

# State Estimation and Voltage Stability Monitoring using ILP PMU Placement

R. Sudha, Deepak Jain, Umang Lahoty, Swati Khushalani, Nivedita G., Jayabarathi T.

**Abstract:-** This paper shows various cases under which optimal PMU placement is done. Zero injection busses are also considered for the placement problem which reduces the number of PMU required. For this topology method is used. In case of failure of single PMU, the reliability of the system should be improved. For this the problem formulation is modified according to which each bus is observed by at least two PMUs. The PMU placement is then used to get data for state estimation. The results-voltages and phase angles of bus system are compared with and without PMU using two algorithms- WLS and LAV. It is found that LAV is better algorithm than WLS and errors are reduced if the PMU measurements are included. The PMU data is also used for the voltage stability analysis using two indices-FVSI and LQP. Contingency analysis is done using these under different operating conditions to get an idea of stressful situation of lines randomly chosen.

**Index Items:-** PMU, WLS and LAV, using two indices-FVSI and LQP.

## I. INTRODUCTION

Phasor Measurement Units (PMUs) are the most accurate and advanced instrument utilizing synchronous measurement technology available to power system engineers and system operators. Many researchers have dedicated their attention to application of PMUs in power systems observability. When placed at the bus, a PMU can measure the phasor voltage of the bus, and different number of phasor currents of branches incident to that bus, depending on the type of PMU. It is well known that having a PMU installed at each and every bus in the system, the state of the system can be directly measured through PMUs, and the entire system would be observed by PMUs. However, the cost of PMUs and their installation does not allow utility companies to install PMUs at every bus. Costly procedure of purchasing and installation of PMUs motivated the optimal placement of the PMUs in the system. In other words, utility companies are looking for the minimum number of PMUs that can observe the entire system. A power system is called observable if the state of the system can be uniquely identified. Our study shows that for the optimal PMU placement problem, multiple solutions with the same cost exist. To compare these solutions qualitatively, We introduce a performance index SORI (system observability redundancy index). If zero injection busses are also modeled in the PMU placement problem, the total number of PMUs can be further reduced.

**Manuscript received on April 26, 2012.**

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To enhance the reliability of system monitoring, if a bus is observed by at least two PMUs instead of one, the loss of one will still keep the system observable [3].

The benefits of synchronized phasor measurements have been well recognized in the field of state estimation and system voltage monitoring. They are the potential applications that have their merits in the field of real time monitoring and control of power system. Our objective is to create a state estimator model to perform better security analysis of power system. The aim is to outline the work being done in the field of state estimation using PMU data. This is done by comparing the results of state estimation-voltages and angles on each bus of an IEEE 14 bus system using two algorithms i.e. WLS and LAV.

By continuously monitoring the power system, the reliability of a power grid can be increased by detecting faults at an early stage and preventing power outages. Voltage collapse proximity indicators based voltage security monitoring system using synchronized phasor measurements is another application. The objective is to monitor voltage stability utilizing PMU readings. The voltage stability analysis is performed using two line stability indices-FVSI and LQP and finally to perform post contingency analysis at the operating load condition which includes single load change, multiple load change and transmission line outages.

## II. OPTIMAL PMU PLACEMENT

Phasor measurement units (PMU) allow the state estimator to get the idea of following parameters:

1. Voltage magnitude and phase angle of each bus.
2. Branch current phasor emerging from that bus.

However installing PMU at each bus is not possible due to cost constraint. Hence there is a need of optimal placement of PMUs. This paper aims at the placement of minimum number of PMUs with complete and maximum observability. An IPL approach for this purpose is used.

### A. ILP Approach

Integer linear programming is an optimization technique in which variables can assume only integer values. Other convention methods like graph theory and simulated annealing method of PMU placement have complexities due to non-linearity. This disadvantage is overcome by using ILP which uses linear objective function and constraints.

There are three common variations of ILP. Pure ILP; where all variables assume integer values, Mixed ILP; where some variables assume integer values and Binary ILP; where all variables assume either 1 or 0 i.e., yes or no. In this paper the focus is on Binary ILP which means either PMU is placed (value is 1) or PMU is not placed (value is 0).



Optimal PMU placement using ILP approach is introduced in [1], [2].

For an n-bus system we introduce a PMU placement vector  $d$ , which contains elements from  $d_1$  to  $d_n$ .  $d_i$  represents the placement of PMU at  $i^{th}$  bus.

If  $d_i=1$ , PMU is placed

If  $d_i=0$ , PMU is not placed at  $i^{th}$  bus

Hence, the formulation of ILP can be done as follows:

(1) is the function which minimizes the summation of PMUs placed in the system which makes it the objective of the problem to be optimized.

(2) represents the primary constraint of the problem which makes sure that each bus is observed by at least one PMU.

