A Fuzzy Based Two Warehouses Inventory Model for Deteriorating Items

A. K. Malik, Yashveer Singh, S. K. Gupta

Abstract-: In real life situations, especially for new products, the probability is not known due to lack of historical data and adequate information. Then these parameters and variables are treated as fuzzy parameters. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences and natural sciences. Over the years there have been successful applications and implementations of fuzzy set theory in production management. In this study, a fuzzy based two warehouses inventory model has been developed with exponential demand. Deterioration rates of two warehouses are considered to be different due to change in environment. The holding cost in RW is assumed to be higher than those in OW. To reduce the inventory costs, it will be economical for firms to store goods in OW before RW, but clear the stocks in RW before OW. The parameters such as holding costs, ordering cost and deteriorating cost for two warehouses are considered as fuzzy number. We considered the triangular fuzzy number to represents the fuzzy parameters. The total inventory cost is obtained in crisp environment as well as fuzzy sense with the help of Signed distance method.

Keywords:- Exponential demand, linear deterioration, Fuzzy model, Crisp model, Signed distance.

I. INTRODUCTION

If we consider the real world inventory systems we find that the exits parameters and variables which are uncertain or almost uncertain. When these uncertainties are significant, they are usually treated by probability theory. Of course, to address such an uncertainty, we need to prescribe an appropriate probability distribution. In some cases, uncertainties can be defined as fuzziness or vagueness, which are characterized by fuzzy numbers of the fuzzy set theory. One of the most important concerns of the management is to decide when and how much to order or to manufacture so that the total cost associated with the inventory system should be minimum. This is somewhat more important, when the inventory undergo decay. Medicine, electronic goods, chemicals, pharmaceuticals etc. deteriorate through a gradual loss of potential or utility with the passage of time. Thus deterioration of physical goods in stock is very realistic feature. **Ghare** and **Schrader (1963)** developed the first time an inventory model for deteriorating items. **Covert** and **Philip (1973)** extended the above inventory model with a two-parameter Weibull distribution.

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Hartley (1976) discussed an inventory model with two storage facilities. It is generally assumed that the holding cost in the RW is greater than the same in the OW. Hence, the items are stored first in the OW, and only excess of stock is stored in the RW. Further, the items of the RW are released first, and then the items of the OW. Bhunia and Maiti (1998) established an inventory model for deteriorating items with linear demand and shortages under two-warehouse. Singh and Malik (2009) discussed a two warehouse production inventory systems with exponential demand and variable deterioration. Lee and Hsu (2009) discussed a two-warehouse inventory model for deteriorating items with time-dependent demand.

In the crisp environment, all parameters in the total inventory cost such as holding cost, ordering cost, set-up cost, purchasing cost, deterioration rate, demand rate and production rate etc. are known and have definite value without ambiguity. Some of the business situations fit such conditions, but in most of the situations and in the day-by-day changing market scenario the parameters and variables are highly uncertain or imprecise. The concept of soft computing techniques (fuzzy logic) first introduced by Zadeh (1965). The invention of fuzzy set theory by the need to represent and capture the real world problem with its fuzzy data due to uncertainty. Instead of ignoring or avoiding uncertainty, Zadeh discussed a set theory to remove this uncertainty. It is to use hybrid intelligent methods to quickly achieve an inexact solution rather than use an exact optimal solution via a big search. Fuzzy set theory is being recognized as an important problem modeling and solution technique. Bellman and Zadeh (1970) discussed the difference between randomness and fuzziness by showing that the former deals with uncertainty regarding membership or non-membership of an element in a set while later is concerned with the degree of uncertainty by which an element belongs to a set. Silver and Peterson (1985) developed on decision systems for inventory management and production planning.

Zimmermann (1985) gives a review on applications of fuzzy set theory. Park (1987) discussed the economic order quantity model (EOQ) in which trapezoidal fuzzy numbers are used to model ordering costs and inventory holding cost. Lee et al. (1990) suggest the application of fuzzy set theory to lot-sizing in material requirements planning. Yao and Lee (1999) presented a fuzzy inventory model with and without backorder for fuzzy order quantity with trapezoidal fuzzy number. Hsieh (2002) considered two fuzzy production-inventory models: one for crisp production quantity with fuzzy parameters and the other one for fuzzy production quantity.

