Image Denoising by Multiscale - LMMSE in Wavelet Domain and Joint Bilateral Filter in Spatial Domain

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Abstract*— This paper deals with LMMSE-based denoising scheme with a wavelet interscale model and Joint bilateral Filter in spatial domain. The proposed algorithm consists of two stages .In the first stage, a vector is represented by the wavelet coefficients at the same spatial locations at two adjacent scales and the LMMSE is applied to the vector. Compare to Orthogonal Wavelet Transform (OWT), Overcomplete Wavelet Expansion (OWE) provides better results hence it is employed. While applying the LMMSE rule, the important features in an image like edges, curves and textures can be identified. Also spatial domain method output provides a high quality denoising image than wavelet method with fewer artifacts; hence this wavelet domain output as a reference image for the Joint Bilateral Filter (JBF) .By using this reference image and the non-linear combination of information of adjacent pixel, the edge details of the images can be preserved in a well manner. The experimental results prove that the proposed approach is competitive when compared to other denoising methods in reducing various types of noise. Also the proposed algorithm outperforms other methods both visually and in case of objective quality peak-signal-to-noise ratio (PSNR).

***Index Terms*— Image Denoising; Joint Bilateral Filter; Multiscale LMMSE; Interscale Wavelet Model.**

I. INTRODUCTION

In image processing system, the acquisition techniques and systems introduce various types of noises and artifacts in the digital image that leads to poor quality image. For example Magnetic Resonance Imaging (MRI) is the most common tool for diagnosis in medical field and it is often affected by various types of noises during image acquisition process. Besides the noisy image produces undesirable visual quality, it also lowers the visibility of low contrast objects. Hence noise removal is essential in medical imaging applications in order to enhance and recover fine details which are hidden in the data. In many occasions, noise in digital images is found to be additive in nature with uniform power in the whole bandwidth and with Gaussian probability distribution. Such a noise is referred to as Additive White Gaussian Noise (AWGN). It is difficult to suppress AWGN since it corrupts almost all pixels in an image.

During the early days, AWGN is suppressed using linear spatial domain filters such as mean and wiener filters [1-8].Linear techniques possess mathematical simplicity but have the disadvantage of yielding blurring effect. They also do not perform well in the presence of signal dependent noise. The Image denoising methods can be done either by transform domain methods or by spatial domain methods. Transform domain methods first transform an image from the spatial domain into a different domain and suppress noise in the transform domain. Hence to perform a meaningful and useful task, a suitable transform is used. e.g. Discrete Fourier Transform (DFT) [1], Discrete Cosine Transform (DCT) [7], Discrete Hartley Transform (DHT) [8], Discrete Wavelet Transform (DWT) [9-22] etc.

As already mentioned, wavelet transform has always considered as a powerful denoising tool. These wavelet based methods mainly depend on thresholding the discrete wavelet transform (DWT) coefficients, which have been affected by AWGN. Two standard thresholding rules are hard-thresholding and soft-thresholding. In both cases, the coefficients that are below a certain threshold are set to zero. In hard thresholding, the remaining coefficients are left unchanged. In soft thresholding, the magnitudes of the coefficients above threshold are reduced by an amount equal to the value of the threshold. In both cases each wavelet coefficient is multiplied by a given shrinkage factor, which is a function of the magnitude of the coefficient. In wavelet thresholding, an image has been decomposed into its approximation (low-frequency) and detail (high-frequency) subbands; since most of the image information is concentrated in approximation sub band, the remaining detail subbands are processed with hard or soft thresholding operations. The difficult task in wavelet thresholding is the selection of threshold. Various threshold selection methods have been proposed, such as, VisuShrink [10], SureShrink [11] and Baye's Shrink [12]. In the VisuShrink method, a universal threshold which is a function of the noise variance and the number of samples is developed based on the minimal error measure. The threshold value in the Sure Shrink approach is optimal one in terms of the Stein's unbiased risk estimator. The Baye's Shrink approach determines the threshold value in a Bayesian rule, through modeling the distribution of the wavelet coefficients as Gaussian. These shrinkage methods have further improved with the help of interscale and intrascale correlations of the wavelet coefficients [14]–[18]. Originally, Donoho and Johnstone proposed the use of a universal threshold method uniformly throughout the whole wavelet decomposition [19], [20]. Then the implementation of the wavelet tree was found to be more effective [21] – [23]. An adaptive wavelet threshold method is applied with the help of Baye's Shrink (BS) for wavelet

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thresholding method [22], [23].
Edges can be located very effectively in the wavelet transform Domain [24] and a spatially selective noise filtration technique based on the direct spatial correlation of the wavelet transform at several adjacent scales is implemented. The dyadic wavelet transform (DWT) which equals the Canny edge detector[25] helps to Characterize the important features like edges, curves and textures.