Formulation of ILP:

(1) Objective function:

$$\min \sum_{i=1}^n d_i$$

(2) Constraints

$$Pd \geq e$$

$n \rightarrow$  size of the bus system.

$d \rightarrow$  vector of length of  $n$  with each element representing possibility of PMU placement at  $i^{th}$  bus i.e.,

$$d = [d_1 \ d_2 \ d_3 \ \dots \ d_n]^T$$

$e \rightarrow$  unit vector of length  $n$  i.e.,

$$e = [1 \ 1 \ 1 \ \dots \ 1]^T$$

$P \rightarrow$  connectivity matrix formed using the line data of the bus system i.e.,  $P_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or } i \text{ and } j \text{ are adjacent buses;} \\ 0 & \text{otherwise.} \end{cases}$

Fig. 1.1 Formulation of ILP

This ILP formulation does not include zero injection and PMU outage consideration.

### B. Zero injection bus consideration

Zero injection busses are those busses from which no current is injected into the system [3]. Since, no current is being injected into the system the adjacent busses can be assumed to be directly connected. This can be explained using the Kirchhoff's voltage law. If the adjacent busses to the zero injection bus are observable, then no separate PMU is required to observe the zero injection bus. This is explained in [3]. Hence, on consideration of zero injection busses, no. of PMUs can be reduced.

For solving the problem we use topology method [4]. The approach is to merge the zero injection bus with any of its neighbors which is arbitrarily done. Different choices of neighbors will lead to different PMU placement strategies; however the number of PMUs will be always the same. This would modify the connectivity matrix. This is explained by,

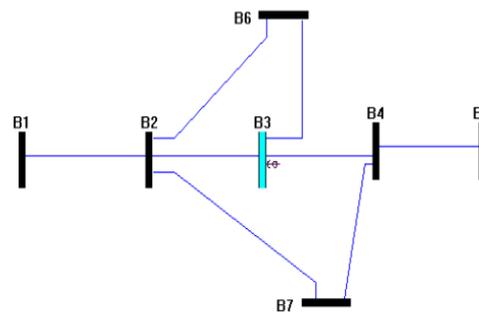


Fig. 1.2 7-bus system before merge

Corresponding connectivity matrix,

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Now randomly merging with bus 6, after merging the bus system is

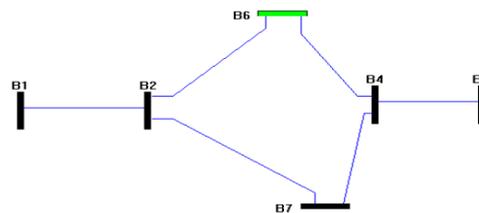


Fig. 1.3 7-bus system after merge

Corresponding matrix will be,

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

After the modified matrix is obtained the ILP is solved using this connectivity matrix with the method explained later.

### C. Maximum observability

Observability means the bus is observable by at least one PMU. The whole system is observable when each bus is made observable by minimum number of PMUs. Now with this optimal number of PMUs there can be different possibilities for placement. This is decided by the maximum observability. The total of observability with each PMU is calculated and maximum of them gives the optimal placement. This is termed as SORI and can be explained using [3].

The maximization makes the larger portion of system observable, in case of any, emergency or outage.

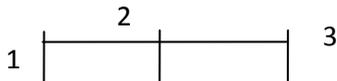
#### D. PMU outage

In day to day world, fault occurrence in power system is very common. Hence the PMU can become faulty in a bus system. To overcome this problem we should consider the PMU outage case. The observability for each bus in this case would increase from one to two. This will enhance the system reliability.

#### E. Solving of ILP

The solving of ILP can be explained using a case of a 3-bus system,

Let the system be,



First step is to construct connectivity matrix,

$$p = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Then taking the input for the number of busses,

$$N=3$$

Now forming binary matrix of size  $n \times 2^n \rightarrow 3 \times 8$ ,

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

So by multiplying  $p$  with each column of  $d$  we get the following  $z$  matrix,

$$Z = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 & 1 & 2 & 2 \\ 0 & 1 & 1 & 2 & 1 & 2 & 2 & 3 \\ 0 & 1 & 1 & 2 & 0 & 1 & 1 & 2 \end{bmatrix}$$

Next step is to eliminate the elements of unobservability. Hence making the columns with 0 as 0, we get

$$Z = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 0 & 0 & 2 & 2 & 3 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

In the next step, we find the columns with maximum number of 1s. In this case maximum number of 1s is three and only 1 column contain it i.e. 3.

Column 3 in  $d$  matrix refer to the following vector,

$$D(3) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Hence, we get no of PMU required = 1.

After finding the number of PMU, we find the optimal position for maximum observability. For this, we make all the columns of  $d$  equal to 0 which don't have number of 1s equal to 1.