He used the graded mean integration representation method for defuzzifying the fuzzy total inventory cost. Y.W. Zhou (2003) proposed a Multi-warehouse Inventory Model for Items with Time-varying Demand and Shortages. Papadrakakis and Lagaros (2003) discussed about soft computing methodologies for structural optimization. Sundarraj and Talluri (2003) developed a multi-period optimization model for the procurement of component-based enterprise information technologies. Chang et al. (2004) presented a lead-time production model based on continuous review inventory systems, where the uncertainty of demand during lead-time was dealt with probabilistic fuzzy set and the annual average demand by a fuzzy number only. Chang et al. (2006) presented a model in which they considered a lead-time demand as fuzzy random variable instead of a probabilistic fuzzy set. Mahapatra and Maiti (2006) formulated a multi-item, multi-objective inventory model for deteriorating items with stock- and time-dependent demand rate over a finite time horizon in fuzzy stochastic environment. Mahata and Goswami (2006) developed a fuzzy production-inventory model with permissible delay in payment. They assumed the demand and the production rates as fuzzy numbers and defuzzified the associated cost in the fuzzy sense using extension principle. Dutta et al. (2007) considered a continuous review inventory system, where the annual average demand was treated as a fuzzy random variable. The lead-time demand was also assessed by a triangular fuzzy number. Maiti and Maiti (2007) developed multi-item inventory models with stock dependent demand, and two storage facilities were developed in a fuzzy environment where processing time of each unit is fuzzy and the processing time of a lot is correlated with its size. Yung et. al (2007) discussed on procurement planning of time-variable demand in manufacturing system based on soft computing techniques. Singh and Singh (2008) developed the fuzzy inventory model for finite rate of replenishment using the signed distance method. Halim et al. (2008) developed a fuzzy inventory model for perishable items with stochastic demand, partial backlogging and fuzzy deterioration rate. The model is further extended to consider fuzzy partial backlogging factor. Goni and Maheswari (2010) discussed the retailer's ordering policy under two levels of delay payments considering the demand and the selling price as triangular fuzzy numbers. They used graded mean integration representation method for defuzzification. Halim et al. (2010) addressed the lot sizing problem in an unreliable production system with stochastic machine breakdown and fuzzy repair time. They defuzzified the cost per unit time using the signed distance method.

In this paper we develop an inventory model with two warehouses for deteriorating items. Here two warehouse, one is own warehouse (OW) and other is rented warehouse (RW). The holding cost of rented warehouse is higher than own warehouse. Demand rate is taken be exponentially. The holding costs, ordering cost and deteriorating cost for two warehouses are considered as fuzzy number. The triangular and trapezoidal both types of fuzzy number are used for represents the fuzzy parameters. The total inventory costs are obtained in crisp and as well as fuzzy sense with the help of Signed distance method.

II. NOTATION AND ASSUMPTIONS

The inventory model is based on the following assumptions:

- A the ordering cost per order
- h the inventory holding cost in RW per unit per time unit
- g the inventory holding cost in OW per unit per time unit
- C_d the deteriorating cost per unit per time unit
- t_1 the time at which the inventory level in RW reaches zero, $t_1 \geq 0$
- T = $(t_1 + t_2)$ the length of cycle time.
- R the maximum inventory level in only RW during $[0, t_1]$.
- W the maximum inventory level in OW during $[0, T]$.
- IM the maximum positive inventory level in RW and OW during $[0, T]$.
- $I_R(t)$ the level of inventory in RW at time t, $0 \leq t \leq t_1$
- $I_{O1}(t)$ the level of inventory in OW at time t, $0 \leq t \leq t_1$
- $I_{O2}(t)$ the level of inventory in OW at time t, $t_1 \leq t \leq t_1 + t_2$
- TC the total cost per time unit.

{~ Sign represents the fuzziness of the parameters}

In addition, the following notations are used throughout in this paper:

- The inventory system deals with single item.
- Shortages are not allowed.
- The demand rate is $\mu e^{\lambda t}$.
- The deterioration rate in RW is $\alpha + \beta t$ and in OW is $a + bt.$
- The lead – time is zero or negligible.
- ***Triangular Fuzzy Number:*** Let k=(k_1, k_2, k_3) is a triangular fuzzy number, where k_1=k-Δ_1, k_2 =k, k_3 = k+Δ_2. The membership function of k is

$$\mu_{\tilde{k}}(\tilde{k}) = \begin{cases} \dfrac{k-k_1}{k_2-k_1} & k_1 \leq k \leq k_2 \\ \dfrac{k_3-k}{k_3-k_2} & k_1 \leq k \leq k_2 \\ 0 & otherwise \end{cases}$$

