

# Calculation of Induction Motor Model Parameters using Finite Element Method

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**Abstract:** The paper attempts to model a three phase squirrel cage induction motor and using Finite Element Method to obtain the finite element field solutions. The linear time harmonic vector field potential solution is used for the inductance determination. The Finite Element Analysis software used is FEMM.

**Index Terms:** FEM, FEMM, Finite Element Method, Induction Motor, Motor Modelling, Motor Parameters.

## I. INTRODUCTION

Finite Element Methods are widely used today to correctly compute operating properties and characteristics of electrical machines without physically constructing a prototype, thus saving on costs and enabling us to try out changes to parameters so as to design a machine with the optimum parameters. There are varied tools for finite element analysis, the more popular ones being AnsysMultiphysics, COMSOL Multiphysics, ElecNet etc.

The FEM methods have been successfully used in the past to design synchronous permanentmagnet machines and synchronous generators. The two main methods used are as follows. Rotor and stator currents are calculated using the circuit models of induction motor. This is known as the circuit approach. [1][2] The alternative approach, involves calculating the stator currents like the previously mentioned authors but they calculate rotor currents directly as eddy currents. [3] [4]

Both approaches have their advantages and disadvantages but by combining them it has been possible to eliminate the principle disadvantage of the consideration of the actual current distribution in rotor bars.

The purpose of this paper is to attempt to describe a method for the determination of the parameters of the two phase induction motor model under different operating conditions. The choice of magnetic excitations will be confirmed by the derivation of a corresponding two axis equivalent circuit model in the general frame of reference that considers saturation as well. This will make it possible to determine the magnetizing inductance as well as the real values of the stator and rotor leakage inductances which cannot be separated arbitrarily with classic measurements. The method is not completely new [5] but eliminates some of

the drawbacks of the current methods and so may be considered as an alternative approach. [6]

## II. MOTOR MODEL

We first decide on a motor model to be used. The model shown in figure 1 is a simple per-phase model used for the purpose of this paper.

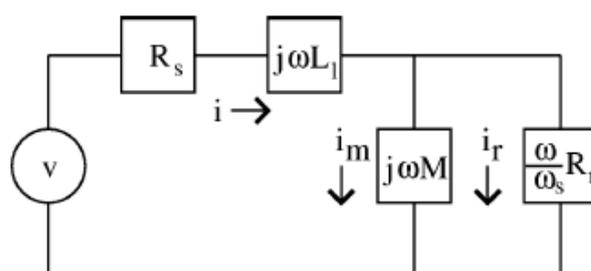


Fig: 1

The model represents a single phase of an induction motor at steady state i.e. constant frequency and constant speed. The symbols in the model represent the following:

$R_s$  = Stator Resistance

$L_l$  = Inductance

$M$  = Flux linking Stator to Rotor

$R_r$  = Rotor Resistance

$\omega$  = Applied electrical frequency

$\omega_s$  = Difference between electrical frequency of Stator and mechanical frequency of Rotor (Slipfrequency) for a motor which has a stator with p pole pairs, the slip frequency can be calculated as:

$$\omega_s = \omega - p\omega_r$$

where  $\omega_r$  = mechanical rotor speed.

The voltage source (v) shown in the per-phase voltage and i is the per phase current, both in RMS.

Now to perform the modeling of the motor, the parameters simply need to be found.

## III. MOTOR EQUATIONS

The motor impedance (Z) can be easily calculated from figure1.

$$Z = R_s + j\omega \left[ L_l + \frac{M}{1 + j\tau\omega_s} \right]$$

Where  $\tau$  is the rotor time constant ( $=M/R_r$ )

Voltage is then related to the current as

$$v = Z i$$

Per phase flux linkage:

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One can note that the second term in the motor impedance is linked with  $j\omega$ . This implies that this is a voltage contribution that has to do with the variation of the flux at a frequency  $\omega$ . We can then imply that the flux  $\phi$ , linking any phase is:

$$\phi = \left[ L_1 + \frac{M}{1 + j\tau\omega_s} \right] i$$

Dividing by current, we can obtain a slip frequency dependant inductance. Separating this into real and complex components we have,

$$L(\omega_s) = \left( L_1 + \left( \frac{M}{1 + (\tau\omega_s)^2} \right) \right) - j \left( \frac{\tau\omega_s M}{1 + (\tau\omega_s)^2} \right)$$

Since flux linkage is easily calculated with high accuracy using FEMM, this equation will later enable us to use calculated flux linkages at different motor slip frequencies to identify the parameters of the motor.

#### IV. IDENTIFICATION OF MODEL PARAMETERS VIA FINITE ELEMENT METHOD

The formulas developed above can be used as the basis for a finite element identification of motor parameters. Using FEMM, the rotor does not move, however this does not present a problem as far as parameter identification goes as in the zero slip case, the frequency simply degenerates to  $\omega_s = \omega$

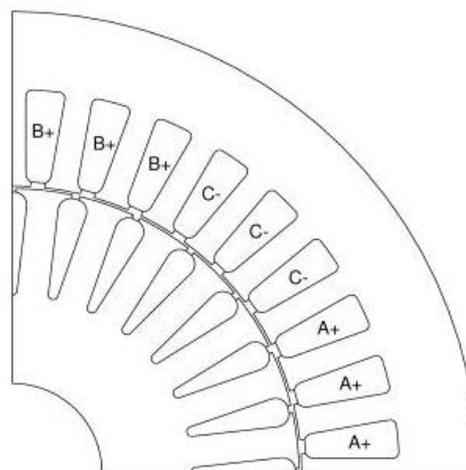
Now, one could either attempt to base parameter identification on torque results analyzed by using a constant stator current over a range of frequencies. The alternative method and the one which we will use is to attempt to fit against inductance results. We choose this approach because the program obtains flux linkage by performing a volume integral that is closely related to the computation of stored energy, a quantity that FEMM computes with an extremely high degree of accuracy.