We get,

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now the same steps are followed from the multiplication of  $p$  and  $d$  after which we get the  $y$  matrix as,

$$Y = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So the SORI for this case will be the sum of elements of column 3 which is,

$$SORI = 3$$

So for the 3 bus system the optimal position is at bus 2.

Now for the 14 bus system the following results are obtained,

1. No of PMUs = 4
2. Initial SORI = 14
3. Final SORI = 19
4. PMU placement = 2,6,7,9

The whole explanation of 14 bus system, its placement, system estimation and voltage monitoring is explained later.

### III. STATE ESTIMATION

State estimation in power systems refers to the collection of a redundant set of measurements from around the system and computing a state vector of the voltage at each observed bus[5]. This paper presents the mathematical basis for analyzing state estimation techniques and studies modification of these algorithms to include phasor measurements to improve the quality of the measurement. Two methods have been used to solve state estimation problem – WLS (weighted least square) and LAV (least absolute value).

The WLS method minimizes the weighted sum of squares of the difference between the measured and estimated values. The weight for each measurement is obtained from the accuracy of the device which is termed as the standard deviation of the measurement[5]. More accurate measurements are given more weight so that the estimation procedure influences the solution based on the measurements of greater accuracy. Full Newton Raphson (NR) method is used in linearizing and iteratively solving the states it. The state estimator becomes WLS estimator by including the measurement covariance matrix[6].

In LAV method, the objective function for minimization in this method is the sum of the absolute values of the difference between measured and estimated quantities with constraints on equations for measurements[5]. Linear programming technique is used to formulate and solve the problem as a linear programming problem using Simplex method.



The problem for any state estimation procedure is to solve for the system states (bus voltages and angles) based on available data. The state estimation problem is shown by equation 3.1 [5].

$$Z_i = h_i(x) + e_i \quad 3.1$$

Where,  $i=1, 2, 3 \dots m$   
 $m$ =number of measurements  
 $e_i$  = error in  $i$ th measurement  
 $z_i$  =  $i$ th measurement  
 $h_i(x)$  =function relating state variables with measurements.  
 $X$ =state variables (all bus voltages and angles except).

The exact values of measurements are obtained from a power flow program. A random error has been introduced in the measurements using equation 3.2[5].

$$Z_i = A_i * (1 + RND * \sigma_i) \quad 3.2$$

Where,  $z_i$  = simulated measured value.  
 $i=1, 2, 3 \dots m$ .  
 $m$ = number of measurements.  
 $RND$ =Random number with normal distribution & zero mean which is between -1 to +1.  
 $A_i$ =Actual value from power flow program.  
 $\sigma_i$  = Standard deviation of  $i$ th measurement.  
 The assumed values of standard deviation for different types of measurements are taken from [5].  
 $L1$  norm and error index [5] have been used to measure the accuracy of the algorithms. Smaller the value of norm and error index, better the performance of the algorithm.

**A. State Estimation Without PMU**

The steps involved in state estimation methods are:-

1) *Formulation of Ybus, Weight Matrix and Measurement vector*- The Ybus is formed by using the branch data. The weight vector is formed by taking the inverse of the variance (square of standard deviation) of the measurements. The error has been initialized to be greater than the tolerance before the first iteration.

2) *Formation of Measurement function*:- The measurement function is formed using following equations[5]-

$$h1(x) = V_k \quad 3.3$$

$$h21(x) = V_i * V_j * Y_{ij} * \cos(\delta_i - \delta_j - \Phi_{ij}) - V_i * V_i * Y_{ij} * \cos(\Phi_{ij}) \quad 3.4$$

$$h22(x) = V_i * V_j * Y_{ij} * \sin(\delta_i - \delta_j - \Phi_{ij}) + V_i * V_i * Y_{ij} * \sin(\Phi_{ij}) \quad 3.5$$

$$h2(x) = [h21(x); h22(x)] \quad 3.6$$

$$h31(x) = V_i * \Sigma V_j * Y_{ij} * \cos(\delta_i - \delta_j - \Phi_{ij}) \quad 3.7$$

$$h32(x) = V_i * \Sigma V_j * Y_{ij} * \sin(\delta_i - \delta_j - \Phi_{ij}) \quad 3.8$$

$$h3(x) = [h31(x); h32(x)] \quad 3.9$$

$$h(x) = [h1(x); h2(x); h3(x)] \quad 3.10$$

Where,  
 $h(x)$  = Measurement function.  
 $X$  = States of the system ( $V$  and  $\delta$ )  
 $k$  = number of the iteration ( $k \leq 100$ ).  
 $V_k$  = voltage in  $k$ th iteration.  
 $\Delta$  = Bus angle  
 $n$ = number of states  
 $i$ = From bus number  
 $j$ = To bus number  
 $Y$  = Magnitude of Ybus element  
 $\Phi$  = Angle of Ybus element.