III. MATHEMATICAL MODEL (CRISP MODEL)

The inventory level at any instant of time during $[0, T]$ is described by the following differential equation:

$$\frac{dI_R(t)}{dt} + (\alpha + \beta t)\, I_R(t) = -\mu e^{\lambda t} \quad 0 \leq t \leq t_1 \quad \ldots.$$
(1)

$$\frac{dI_{O1}(t)}{dt} + (a + bt)\, I_{O1}(t) = 0 \quad 0 \leq t \leq t_1 \quad \ldots.$$
(2)

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$$\frac{dI_{O2}(t)}{dt}+(a+bt)I_{O2}(t)=-\mu e^{\lambda t} \quad t_1 \le t \le T \quad \ldots (3)$$

With boundary conditions $I_R(0)=R$, $I_{O1}(0)=W$ and $I_{O2}(T)=0$.
Solutions of above equations are:

$$I_R(t)=e^{-\left(\alpha t+\beta\frac{t^2}{2}\right)}\left[R-\mu\left\{\begin{array}{l} t+(\alpha+\lambda)\frac{t^2}{2} \\ +\frac{\beta t^3}{6}+\beta(\alpha+\lambda)\frac{t^4}{8}+\ldots\ldots \end{array}\right\}\right] \quad \ldots (4)$$

$$I_{O1}(t)=W\, e^{-\left(at+b\frac{t^2}{2}\right)} \quad \ldots (5)$$

$$I_{O2}(t)=e^{-\left(at+b\frac{t^2}{2}\right)}\mu\left[\begin{array}{l}(T-t)+\frac{(a+\lambda)}{2}(T^2-t^2) \\ +\frac{b}{6}(T^3-t^3)+\frac{b(a+\lambda)}{8}(T^4-t^4)+\ldots\ldots\end{array}\right] \quad \ldots (6)$$

The maximum positive inventory is

$$IM=I_R(0)+I_{O1}(0)=R+W \quad \ldots (7)$$

According to given conditions at $t=t_1$, $I_{O1}(t_1)=I_{O2}(t_1)$

$$W=\mu\left[\begin{array}{l} t_2+\frac{(a+\lambda)}{2}(t_2^2+2t_1t_2)+\frac{b}{6}(t_2^3+3t_1^2t_2+3t_2^2t_1) \\ +\frac{b(a+\lambda)}{8}(t_2^4+4t_1t_2^3+4t_2t_1^3+6t_1^2t_2^2)+\ldots\ldots\end{array}\right] \quad \ldots (8)$$

The equation is showing that relation between t_1 & t_2.
Next, the total relevant inventory cost per cycle consists of the following elements:

1. **Ordering cost per cycle is** OC=A ….. (9)
2. Inventory **holding cost** per cycle in **RW** is given by

$$HC_{RW}=h\int_0^{t_1} I_R(t)\,dt$$

$$=h\left[\begin{array}{l} R\left(t_1-\frac{\alpha t_1^2}{2}-\beta\frac{t_1^3}{6}\right) \\ -\mu\left(\frac{t_1^2}{2}+\lambda\frac{t_1^3}{6}-\alpha\frac{t_1^3}{6}-\frac{\beta t_1^4}{12}\right)\end{array}\right] \quad \ldots (10)$$

3. Inventory **holding cost** per cycle in **OW** is given by

$$HC_{OW}=g\left(\int_0^{t_1} I_{O1}(t)dt+\int_{t_1}^{t_1+t_2} I_{O2}(t)dt\right)$$

$$=g\left\{W\left(t_1-\frac{a}{2}t_1^2-b\frac{t_1^3}{6}\right)\right.$$

$$\left.+\mu\left[\frac{t_2^2}{2}+\frac{(a+\lambda)}{2}\left\{\frac{2}{3}t_2^3+t_1t_2^2\right\}+\frac{b}{6}\left\{\frac{3}{4}t_2^4+2t_1t_2^3\right\}\right]\right\} \quad \ldots (11)$$