A wavelet based multiscale products thresholding method [26] which fusing both dyadic wavelet transform and adaptive multiscale product thresholding is proposed. In the multiscale products, edges can be effectively distinguished from noise.

Another spatial domain method known as Bilateral Filter(BF) is proposed in [27].It helps in edge preservation and smoothing. Bilateral filtering smooths images while preserving edges with the nonlinear combination of nearby pixel values. In Multiresolution Bilateral Filtering [27] for Image Denoising, the bilateral filter is combined with wavelet thresholding to provide a image denoising framework, which helps in removing noise in real noisy images [9].In the hybrid model [28] it clearly shows the use of bilateral filters in combination with wavelet thresholding filters on subbands of a decomposed image deteriorates the performance. Additionally Wavelet Transform decorrelates signals in a well manner, strong intrascale and interscale dependencies between wavelet coefficients exist. The performance of denoising would be significantly improved if such dependencies could be efficiently modeled and exploited. In [30] they classified the wavelet statistical models into intrascale, interscale and hybrid ones.

But herein this proposed work, the Joint Bilateral Filter [31] is combined with LMMSE-based denoising scheme with a wavelet interscale model [32] to get an improved peak-signal-to-noise ratio (PSNR).The main contribution in this proposed work is that during the first stage, the noisy image is implemented with multiscale LMMSE in wavelet domain and takes this image as a reference image for the next stage. With the help of this reference image, Joint Bilateral Filter (JBF) is applied in the second stage so that the edge details of the images can be preserved in an effective manner.

The experimental results proves that the proposed method could reduce the noise in a well manner while preserving edge details in an image as compared with other spatial and transform domain methods.

The paper is organized as follows. Section II introduces Multiscale LMMSE Interscale Model. Joint Bilateral filter is discussed in Section III. In Section IV, Proposed Method is discussed. The Section V presents the experimental results and discussion while concluding remarks are given in Section VI.

II. MULTISCALE LMMSE INTERSCALE MODEL

A. *Overcomplete Wavelet Expansion (OWE)*

Orthogonal Wavelet Transform (OWT) sometimes cause visual artifacts in threshold-based denoising [11] – [22] and it has been observed that the OWE achieves better results in noise suppression and artifacts reduction. Instead of down sampling of wavelet coefficients in OWT the restored image by OWE is an average of several circularly shifted denoised versions of the same signal by OWT by which the additive white Gaussian noise is well suppressed.[30] – [31].

B. *Multiscale LMMSE Interscale Model*

One of the most popular approaches for denoising is Wavelet Thresholding. Several Wavelet thresholding techniques have proved to be effective in denoising [14]–[23]. Non-significant wavelet coefficients below a preset threshold value are discarded as noise and the image is reconstructed from the remaining significant coefficients. Compared to linear filter domain denoising methods that blur images as well as smoothing noise, the nonlinear wavelet thresholding method helps to preserve edge details. A wavelet based multiscale products thresholding method provides better results than the normal shrinkage methods [14] – [23]. While multiplying the adjacent wavelet scales it helps to sharpen the edge structures and identified significant pixels from the multiplication in a iterative manner.
The wavelet transform of noisy image is defined as,

$$f = g + \varepsilon \qquad (1)$$

The above equation can be written as,

$$D_i^d f = D_i^d g + D_i^d \varepsilon \qquad (2)$$

The original image is corrupted with additive Gaussian white noise and it is written as,

$$g = f + \varepsilon \qquad (3)$$

After apply OWE the transformed image is written as

$$w_i = x_i + v_i \qquad (4)$$

Where w_i represents the coefficients at scale i.
Instead of using soft/hard thresholding, LMMSE method is applied. Hence the LMMSE of x_i is given by

$$\hat{x}_i = c' . w_i \qquad (5)$$

$$c = \frac{\sigma_{x_i}^2}{\sigma_{x_i}^2 + \sigma_i^2}$$

The variance of x_i *and* v_i are given by$\sigma_{x_i}^2, \sigma_i^2$respectively.
The term w_{i+1}^D is expressed in the form of

$$w_{i+1}^D = s_0 * L_i^D \qquad (6)$$

Where $*$ represents the convolution operator and filter L_i^D (Diagonal Component) is given by,