The approach, thus, is as follows:

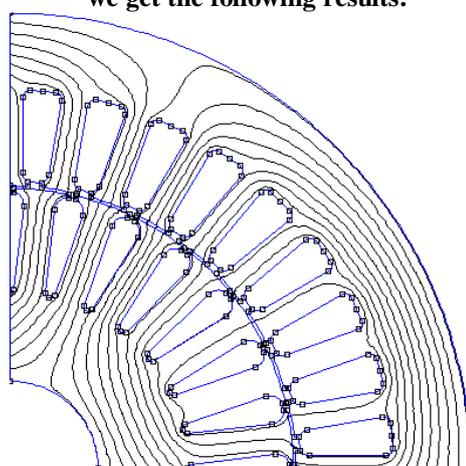
- To formulate a finite element model of interest
- Apply 3 phase currents to the stators over a range of frequencies
- For each analysis, evaluate the flux linkage of one phase that is required to fit the parameters of that model
- Perform a regression analysis to get the parameter values

#### V. FINITE ELEMENT MODEL

The particular motor of interest is intended to be a 2 HP motor running of a 220 V<sub>rms</sub> line-to-line, 50 Hz, 3-phase supply. This motor is a 4-pole machine (*i.e.*  $p = 2$ ), implying that it will be running at slightly less than 1500 RPM. The winding configuration for one pole of the machine is pictured below in Figure 2. There are a total of 36 slots on the stator and 28 slots on the rotor. A total of 44 turns sit inside each slot (*i.e.* so that a phase current of 1 A would place a total of 44 Amp\*Turns in a slot). The rotor's diameter is 80 mm, and the clearance between the rotor and stator is 0.375 mm. The length of the machine in the into-the-page direction is 100 mm.



**Fig 2: Example Induction Motor Winding configuration Performing the analysis in FEMM to get the flux linkage, we get the following results:**



**Fig 3: Plot of Real component of A**

**Table 1: Inductance values at different frequencies**

Frequency, Hz	Real Inductance	Imaginary Inductance
0.25	0.3113897	-0.078562
0.5	0.2644535	-0.130207
0.75	0.2126343	-0.15379
1	0.1683203	-0.158362
1.25	0.1342043	-0.15323
1.5	0.1088912	-0.144139
1.75	0.0902097	-0.13398
2	0.076302	-0.124052
2.25	0.0657982	-0.114864
2.5	0.057736	-0.106568
2.75	0.0514475	-0.099154
3	0.0464669	-0.092553

#### VI: OBTAINING MOTOR PARAMETERS FROM FEM RESULTS



Regression analysis is performed on the results to obtain the inductance.

The imaginary part of the inductance is obtained from the above equation as:

Taking  $c_1 = \tau M$  and  $c_2 = \tau^2$ , we get:

$$(\omega_s)c_1 + (L_1\omega_s^2)c_2 = -L_1$$

Thus we get a linear equation with  $c_1$  and  $c_2$  as parameters. Values of these parameters need to be obtained that fit all the values of  $\omega_s$  and  $L_1$ . The 12 equations are represented using the following matrices:

$$m = [\omega_s L_1 \omega_s^2]$$

$$b = [-L_1]$$

thus we get:

$$m \begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = b$$

This is easily solved in MATLAB to get  $c_1$  and  $c_2$  using:

$$\begin{Bmatrix} c_1 \\ c_2 \end{Bmatrix} = (m^T m)^{-1} m^T b$$

Thus we get  $c_1 = 0.0523$  and  $c_2 = 0.0272$  and from that we can imply that:

$$\tau = 0.1649 \text{ sec}$$

$$M = 0.3171 \text{ H}$$

Similarly, we can use the real part of inductance for calculating the leakage inductance. We already know the values of  $M$  and  $\tau$  so we simply rearrange to get the leakage inductances and take the average.

Equation of the real part of the inductance:

$$Re(L) = L_r = \left( L_1 + \frac{M}{1 + (\omega_s)^2} \right)$$

Rearranged to find  $L_1$ :

$$L_1 = L_r - \frac{M}{1 + (\omega_s)^2}$$

The average  $L_1$  is obtained as 0.0158H from the data given above.

Thus we successfully obtain the motor parameters as:

$$M = 0.3171 \text{ H}$$

$$\tau = 0.1649 \text{ s}$$

$$L_1 = 0.0158 \text{ H}$$

## VII. CONCLUSION

With the proposed magnetic field analysis based on the finite element method, it is possible to determine the motor parameters. The proposed method ensures the required separation of the rotor and stator leakage inductances in both saturated and unsaturated case. The described model of the induction motor with the complete known set of parameters can be used in the advanced control synthesis of the induction motors.

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