4. **Deterioration cost** per cycle in **RW** is given by

$$DC_{RW}=C_d\int_O^{t_1}(\alpha+\beta t)I_R(t)\,dt$$

$$=C_d\alpha\left[\begin{array}{l} R\left(t_1-\frac{\alpha t_1^2}{2}-\beta\frac{t_1^3}{6}\right) \\ -\mu\left(\frac{t_1^2}{2}+\lambda\frac{t_1^3}{6}-\alpha\frac{t_1^3}{6}-\frac{\beta t_1^4}{12}\right)\end{array}\right]$$

$$+C_d\beta\left[\begin{array}{l} R\left(\frac{t_1^2}{2}-\frac{\alpha t_1^3}{3}-\beta\frac{t_1^4}{8}\right) \\ -\mu\left(\frac{t_1^3}{3}-\alpha\frac{t_1^4}{8}+\lambda\frac{t_1^4}{8}-\frac{\beta t_1^5}{15}\right)\end{array}\right] \quad \ldots (12)$$

6. **Deterioration cost** per cycle in **OW** is given by

$$DC_{OW}=C_d\left\{\int_0^{t_1}(a+bt)I_{O1}(t)dt+\int_{t_1}^{t_1+t_2}(a+bt)I_{O2}(t)dt\right\}$$

$$=C_d\left\{W\left(at_1+b\frac{t_1^2}{2}\right)\right.$$

$$\left.+\mu\left[a\left(\frac{(t_1+t_2)^2}{2}-t_1t_2-\frac{t_1^2}{2}\right)+b\left\{\frac{(t_1+t_2)^3}{6}-\frac{t_1^2}{2}t_2-\frac{t_1^3}{6}\right\}\right]\right\} \quad \ldots (13)$$

Therefore, the total inventory cost per unit time is given by TC (t_1, t_2)

$$=(1/T)\,[OC+HC_{RW}+HC_{OW}+DC_{RW}+DC_{OW}] \quad \ldots (14)$$

Substituting Equation (9)–(13) in the above equation (14), we get

$$TC(t_1,t_2)$$

$$=\frac{1}{T}\left(A+h\left[\begin{array}{l} R\left(t_1-\frac{\alpha t_1^2}{2}-\beta\frac{t_1^3}{6}\right) \\ -\mu\left(\frac{t_1^2}{2}+\lambda\frac{t_1^3}{6}+\alpha\frac{t_1^3}{6}+\frac{\beta t_1^4}{12}\right)\end{array}\right]\right.$$

$$+g\left[\begin{array}{l} W\left(t_1-\frac{a}{2}t_1^2-b\frac{t_1^3}{6}\right) \\ +\mu\left(\frac{t_2^2}{2}+\frac{(a+\lambda)}{2}\left\{\frac{2}{3}t_2^3+t_1t_2^2\right\}+\frac{b}{6}\left\{\frac{3}{4}t_2^4+2t_1t_2^3\right\}\right)\end{array}\right]$$

$$+C_d\alpha\left[\begin{array}{l} R\left(t_1-\frac{\alpha t_1^2}{2}-\beta\frac{t_1^3}{6}\right) \\ -\mu\left(\frac{t_1^2}{2}+\lambda\frac{t_1^3}{6}-\alpha\frac{t_1^3}{6}-\frac{\beta t_1^4}{12}\right)\end{array}\right]$$

$$+C_d\beta\left[\begin{array}{l} R\left(\frac{t_1^2}{2}-\frac{\alpha t_1^3}{3}-\beta\frac{t_1^4}{8}\right) \\ -\mu\left(\frac{t_1^3}{3}-\alpha\frac{t_1^4}{8}+\lambda\frac{t_1^4}{8}-\frac{\beta t_1^5}{15}\right)\end{array}\right]$$

$$\left.+C_d\left\{W\left(at_1+b\frac{t_1^2}{2}\right)+\mu\left[\begin{array}{l} a\left(\frac{(t_1+t_2)^2}{2}-t_1t_2-\frac{t_1^2}{2}\right) \\ +b\left\{\frac{(t_1+t_2)^3}{6}-\frac{t_1^2}{2}t_2-\frac{t_1^3}{6}\right\}\end{array}\right]\right\}\right) \quad \ldots (15)$$

The total relevant inventory cost per unit time is minimum if

$$\frac{\partial TC}{\partial t_1}=0,\ \frac{\partial TC}{\partial t_2}=0,\ \left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial t_2^2}\right)-\left(\frac{\partial^2 TC}{\partial t_1 t_2}\right)^2 \succ 0 \text{ and } \frac{\partial^2 TC}{\partial t_1^2}\succ 0.$$

IV. FUZZY MODEL

In the above model we develop a crisp model in which assumed all the parameters are fixed or as considered the rate of deterioration in both RW and OW assumed is time dependent, but in real life and global market situations they fluctuate with own actual values. So the parameters are not being assumed to constant. To dealing such type of uncertainty consider a fuzzy model in which we assumed holding costs in both warehouses, ordering cost and deteriorating cost is a fuzzy number which is represent by triangular numbers. We discussed in this fuzzy model signed distance method.

