Abstract—A modified Invasive Weed based methodology for optimal measurement of Phasor measurements units (PMUs) for complete observability of Power system is presented in this paper. The prime objective of this Optimization problem is to reduce the number of PMUs and to maximize the redundancy at power system buses. In this paper MIWO (Modified Invasive Weed Algorithm) is implemented for three bush systems namely 7, 9, IEEE 14 standard bus systems. The proposed algorithm is very easy to understand and its result is as satisfactory as results of other algorithm methods.

Keywords: Invasive Weed Algorithm, Phasor Measurement Units, Observability, Optimal Placements.

I. INTRODUCTION

Phasor measurement unit (PMUs) proposed in 1990, is a power system device used for measurement of synchronized voltage and current in a power system[1]. Synchronicity among phasor measurement units (PMUs) is achieved by same-time sampling of voltage and current waveforms using a common synchronizing signal from the global positioning satellite (GPS). The unique ability to calculate synchronized phasors makes the PMU one of the most important measuring devices in the future of power system monitoring and control[2]. The importance of PMUs in Power system monitoring and control has been very well recognized. The measurement and analysis done with PMUs is superior than all the conventional methods as it calculates phasors using Discrete Fourier Transform (DFT). PMUs measures the voltage phasors of the node on which it is placed and current phasors of the incident branches with greater accuracy[3]. This ability of PMU makes it possible to observe the nodes which are connected to the node on which PMU is placed. As PMU measures both voltage as well as current phasors so, there is no need to place PMU at all nodes to make the whole system observable. This creates the problem placing appropriate number of PMU at strategic positions to make the whole system observable with minimum number of PMU.

II. PMU PLACEMENT PROBLEM FORMULATION

A PMU placed at any given bus measures the voltage phasor as well as current phasors of the lines incident to that bus. Thus, by placing PMUs at strategic bus location, entire system can be made observable. The objective of PMU placement problem is to make the entire system observable by placing the minimum number of PMUs at strategic points. The PMU placement problem can be given by:

\[
\text{Min } \sum_{i} w_i x_i
\]

s.t. \( f(X) \geq 1 \)

Where X is binary decision variable whose elements are given as:

\[
x_i = \begin{cases} 
1 & \text{if a PMU is placed at bus} \\
0 & \text{otherwise} 
\end{cases}
\]

\( W_i \) represents the cost variable and \( f(X) \) represent the vector whose value is no-zero, when the particular bus is solvable by given phasor measurement and zero otherwise. The entire procedure of building seven bus constraints in a PMU placement problem is explained with the help the seven bus system. The seven bus system is given in the figure (Figure 1) [3].
Optimal Placement of Phasor Measurement Unit Using Modified Invasive Weed Algorithm

Initially a bus connectivity matrix A is formed with the help of the criterion given below:

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}
\]

In this case constraints are formed as:

\[
f_1 = x_1 + x_2 \\
f_2 = x_1 + x_2 + x_3 + x_6 + x_7 \\
f_3 = x_2 + x_3 + x_4 + x_6 \\
f_4 = x_3 + x_4 + x_5 + x_7 \\
f_5 = x_4 + x_5 \\
f_6 = x_2 + x_3 + x_6 \\
f_7 = x_2 + x_4 + x_7 \\
\]

\[
f(X) = \begin{cases}
1 & \text{if } f_1 \lor f_2 \lor f_3 \lor f_4 \lor f_5 \lor f_6 \lor f_7 \\
0 & \text{otherwise}
\end{cases}
\]

In the following constraints + signifies OR and every vector function i.e. f1 to f7 has to be greater than 1 in order to make the system observable. From the constraints it can be concluded that if a PMU is placed at bus 2, it will make buses at 1, 2, 3, 6 and 7 observable.

**III. INVASIVE WEED OPTIMIZATION**

The basic idea of MIWO (Modified Invasive weed Algorithm) has been taken from Invasive Weed Algorithm. For stimulating colonizing behaviour of weeds, some properties of the process is considered [4]. They are:

1) A finite number of seeds is spread over the entire search area.
2) Each seed will develop and grow into a flowering plant and produce seeds depending on the fitness.
3) All the newly produced seeds are randomly dispersed to the whole search area to produce new plants.
4) This process gets repeated until maximum number of plants is reached; now only plants with best fitness can survive and produce seeds and others will get eliminated. The process will continue until maximum number of iterations is reached and hopefully the plant with the best fitness is closest to the optimal solution.

**IV. MODIFIED INVASIVE WEED OPTIMIZATION (MIWO)**

Some modifications are done in the basic Invasive Weed algorithm according to the needs of our problem. In PMU placement problems all the seeds grows to plants at same time, so there is no need for further reproduction. Due to this reason there is no need of the Dispersion process. So, basically Modified Invasive Weed Algorithm is a three step process.

1) A finite number of seeds are being spread out over the search area.
2) Every seed grows to a flowering plant, having different fitness values.
3) The plants with poor fitness are selectively eliminated.