$$L_i^D - H_0 * H_o' * \ldots * H_{i-1} * H_{i-1}' * G_i * G_i' \qquad (7)$$

Similarly for horizontal and vertical direction it is given by,

$$w_{i+1}^H = s_o * L_i^H, \qquad w_{i+1}^v = s_o * L_i^v \qquad (8)$$

$$L_i^H = H_0 * H_o' * \ldots * H_{i-1} * H_{i-1}' * G_i * H_i'$$

$$L_i^v = H_0 * H_o' * \ldots * H_{i-1} * H_{i-1}' * H_i * G_i' \qquad (9)$$

In all directions (diagonal, horizontal and vertical) the noise standard deviation of v_i is calculated by,

$$\sigma_i = \|L_{i-1}\| \sigma \qquad (10)$$

Where L_{i-1}represents the corresponding filter.
The standard deviation $\sigma_{x_i}^2$ of noiseless imagex_i is estimated by,

$$\hat{\sigma}_{x_i}^2 = \sigma_{w_i}^2 - \sigma_i^2 \quad (11)$$

$$\sigma_{w_i}^2 = \frac{1}{A.B}\sum_{a=1}^{A}\sum_{b=1}^{B} w_i^2(a,b) \quad (12)$$

Where A and B are the numbers of input image rows and columns. The LMMSE-based wavelet denoising schemes proposed in [9] - [10] achieved better results. These two methods exploited the wavelet intrascale dependencies. Hence to achieve an interscale wavelet model which is based on LMMSE, the wavelet adjacent scales are strongly correlated and these interscale dependencies can be exploited for better denoising results. The wavelet based represented images are similar across scales and especially among the adjacent scales. In wavelet domain, the noise level decrease rapidly along scales, while signal structures are strengthened with scale increasing. Therefore coarser scale information is being used to improve finer scale estimation.

When the input image is decomposed into 'i' scales. Scale i is strongly correlated with scale (i+1), but its correlations with scales i+2, i+3…….. decreases rapidly. Assemble the points with the same orientation at scales i and i+1 as a vector.

$$\vec{w}_i(a,b) = [w_i(a,b)w_{i+1}(a,b)]^T \quad (13)$$

$$\vec{w}_i = \vec{x}_i + \vec{v}_i \quad (14)$$

Where as

$$\hat{x}_i(a,b) = [x_i(a,b)x_{i+1}(a,b)]^T$$
$$\hat{v}_i(a,b) = [v_i(a,b)v_{i+1}(a,b)]^T$$

The LMMSE of $\hat{\vec{x}}_i$ is given by,

$$\hat{\vec{x}}_i = P_i(P_i + R_i)^{-1}\vec{w}_i \quad (15)$$

Here the covariance matrices of x_i and v_i are represented by P_i and R_i respectively.

$$P_i = E[\vec{x}_i\vec{x}_i^T] = E\begin{bmatrix} x_i^2 & x_i x_{i+1} \\ x_i x_{i+1} & x_{i+1}^2 \end{bmatrix}$$

$$R_j = E[\vec{v}_i\vec{v}_i^T] = E\begin{bmatrix} v_i^2 & v_i v_{i+1} \\ v_i v_{i+1} & v_{i+1}^2 \end{bmatrix}$$

The correlation coefficient value is given by,

$$\rho_{i,i+1} = \frac{\sqrt{\sum_l \sum_k L_i(r,s) L_i(r,s)}}{\|L_{i-1}\|.\|L_i\|} \quad (16)$$

v_i and v_{i+1} represents the joint gaussian distribution function. Therefore the density is expressed as

$$p(v_i, v_{i+1}) = \frac{1}{2\pi\sigma_i\sigma_{i+1}\sqrt{1-\rho_{i,i+1}^2}} * e^{-\frac{1}{2(1-\rho_{i,i+1}^2)}\left[\frac{v_i^2}{\sigma_i^2} - \frac{2\rho_{i,i+1}v_i v_{i+1}}{\sigma_i\sigma_{i+1}} + \frac{v_{i+1}^2}{\sigma_{i+1}^2}\right]} \quad (17)$$

The expected value is given by,

$$E[v_i v_{i+1}] = \rho_{i,i+1}\sigma_i\sigma_{i+1} \quad (18)$$

$$E[x_r x_s] \approx E[w_r w_s] - E[v_r v_s]$$

Where 'r' and's' represents i and i+1 respectively.

$$E[w_r w_s] = \frac{1}{A.B}\sum_{a=1}^{A}\sum_{b=1}^{B} w_r(a,b).w_s(a,b) \quad (19)$$

III. SPATIAL DOMAIN APPROACH

Joint Bilateral Filter (JBF)