$H1(x)$  = Measurement equation for voltage measurements  
 $h21(x)$  = Measurement equation for real power line flow measurements  
 $h22(x)$  = Measurement equation for reactive power line flow measurements.  
 $H31(x)$  = Measurement equation for real power injection measurements.  
 $H32(x)$  = Measurement equation for reactive power injection measurements.  
 Measurement vector is an  $m*1$  matrix, where ‘ $m$ ’ is the number of measurements.

3) *Formation Of Measurement Jacobian*- The Jacobian matrix is formed by taking the partial derivative of measurement function with respect to states. It is given by equation 3.11[5]:-

$$J = \partial h / \partial x \quad 3.11$$

where,  
 $J$ = Measurement Jacobian = [ $J1 J2; J3 J4; J5 J6; J7 J8; J9 J10$ ]  
 $J1 = \partial V / \partial \delta$   
 $J2 = \partial V / \partial V$   
 $J3 = \partial h21(x) / \partial \delta$   
 $J4 = \partial h21(x) / \partial V$   
 $J5 = \partial h22(x) / \partial \delta$   
 $J6 = \partial h22(x) / \partial V$   
 $J7 = \partial h31(x) / \partial \delta$   
 $J8 = \partial h31(x) / \partial V$   
 $J9 = \partial h32(x) / \partial \delta$   
 $J10 = \partial h32(x) / \partial V$

The measurement Jacobian matrix is of the size  $m*n$  ( $m$ =number of measurements and  $n$ =number of states).

4) *Solving For States Without PMU*- In WLS method the equation obtained after minimization is given by 3.12[5]:-

$$F(x) = H^T * R^{-1} * [z - h(x)] \quad 3.12$$

Where  
 $H$  = Measurement Jacobian ‘ $J$ ’  
 $R^{-1}$  = Weight Matrix  
 $h(x)$  = Measurement function  
 Equation 3.12 represents a non-linear equation which is solved by the Newton Raphson (NR) method. The Jacobian for NR method  $J_n$  is given equation 3.13[5].  
 $J_n = - H^T * R^{-1} * H \quad 3.13$

The states are updated using equation 3.14[1]

$$x^{k+1} = x^k + (H^T * R^{-1} * H)^{-1} * H^T * R^{-1} * [z - h(x)] \quad 3.14$$

Where  $x^{k+1}$  = Updated state in  $k+1$  iteration

The error in every iteration is given by equation 3.15[5]

$$Err = \text{Maximum (Absolute Value of } (H^T * R^{-1} * [z - h(x)])) \quad 3.15$$

In LAV method the steps for solving the states using the Simplex LP method are shown by the following set of equations.



Minimize  $d^T * X$   
 Subject to  $Aeq * X = beq$   
 Where,  $d = [cpp; cpp; cpp1; cpp1]$   
 $Aeq = [J -J \text{Im}1 -\text{Im}1]$   
 $beq = z_i - h_i(x)$   
 $cpp = A$  zero vector of length equal to number of states.  
 $Cpp1 =$  Vector containing ones in all the entries with size of  $m * 1$  ( $m =$  no. of measurements)  
 $\text{Im}1 =$  Identity matrix of order 'm'.

The matrices  $Aeq$ ,  $beq$ , and  $d$  are given to the Simplex method based linear programming function in MATLAB[7]. The output of the linear programming problem is the vector  $X$ . The changes in values of states contained in  $X$  are added to the values obtained in the previous iteration to get the updated values for state variables.

### B. State Estimation With PMU

The method to include phasor measurements in weighted least square method have been explained below. They are given more weight compared to the classical measurements.

$S = [Zs; Zr]$   
 $Zr = T * Zp$   
 $Zr = [V_{real}; V_{imag}; I_{real}; I_{imag}]$

Where,  
 $S =$  Measurement set with both phasor and classical state estimation measurements.

$Zs =$  Classical state estimation measurements  
 $Zr =$  Phasor measurements in rectangular co-ordinates  
 $Zp =$  Phasor measurements in polar co-ordinates  
 $T =$  Transformation matrix to convert polar to rectangular co-ordinates. Equations 3.16 through 3.18 [1] are used to form the measurement function, measurement Jacobian and weight matrix including phasor measurements.

$H_{total} = [h; h_{pmu}]$  3.16  
 $H_{total} = [H; H_{pmu}]$  3.17  
 $W_{total} = [W_{0m * u}; W_{0u * m} W_{pmu}]$  3.18

Where,  
 $h_{total} =$  the measurement function with calculated values of both phasor and traditional measurements.