V. SIGNED DISTANCE METHOD

This method we used triangular fuzzy number for holding costs, ordering cost and deteriorating cost. Suppose the following fuzzy numbers:

(1) $h \in [h-\Delta_1, h+\Delta_2]$, where $0<\Delta_1<h$ and $0<\Delta_1\Delta_2$

(2) $g \in [g-\Delta_3, g+\Delta_4]$, where $0<\Delta_3<g$ and $0<\Delta_3\Delta_4$

(3)$A \in [A-\Delta_5, A+\Delta_6]$, where $0<\Delta_5<A$ and $0<\Delta_5\Delta_6$

(4) $C_d \in [C_d -\Delta_7, C_d +\Delta_8]$, where $0<\Delta_7< C_d$ and $0<\Delta_7\Delta_8$.

The signed distance of the above fuzzy numbers is

(1) $d(\tilde{h},0) = h + \frac{1}{4}(\Delta_2 - \Delta_1)$

(2) $d(\tilde{g},0) = g + \frac{1}{4}(\Delta_4 - \Delta_3)$

(3) $d(\tilde{A},0) = A + \frac{1}{4}(\Delta_6 - \Delta_5)$

(4) $d(\tilde{C}_d,0) = C_d + \frac{1}{4}(\Delta_8 - \Delta_7)$

Using equation (15), we have

$$\tilde{TC} = (TC_1, TC_2, TC_3)$$

$$TC_1(t_1,t_2) = \frac{1}{T}\left\{\begin{aligned}&(A-\Delta_5)\\ &+(h-\Delta_1)\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} + \alpha\frac{t_1^3}{6} + \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+(g-\Delta_3)\left[\begin{aligned}&W\left(t_1 - \frac{a}{2}t_1^2 - b\frac{t_1^3}{6}\right)\\ &+\mu\left(\frac{t_2^2}{2} + \frac{(a+\lambda)}{2}\left\{\frac{2}{3}t_2^3 + t_1t_2^2\right\} + \frac{b}{6}\left\{\frac{3}{4}t_2^4 + 2t_1t_2^3\right\}\right)\end{aligned}\right]\\
&+(C_d-\Delta_7)\alpha\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} - \alpha\frac{t_1^3}{6} - \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+(C_d-\Delta_7)\beta\left[\begin{aligned}&R\left(\frac{t_1^2}{2} - \frac{\alpha t_1^3}{3} - \beta\frac{t_1^4}{8}\right)\\ &-\mu\left(\frac{t_1^3}{3} - \alpha\frac{t_1^4}{8} + \lambda\frac{t_1^4}{8} - \frac{\beta t_1^5}{15}\right)\end{aligned}\right]\\
&+(C_d-\Delta_7)\left\{\begin{aligned}&W\left(at_1 + b\frac{t_1^2}{2}\right)\\ &+\mu\left[\begin{aligned}&a\left(\frac{(t_1+t_2)^2}{2} - t_1t_2 - \frac{t_1^2}{2}\right)\\ &+b\left\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^2}{2}t_2 - \frac{t_1^3}{6}\right\}\end{aligned}\right]\end{aligned}\right\}\end{aligned}\right\}$$