The major defect in the bilateral filter [26] in image denoising is that the edge could not be preserved in an effective manner and the denoised image provides a blurred appearance. The important image features are preserved by wavelet-based LMMSE denoising, so use this as a reference image, like the flash image in [23]. In this type of filter, the basic bilateral filter is modified to compute the edge stopping function using reference image. By this method, the edge-stopping function could be estimated more accurately. In the Joint Bilateral filter [24], the parameters, σ_c and σ_e vary with respect to the reference image quality and the noise level. At a particular pixel location *n*, the bilateral filter output is calculated as follows,

Mathematically, at a particular pixel location *v*, the joint bilateral filter output is calculated as follows,

$$I'(y) = \frac{\sum_{x\in N(y)} e^{-\frac{\|y-x\|^2}{2\sigma_e^2}} H \quad I(x)}{\sum_{x\in N(y)} e^{-\frac{\|y-x\|^2}{2\sigma_e^2}} e^{-\frac{|I(y)-I(x)|}{2\sigma_c^2}}} \quad (20)$$

And $\quad H = e^{-\frac{|J(y)-J(x)|}{2\sigma_c^2}}$

Here J represents the wavelet denoising reference image and σ_c determines the threshold value which varies in intensity from the central pixel according to the reference image. With the help of reference image, the denoising is performed. In the joint bilateral filter, the edge-stopping function is calculated as,

$$\sigma_c = \sqrt{\sigma_n} + 3 \quad (21)$$

IV. PROPOSED METHOD

In our proposed method, the noisy image is passed through Bilateral Filter (BF) and some amount of noise is reduced but the image becomes blurred, hence adaptive wavelet thresholding is applied with multiscale product rule in the second stage. Rather than using OWT, OWE is employed. After apply the OWE, wavelet interscale dependencies is exploited by multiply the adjacent wavelet subbands to preserve edge structures while suppressing noise. This scheme multiplies the adjacent wavelet subbands to strengthen the significant features in the image and then applies the thresholding to the multiscale products rather than to its coefficients and it helps for better preservation of edge details in the image. With the help of this reference image (wavelet domain output), Joint Bilateral Filter (JBF) is applied so that the edge details of the images can be preserved in an effective manner. The performance of the proposed image denoising algorithm and the reconstructed image quality are measured using Mean Squared Error (MSE) and Peak Signal to Noise Ratio (PSNR), given in equations (22) and (23) respectively.

$$MSE = \frac{1}{AB}\sum_{a=1}^{A}\sum_{b=1}^{B}(x(a,b) - \hat{x}(a,b))^2 \quad (22)$$

$$PSNR = 10 log \frac{255^2}{MSE} \qquad (23)$$

Where $x(a,b)$ denotes the sample of original image and $\hat{x}(a,b)$ denotes the sample of distorted image. A and B are number of pixels in row and column directions respectively.