$H_{total} =$  Measurement Jacobian for both phasor and traditional measurements

$H =$  Measurement Jacobian for traditional measurements  
 $H_{pmu} =$  Measurement Jacobian for PMU measurements  
 $W_{total} =$  Weight matrix of both PMU and traditional measurements

$W =$  Weight matrix of traditional measurements  
 $W_{pmu} =$  Weight matrix of PMU measurements  
 $0_{m * u} =$  Zero matrix of order  $m * u$   
 $m =$  Number of traditional measurements  
 $u =$  Number of PMU measurements

$\Delta x = (H_{total}^T * W_{total} * H_{total})^{-1} * H_{total}^T * W_{total} * [S - h_{total}]$

The updated state is obtained by adding the value of  $\Delta x$  to the previous states. The process is repeated till the error becomes less than the tolerance. The measurement equations and measurement Jacobian for phasor measurements is given by the next set of equations.

$H1_{pmu} = V_{real} = |V| * \cos(\delta)$   
 $h2_{pmu} = V_{imag} = |V| * \sin(\delta)$   
 $h3_{pmu} = I_{real} = [(|Vi| * \cos(\delta_i) - |Vj| * \cos(\delta_j)) * g_{ij}] - [(|Vi| * \sin(\delta_i) - |Vj| * \sin(\delta_j)) * b_{ij}]$

$h4_{pmu} = I_{imag} = [(|Vi| * \cos(\delta_i) - |Vj| * \cos(\delta_j)) * b_{ij}] + [(|Vi| * \sin(\delta_i) - |Vj| * \sin(\delta_j)) * g_{ij}]$   
 $h_{pmu} = [h1_{pmu}; h2_{pmu}; h3_{pmu}; h4_{pmu}]$   
 $H_{pmu} = \partial h_{pmu} / \partial x$

## IV. SYSTEM VOLTAGE MONITORING

Voltage stability problems have been receiving increased attention in recent years. The problems pose serious consequences such as excessive voltage drop or dynamic instability. Since power systems are operated under increasingly stressed condition, the ability to maintain voltage stability has become a growing concern, and good measures to improve the reactive power and voltage level control are required[8].

### A. Voltage Collapse Proximity Indicators

Voltage collapse is the process by which voltage instability leads to loss of voltage in a significant part of a power system[8]. System voltage monitoring is one of the fields in which the readings of PMU's can be utilized. Voltage stability indices will indicate how far an operating point is from voltage instability. The two types of line stability indices that are used for the analysis are Line Stability Index, FVSI and Line Stability Index, LQP.

1) *Fast Voltage Stability Index*:- FVSI is given by [8]

$$FVSI = \frac{4Z^2 Q_r}{V_s^2 X}$$

where  $Z$  is the line impedance,  $X$  is the line reactance,  $Q_r$  is the reactive power flow at the receiving end and  $V_s$  is the sending end voltage. The line that will give index value closest to 1 will be the most critical line of the bus and might lead to system wide instability scenario. This index can also be used to determine the weakest bus on the system. The weakest bus is determined based on the maximum load allowed on a load bus. The most vulnerable bus in the system is the bus with the smallest maximum permissible load.

2) *LINE STABILITY INDEX*:- LQP is given by

$$LQP = 4 \left( \frac{X}{V_s^2} \right) \left( \frac{X}{V_s^2} P_s^2 + Q_r \right)$$

where  $X$  is the line reactance,  $Q_r$  is the reactive power flow at the receiving bus,  $V_s$  is the voltage on sending bus and  $P_s$  is the active power flow at the sending bus. Operating at secure and stable conditions requires the value of LQP index to be maintained less than 1[10].

2.27

### B. CONTINGENCY ANALYSIS

The contingency analysis is performed at the operating load condition to avoid misranking of contingencies for the system that operates close to maximum load. The analysis is carried out for the two indices based on following criteria:-

2.28

2.29



1) *Single Load Change*:- The load is increased at one particular bus keeping the load on other busses constant. Several combinations selected are listed below:

- Single load change with real load only
- Single load change with reactive load only
- Single load change with real and reactive load.

2) *Multiple Load Change*:- A practical power system network actually possesses hundreds of nodes and thousands of lines connected to them. For any particular instance, some busses may simultaneously undergo change in load. In this case the load is increased on a few busses until just before the power flow solution diverges.

3) *Transmission Line Outage*:- The critical lines are evaluated when an outage occurs at all the load busses one at a time. The outage is created at a particular load bus by increasing the real and reactive load at that bus to a specifies value.

V. CASE STUDY

This case study is for IEEE 14 bus system. In this section we have done PMU placement, state estimation and system voltage monitoring.