$$TC_2(t_1,t_2) = TC(t_1,t_2)$$

$$TC_1(t_1,t_2) = \frac{1}{T}\left\{\begin{aligned}&(A+\Delta_6)\\ &+(h+\Delta_2)\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} + \alpha\frac{t_1^3}{6} + \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+(g+\Delta_4)\left[\begin{aligned}&W\left(t_1 - \frac{a}{2}t_1^2 - b\frac{t_1^3}{6}\right)\\ &+\mu\left(\begin{aligned}&\frac{t_2^2}{2} + \frac{(a+\lambda)}{2}\left\{\frac{2}{3}t_2^3 + t_1t_2^2\right\}\\ &+\frac{b}{6}\left\{\frac{3}{4}t_2^4 + 2t_1t_2^3\right\}\end{aligned}\right)\end{aligned}\right]\\
&+(C_d+\Delta_8)\alpha\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} - \alpha\frac{t_1^3}{6} - \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+(C_d+\Delta_8)\beta\left[\begin{aligned}&R\left(\frac{t_1^2}{2} - \frac{\alpha t_1^3}{3} - \beta\frac{t_1^4}{8}\right)\\ &-\mu\left(\frac{t_1^3}{3} - \alpha\frac{t_1^4}{8} + \lambda\frac{t_1^4}{8} - \frac{\beta t_1^5}{15}\right)\end{aligned}\right]\\
&+(C_d+\Delta_8)\left\{\begin{aligned}&W\left(at_1 + b\frac{t_1^2}{2}\right)\\ &+\mu\left[\begin{aligned}&a\left(\frac{(t_1+t_2)^2}{2} - t_1t_2 - \frac{t_1^2}{2}\right)\\ &+b\left\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^2}{2}t_2 - \frac{t_1^3}{6}\right\}\end{aligned}\right]\end{aligned}\right\}\end{aligned}\right\}$$

The total inventory fuzzy cost $\tilde{TC}$ per unit time by signed distance method

$$d(\tilde{TC}) = \frac{1}{T}\left\{\begin{aligned}&\frac{1}{4}\left\{\begin{aligned}&(\Delta_6-\Delta_5)\\ &+(\Delta_2-\Delta_1)\end{aligned}\right\}\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} + \alpha\frac{t_1^3}{6} + \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+\frac{1}{4}(\Delta_4-\Delta_3)\left[\begin{aligned}&W\left(t_1 - \frac{a}{2}t_1^2 - b\frac{t_1^3}{6}\right)\\ &+\mu\left(\frac{t_2^2}{2} + \frac{(a+\lambda)}{2}\left\{\frac{2}{3}t_2^3 + t_1t_2^2\right\} + \frac{b}{6}\left\{\frac{3}{4}t_2^4 + 2t_1t_2^3\right\}\right)\end{aligned}\right]\\
&+\frac{1}{4}(\Delta_8-\Delta_7)\alpha\left[\begin{aligned}&R\left(t_1 - \frac{\alpha t_1^2}{2} - \beta\frac{t_1^3}{6}\right)\\ &-\mu\left(\frac{t_1^2}{2} + \lambda\frac{t_1^3}{6} - \alpha\frac{t_1^3}{6} - \frac{\beta t_1^4}{12}\right)\end{aligned}\right]\\
&+\frac{1}{4}(\Delta_8-\Delta_7)\beta\left[\begin{aligned}&R\left(\frac{t_1^2}{2} - \frac{\alpha t_1^3}{3} - \beta\frac{t_1^4}{8}\right)\\ &-\mu\left(\frac{t_1^3}{3} - \alpha\frac{t_1^4}{8} + \lambda\frac{t_1^4}{8} - \frac{\beta t_1^5}{15}\right)\end{aligned}\right]\\
&+\frac{1}{4}(\Delta_8-\Delta_7)\left\{\begin{aligned}&W\left(at_1 + b\frac{t_1^2}{2}\right)\\ &+\mu\left[\begin{aligned}&a\left(\frac{(t_1+t_2)^2}{2} - t_1t_2 - \frac{t_1^2}{2}\right)\\ &+b\left\{\frac{(t_1+t_2)^3}{6} - \frac{t_1^2}{2}t_2 - \frac{t_1^3}{6}\right\}\end{aligned}\right]\end{aligned}\right\}\end{aligned}\right\} \quad \dots (19)$$

Hence find the optimum solution for any particular situation with the help of the above equations. Numerical solution can be obtained with suitable software.

VI. CONCLUSIONS

This paper deals a fuzzy based inventory model for deteriorating items with exponential demand and two warehouse facilities. The proposed model can be used in inventory control of deteriorating items such as fashionable items, medicines, food items, electronic components such as mobile, machines, circuit, toys and fashionable commodities etc. In the future study, it is hoped to further incorporate the proposed model into more realistic assumptions, such as probabilistic demand and holding cost like as stock dependent demand, backorders, quadratic demand, production dependent demand.

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