**V. FLOW CHART OF MIWO**

1. **Distribute seeds in the search area**
   - Seeds develop into flowering plants with different fitness value
   - Eliminate the plants which are most unfit (This will eliminate the possibility of unobservability).
   - Selectively eliminate all the relative unfit plants.
   - Finally, a solution is reached when fittest plant is left
   - Seeds corresponding to fittest plant give optimal number of PMUs
   - Stop
VI. APPLICATION OF MIWO ON 7-BUS TEST SYSTEM

Consider a 7-bus system represented as in the diagram

![Fig 2: 7 Bus System](image)

Initially a binary connectivity matrix $A$ is formed according to the criteria given below:

$$A_{k,m} = \begin{cases} 1 & k = m, \\ 1 & k \text{ and } m \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

The connectivity matrix can also be obtained from the admittance matrix by transforming its entries into binary form:

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

This is the first step in PMU placement problem. After connectivity matrix is formed the next step is initialization of seeds, i.e. a finite number of seeds is distributed in the whole search area. A matrix of order 7X128 is formed. The number of entries is 128, which means 128 seeds are produced which will now develop in a flowering plant. The number is exactly 128 ($2^7$) as it is a 7-bus system and there is just two possible outcomes, 0 (PMU not placed at the respective bus) and 1 (PMU placed at the respective bus). Each seeds now develop into a plant, with each having different fitness values. Each of 128 seeds develop into a plant, so in total, the number of plants is 128. The two initial process of Modified Weed Growth algorithm has been already performed. The last step is elimination. All plants which is most unfit, i.e. matrix with even one zero in any row is eliminated in the first step. This step helps to eliminate all the possible chance of unobservability of any bus. Similarly, all unfit plants are selectively removed and lastly a situation is reached in which optimal number of PMUs is found. For a seven bus system the result is PMUs placed at 2, 5 give us complete observability. It also indicates that optimal number of PMUs in 7-bus system is 2. For finding Optimal Location, a suitable code is applied (case of SORI condition) and the results obtained are PMUs placed at bus 2 and bus 4.

VII. RESULTS AND DISCUSSIONS

The Modified Invasive Weed technique is used to find the optimal no. of PMUs and their optimal position in the test bus system for complete observability. There can be different optimal solution depending upon the bus system. Bus diagram of the 7 bus system, IEEE 9 bus system and IEEE 14 bus system are given in the figures. The optimal number of PMU’s required for 7, 9, 14 buses are 2, 3 and 4 respectively.

The position of PMU’s is as given in the table given below:

<table>
<thead>
<tr>
<th>Bus System</th>
<th>Table I Optimal location of the PMU for different test Bus System</th>
<th>Table II Effect after applying condition of redundancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Bus System</td>
<td>Normal measurement condition without considering the effect of redundancy</td>
<td>Normal measurement condition without considering the effect of redundancy</td>
</tr>
<tr>
<td>7 Bus System</td>
<td>Normal measurement condition considering the effect of redundancy</td>
<td>Normal measurement condition considering the effect of redundancy</td>
</tr>
<tr>
<td>IEEE 9 Bus System</td>
<td>Normal measurement condition without considering the effect of redundancy</td>
<td>Normal measurement condition considering the effect of redundancy</td>
</tr>
<tr>
<td>IEEE 9 Bus System</td>
<td>Normal measurement condition considering the effect of redundancy</td>
<td>Normal measurement condition considering the effect of redundancy</td>
</tr>
<tr>
<td>IEEE 14 Bus System</td>
<td>Normal measurement condition without considering the effect of redundancy</td>
<td>Normal measurement condition considering the effect of redundancy</td>
</tr>
<tr>
<td>IEEE 14 Bus System</td>
<td>Normal measurement condition considering the effect of redundancy</td>
<td>Normal measurement condition considering the effect of redundancy</td>
</tr>
</tbody>
</table>

Effect after applying condition of redundancy

<table>
<thead>
<tr>
<th>Bus System</th>
<th>Location of PMU</th>
<th>No. of times each bus is observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Bus System</td>
<td>2, 5</td>
<td>1,1,1,1,1,1,1,1,1</td>
</tr>
<tr>
<td>7 Bus System</td>
<td>2, 4</td>
<td>1,1,2,1,1,1,1,1</td>
</tr>
<tr>
<td>IEEE 9 Bus System</td>
<td>3, 4, 8</td>
<td>1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1</td>
</tr>
<tr>
<td>IEEE 9 Bus System</td>
<td>4, 6, 8</td>
<td>1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1</td>
</tr>
</tbody>
</table>

Effect after applying condition of redundancy
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VIII. CONCLUSIONS

In this paper integer programming based formulation and the associated solution to the PMU placement in power system is presented. A new algorithm technique i.e. Modified Invasive Weed Algorithm(MIWO) was applied as an optimization tool for finding minimum number of PMUs in 7, IEEE 9 and IEEE 14 bus system. This algorithm can also be used to analyze larger bus systems like IEEE 57 bus system by using TOMLAB. This algorithm focuses on dual objective: (a) to minimize the number of PMU’s needed to maintain complete observability and (b) to maximize the redundancy at all the bushes. The results of MIWO algorithm are so promising that researchers will be encouraged to use MIWO in difficult problems where constraints such as existence of conventional measurements, user-defined measurement redundancy at the bushes and the condition of measurement uncertainty will be taken into account.

REFERENCES