results of the buck converter using conventional, PID controller and the fuzzy controller are presented and compared in this section.
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VI. CONCLUSION

PID and PI controllers and fuzzy controllers were designed and implemented for buck converter. The linear controllers were designed for the converters using frequency response techniques. The PID controller was used for startup transient, while the PI controller was applied during steady state to achieve stable steady-state response. The fuzzy controllers were designed based on an in-depth knowledge of the plant, computer simulations and experimental results. Design and implementation issues and experimental results for the PID and PI controllers and fuzzy controller were compared. The design of linear controllers and fuzzy controllers required quite different procedures. Design of the fuzzy controller did not require a mathematical model, while a small signal model was necessary for the design of PID controllers using frequency response methods. Linear controller design is backed by a long history and wealth of design and analysis tools. More tuning effort was required for fuzzy controllers, and fuzzy control is not a recommended design approach for those who do not have firm grasp or understanding of converter dynamics. Implementation of fuzzy controllers also demanded more computing capability and memory than implementation of linear controllers. Experimental results showed that fast transient response and stable steady-state responses could be achieved for buck converter using fuzzy controllers. For the buck converter, comparable results were obtained using PID and PI controllers and fuzzy controllers. In most cases, the fuzzy controller also yielded superior settling time, particularly under load increases.

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