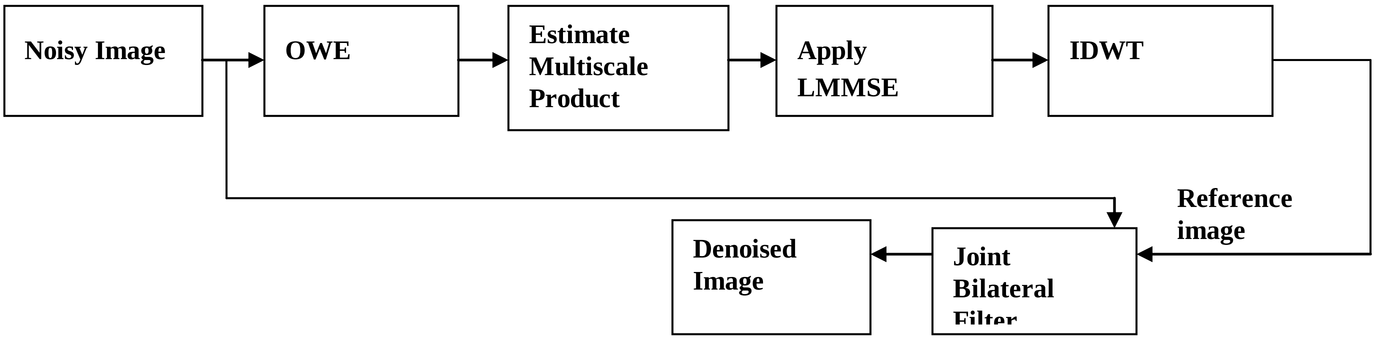

Figure 1. Proposed Method

V EXPERIMENTAL RESULTS AND DISCUSSIONS

Experiments using Bilateral Filtering (BF),Wavelet Thresholding (WT), Multi Resolution Bilateral Filter (MRBF),hybrid model (B+W+B) and proposed method are conducted on a set of standard benchmark (monochrome) images such as Lena, Barbara, House, Boats, Goldhill and Peppers. Those images were added with different noises like Gaussian, Speckle, Salt and Pepper, Riccian and Random noise with standard deviations σ=5,10, 15,20..50. However the results obtained with Gaussian noise added images are shown in Table I. The corresponding pictorial results are shown in Figure 2.The proposed algorithm helps to preserve the edges in the best way while suppressing the noise. The application of Joint Bilateral Filter and multiscale product LMMSE wavelet technique enhances the performance. Hence this hybrid method is recommended as a well competent and efficient model for denoising any type of images.

TABLE I Denoising results on Gaussian Noise added images

TYPE OF FILTER	PSNR(dB)					PSNR(dB)					PSNR(dB)				
	σ=10	σ=20	σ=30	σ=40	σ=50	σ=10	σ=20	σ=30	σ=40	σ=50	σ=10	σ=20	σ=30	σ=40	σ=50
	Lena(512 X512)					**Barbara(512X512)**					**House(512X512)**				
NOISY PSNR	28.1	22.1	18.6	16.1	14.1	28.1	22.1	18.6	16.1	14.1	28.1	22.1	18.6	16.1	14.1
BF	33.6	29.72	27.35	25.48	23.95	31.44	27.12	24.92	23.49	22.37	35.42	31.24	28.42	26.34	24.35
WT	32.1	27.4	24.3	22.2	20.5	30.5	25.9	23.2	21.3	19.7	33.5	27.9	24.7	22.3	20.6
MRBF	33.6	30.7	29.1	27.8	26.6	31.5	27.4	25.5	24.3	23.2	36.7	33.4	31.3	29.6	28.4
B+W+B	32.5	29.8	28.4	27.4	26.6	29.3	25.1	23.6	23.1	22.8	35.7	33.3	31.5	30.2	29.1
Proposed Work	**34.0**	**31.1**	**29.7**	**28.55**	**27.13**	**31.7**	**29.54**	**27.0**	**24.65**	**24.2**	**36.8**	**34.02**	**31.78**	**30.3**	**29.2**
	Boats(512X512)					**Gold hill(512X512)**					**Peppers(512X512)**				
NOISY PSNR	28.1	22.1	18.6	16.1	14.1	28.1	22.1	18.6	16.1	14.1	28.1	22.1	18.6	16.1	14.1
BF	32.2	28.45	26.24	24.60	23.23	30.38	28.45	26.75	25.04	23.72	34.7	30.61	27.81	25.8	24.45
WT	31.2	26.8	23.9	21.8	20.2	31.2	26.8	24.1	21.9	20.3	32.9	27.7	24.5	22.3	20.6
MRBF	32.1	28.8	27	25.7	24.7	31.9	28.9	27.5	26.5	25.6	34.9	31.5	29.6	27.9	26.6
B+W+B	30.4	27.4	25.9	24.9	24.3	30.4	28.1	27.1	26.5	25.9	34.3	31.6	29.4	27.9	26.6
Proposed Work	**32.4**	**29.1**	**27.1**	**25.9**	**25.0**	**32.1**	**29.2**	**27.7**	**26.7**	**26.0**	**35.1**	**31.71**	**29.8**	**28.2**	**27.0**

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a) Noisy image (σ=30) [18.6 dB] b) Bilateral Filter [24.92 dB] c) Wavelet Thresholding [23.2 dB]



d) MRBF [25.5 dB] *e)* Hybrid Model [23.6 dB] *f)* Proposed Method [27.0 dB]

Figure 2. Image Denoising Results on Gaussian Noise added Barbara Image (with PSNR)

VI CONCLUSION

In this paper, a spatially adaptive Joint Bilateral Filter and multiscale products LMMSE wavelet based image denoising framework, which integrating both Joint Bilateral Filter (JBF) and multiscale interscale LMMSE model is presented. The major factor in the performance of the proposed method is the application of JBF and multiscale product based LMMSE, which helps in eliminating the blur effect in digital images. In addition to this, it also helps in preserving the edges. This method multiplies the adjacent wavelet subbands to strengthen the significant features in the image and then applies the LMMSE algorithm to the multiscale products for the better differentiation of edge details in the image. It may be possible to improve the results further by applying Trivariate Shrinkage Filter (TSF), Trilateral Filter and the work in this direction is under progress.

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