For PMU placement, we use the method explained in section II (E). The generalized MATLAB coding for a bus system is implemented. We have considered four cases as follows: [3]

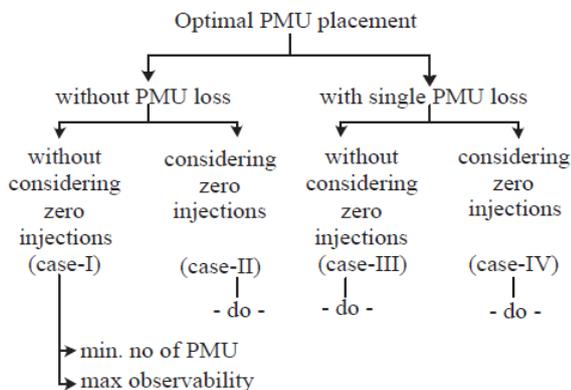


Fig. 5.1 Cases for PMU placement

The results are as follows:

TABLE I

Results of PMU placement

Case	No. of PMU	PMU positions	Initial SORI	Final SORI
Without zero injection	4	2,6,7,9	14	19
Zero injection consideration	3	2,6,9	15	15
PMU outage without zero injection	9	2,4,5,6,7,8,9,11,13	34	39
PMU outage with zero injection	7	2,4,5,6,9,11, 13	31	33

For the state estimation and voltage monitoring we use the result of the case without PMU outage and without zero injection. For state estimation algorithms graphs have been plotted between error index and % redundancy[5] for 5 sets of measurements-42,52,62,72 and 82.

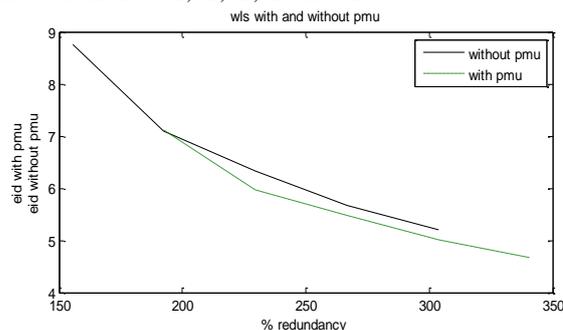


Fig. 5.2 Comparison of WLS with and without PMU

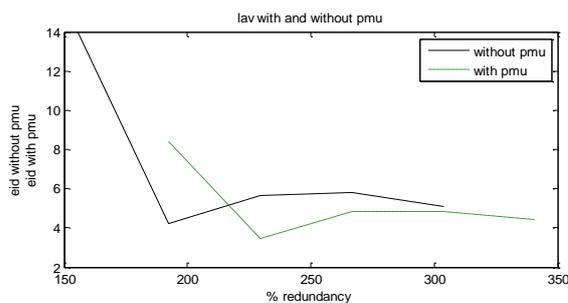


Fig. 5.3 Comparison of LAV with and without PMU

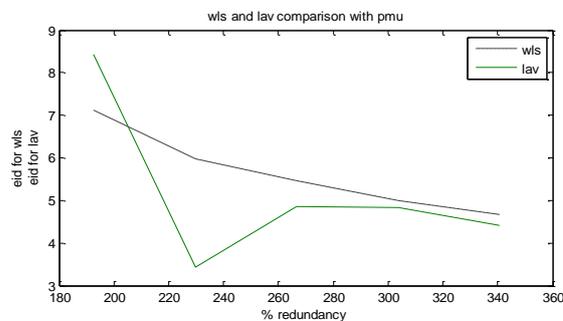


Fig. 5.4 Comparison of WLS and LAV with PMU

For system voltage monitoring graphs have been plotted for both the indices under different contingencies Single load change with real load only is performed at bus 4. Bus 7 was chosen randomly for single load change with reactive load only. Single load change with real and reactive load was performed at bus 10. Busses 11, 12 and 14 were selected randomly for performing multiple load change.

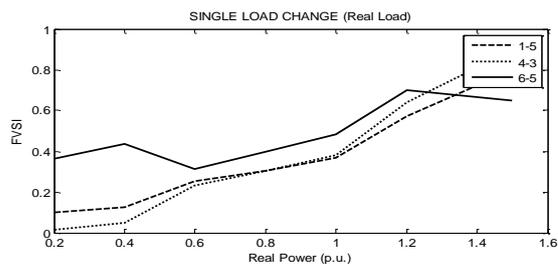


Fig. 5.5 single load change with real load only (FVSI)

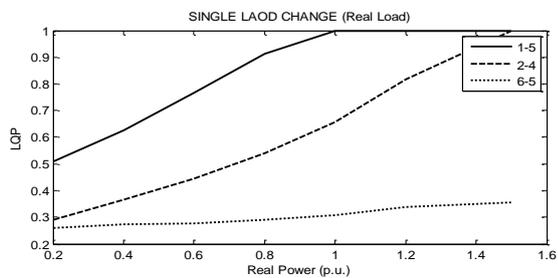


Fig. 5.6 Single load change with real load only (LQP)

