Spectrum Efficiency for Spatially Correlated MIMO OSFBC-OFDM Systems over Various Adaptation Policies

Ch. Siva Rama Krishna, Vidhyacharan Bhaskar

Abstract: In this paper, closed-form expressions for capacities per unit bandwidth for spatially correlated multilayer MIMO-OFDM systems employing Orthogonal Space-Frequency Block Coding (OSFBC) over multipath frequency-selective fading channels are derived for optimal power and rate adaptation, optimal rate adaptation with constant transmit power, channel inversion with fixed rate, and truncated channel inversion adaptation policies. A Signal to Noise Ratio (SNR) based user selection scheme is considered. Closed-form expressions are derived for spatially correlated OSFBC-OFDM system. Optimal power adaptation policy provides the highest capacity over the other adaptation policies. Capacity gains are more prominent for optimal rate adaptation with constant transmit power policy as compared to other adaptation policies.

Keywords: Orthogonal space-frequency block coding; optimal power adaptation; optimal rate adaptation with constant transmit power; channel inversion with fixed rate; truncated channel inversion; outage probability.

I. INTRODUCTION

Multiple-Input–Multiple-Output (MIMO) technology has been recognized as a key approach for improving system performance and channel capacity of wireless communication systems [1]. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) has been considered as a promising technique in future broadband wireless communications. In particular, the MIMO-OFDM system is considered as an attractive solution for broadband wireless communications [2]. A significant advantage of MIMO-OFDM systems is that they allow rate and power allocation (through adaptive modulation) and dynamic resource allocation to the system [3], [4]. Most of the related work that has been done for variable-rate and variable-power allocation in these systems introduces high system complexity, particularly using the well-known water-filling technique [5]–[9]. In [10], the authors proposed performance analysis of scheduling schemes for Rate-Adaptive MIMO OSFBC-OFDM systems. Capacity analysis of multipath fading channels becomes an important and fundamental issue in the design and study of new generation of wireless communication systems due to scarce radio spectrum available and due to the rapidly growing demand for wireless services. Accordingly there have been papers dealing with channel capacity for Rayleigh, Nakagami, Rician, and Generalized Gamma fading channels [11], [12]. The channel capacity per unit bandwidth for different adaptation policies over various fading channels with different diversity schemes is discussed in [13], [14].

This paper derives closed-form expressions for the capacities per unit bandwidth for spatially correlated MIMO-OFDM systems employing OSFBC over multipath frequency-selective fading channels for optimal power adaptation, optimal rate adaptation with constant transmit power, channel inversion with fixed rate, and truncated channel inversion adaptation policies. An SNR based user selection scheme is assumed.

This paper organized as follows. Section 2 derives closed form expressions for capacities per unit bandwidth for spatially correlated MIMO OSFBC-OFDM system for the optimal power and rate adaptation policy, optimal rate adaptation with constant power policy, channel inversion with fixed rate policy, and truncated channel inversion with fixed rate policy respectively. Section 3 presents numerical results for the comparison of capacities per unit bandwidth for various adaptation policies. Finally, Section 4 presents the conclusion.

II. SPECTRUM EFFICIENCY FOR SPATIALLY CORRELATED OSFBC-OFDM SYSTEM

The total energy of the symbol transmitted through the \( n_T \) antennas can be normalized to \( n_T \), therefore the instantaneous SNR per symbol at the receiver of the \( k^{th} \) user can be expressed as

\[
\gamma[k,n] = \frac{\mathbf{v}}{P_{\text{Tx}} R_2} ||H[k,n]||_F^2
\]

where \( \mathbf{v} = \frac{\mathbf{1}}{N_0} \) is the average received SNR per antenna, and \( R_2 \) is the OSTBC code rate [15]. The antennas at the base station and the antennas at the users end have been assumed to be correlated. Similar to the model used in [16], the spatially correlated MIMO channel between the \( k^{th} \) user and the base station on the \( n^{th} \) frequency slot can be modeled as