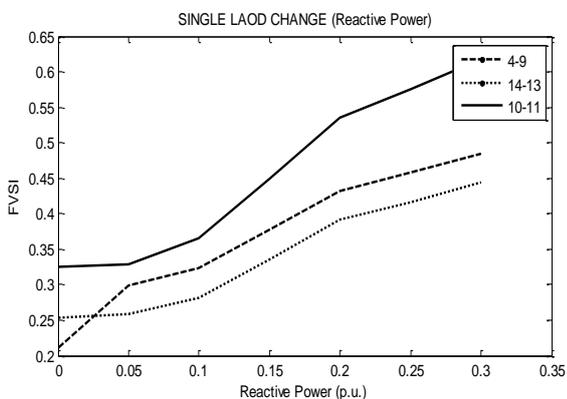


Fig. 5.7 Single load change with reactive load only (FVSI)

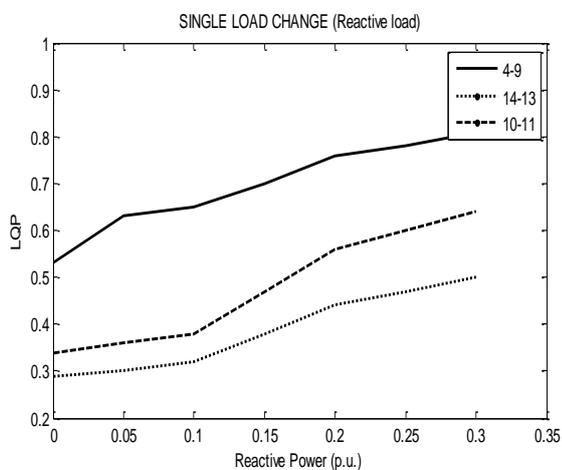


Fig. 5.8 Single load change with reactive load only (LQP)

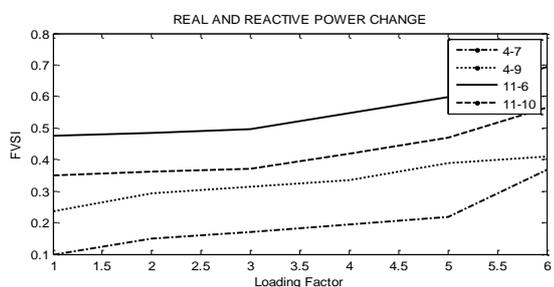


Fig. 5.9 Single load change with real and reactive load (FVSI)

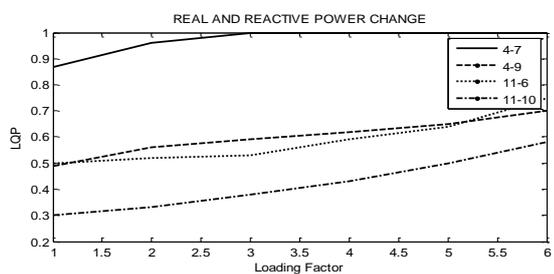


Fig. 5.10 Single load change with real and reactive load (LQP)

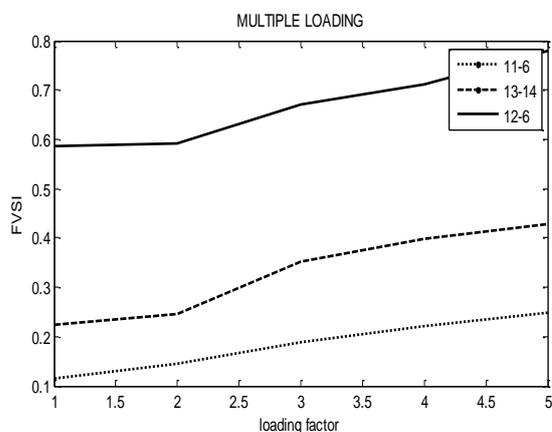


Fig. 5.11 Multiple load change (FVSI)

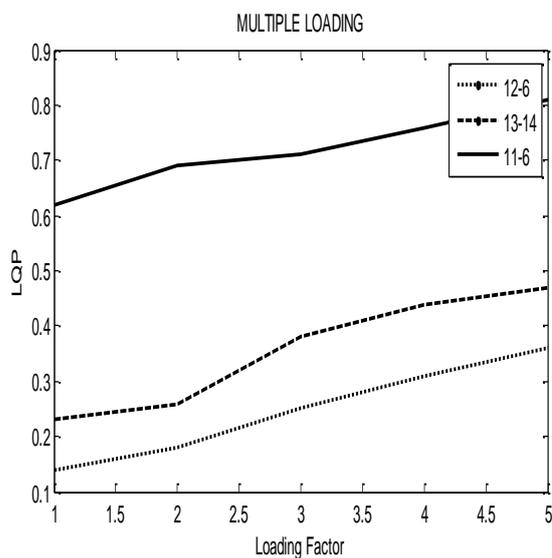


Fig. 5.12 Multiple load change (LQP)

The transmission line outage has been performed on busses 9 and 14 and the results have been tabulated for FVSI values.

TABLE II  
Outage at bus 9

FROM BUS	TO BUS	FVSI
1	5	1
2	3	0.330
4	9	1
6	12	0.424
9	14	1
12	13	0.297
10	11	1
12	6	0.415
2	1	0.386
7	4	0.868
13	6	1
10	9	1

TABLE III Outage at bus 14

FROM BUS	TO BUS	FVSI
1	5	0.96
2	3	0.008
4	9	0.993
6	12	0.584
9	14	1
12	13	0.457
10	11	1
12	6	0.58
2	1	1
7	4	0.377
13	6	1
10	9	0.702

VI. CONCLUSION

1. Optimal placement of PMU is done for four cases namely,
  - a. Without PMU outage
    - i. Without considering zero injection buses
    - ii. Considering zero injection buses
  - b. With PMU outage
    - i. Without considering zero injection buses
    - ii. Considering zero injection buses
2. Optimal PMU placement include
  - a. Minimum number of PMUs
  - b. Optimal positions of PMU with complete and maximum observability.
3. If the zero injection buses are considered the number of PMUs required are reduced when compared with the normal case (without considering zero injection buses). Hence, this is an advantage of this case.
4. Optimal PMU placement has multiple solutions. Two indices bus observability index(BOI) and system observability redundancy index (SORI) are proposed and solution with maximum SORI outcores other solutions. This is shown by improvement from initial SORI to final SORI.

This paper also focused on comparing the state estimation algorithms when there is loss of data from measurement devices in power system. It helps in selecting a better state estimation algorithm during contingencies. It was found that Least Absolute Value algorithm was the best at most of the cases with data loss. The inclusion of few phasor measurements in two of the state estimation algorithms also showed the impact of phasor measurements on the performance of the algorithms. The impact was good at most of the data redundancy levels. Based on the value of FVSI or LQP, an idea can be achieved about the stressful situation of the line. The contingencies included single load change with real power and reactive power only as well as including both, multiple load change and transmission line outages. The voltage collapse prediction method using line stability indices of FVSI and LQP for automatic contingency ranking has shown many advantages. The method is simple and requires very less computational effort. Human error can also be eliminated. Apart from its speed and accuracy, the method is flexible enough for simulating any type of load

modifications in the network as long as the system is in stable state.

REFERENCES

1. A. Abur and F. H. Magnago, "Optimal meter placement for maintaining observability during singlebranch outages," *IEEE Transactions on Power Systems*, vol. 14, no. 4, pp. 1273–1278, Nov. 1999.
2. B. Xu and A. Abur, "Observability analysis and measurement placement for systems with PMUs," in *IEEE Power Systems Conference and Exposition*, vol. 2, Oct. 2004, pp. 943–946.
3. Sanjay Dambhare, DeveshDua, Rajeev Kumar Gajbhiye, S. A. Soman, "Optimal Zero Injection Considerations in PMU Placement: An ILP Approach," 16th PSCC, Glasgow, Scotland, July 14-18, 2008.
4. Thesis by RoozbehEmami, "Enhancement of network monitoring & security analysis using phasor measurement units"
5. S.Kamireddy, "Comparison of State Estimation Algorithms Considering Phasor Measurement Units And Major And Minor Data Loss", M.S. Thesis, *Mississippi State University*, December 2008
6. K.D.Jones, "Three-Phase Linear State Estimation With Phasor Measurements", M.S. Thesis, *Virginia Polytechnic Institute and State University*, May 2011
7. Matlab user Manual obtained from <http://www.mathworks.com/access/helpdesk/help/helpdesk.html>
8. Renuga Verayiah, Izham Zainal Abidin, "A Study on Static Voltage Collapse Proximity Indicators", 2nd IEEE International Conference on Power and Energy (PECon 08), December 1-3, 2008, Johor Baharu, Malaysia.
9. I. Musirin, T.K.A Rahman, "Novel Fast Voltage Stability Index (FVSI) for Voltage Stability Analysis in Power Transmission System," 2002 Student Conference on Research and Development Proceedings, Shah Alam, Malaysia, July 2002.
10. A. Mohamed, G.B. Jasmon, S. Yusoff, "A Static Voltage Collapse Indicator using Line Stability Factors," *Journal of Industrial Technology*, Vol. 7, N1, pp. 73-85, 1989