\[
H[k,n] = R_2^{\frac{1}{2}} [k,n] W[k,n] R_2^{\frac{1}{2}} [k,n],
\]

where \( R_2^{\frac{1}{2}} \) is an \( n_R \times n_R \) matrix representing the correlation between the user and...
receive antennas at the \(k\)th user station, and \(R_z^k\) is a \(n_R \times n_T\) matrix representing the correlation between transmit antennas at the base station. Here, \(W[k,n]\) is a \(n_R \times n_T\) matrix, where its elements are i.i.d. complex Gaussian random variables \(N(0,0.5)\) per dimension.

In this model, the channel covariance matrix is expressed as \(R = R_z^k \otimes R_c^k\), where \((\cdot)^T\) is the transpose operator, and \(\otimes\) denotes the Kronecker product. Assume that there exists \(z\) distinct and nonzero eigenvalues \((\lambda_k, i = 1, 2, ..., z)\) for the covariance matrix \(R\) repeated \(t\) times, such that \(\sum_{i=1}^z \lambda_i = \text{rank}(R)\). Then, the PDF of the SNR is given as [10]

\[
f_y(y) = \sum_{i=1}^z \mu_{ij} \exp \left( -\frac{\eta}{\lambda_i} \right) \exp \left( \frac{-\eta y}{\lambda_i} \right) \tag{2}
\]

where \(\eta = n_T R_z^k\) and

\[
\mu_{ij} = \frac{\mu_{ij}}{(\eta y)^{\lambda_i}} \times \left[ \prod_{k=1}^z \left( 1 + \frac{s}{\lambda_k} \right)^{\lambda_i} \right]_{x=\lambda_i}^{y=q}
\]

where \(\frac{\partial^x y^{-j}}{\partial s^j}\) denotes the derivate of order \(w_j - j\) with respect to \(s\) for \(j = 1, 2, \ldots, w_i\).

Optimal power and rate adaptation (OPRA) policy

Given an average transmit power constraint, the channel capacity of a fading channel with received SNR distribution, \(f_y(y)\), and optimal power and rate adaptation policy (\(\mathcal{C}_{\text{OPRA}}\) bits/s) is given as [13]

\[
\mathcal{C}_{\text{OPRA}} = B \left( \int_{y_0}^{\gamma^*} \log_2 \left( \frac{y}{y_0} \right) f_y(y) dy \right) \tag{3}
\]

where \(B\) (Hz) is the channel bandwidth, and \(y_0\) is the optimal cutoff level SNR below which data transmission is suspended. This optimal cutoff must satisfy the equation [17]

\[
\int_{y_0}^{\gamma^*} \left( \frac{1}{y_0} - 1 \right) f_y(y) dy = 1
\]

To achieve the capacity in (3), the channel fade level must be tracked at both the receiver and transmitter, and the transmitter to adapt its power and rate accordingly, allocating higher power levels and rates for good channel conditions (large), and lower power levels and rates for unfavorable channel conditions (small). Since no data is sent when \(y < y_0\), the optimal policy suffers a probability of outage, \(P_{\text{out}}\), equal to the probability of no transmission, given by

\[
P_{\text{out}} = \int_{y_0}^{\gamma^*} f_y(y) dy = 1 - \int_{0}^{\gamma^*} f_y(y) dy \tag{5}
\]

Substituting (2) into (4), we find that \(y_0\) must satisfy

\[
\sum_{i=1}^z \mu_{ij} \frac{\lambda_i}{y_0} - \mu \lambda_i (j - 1, \nu_y) = \Gamma(m) \tag{6}
\]

where \(m = n_T R_z^k\), \(\Gamma(\cdot) = \int_0^\infty t^{x-1} e^{-t} dt\) is the gamma function (from [18], section 8.35, page 890), and \(\Gamma^c(\cdot, \cdot) = \int_0^\infty t^{x-1} e^{-t} dt\) is the complementary incomplete gamma function (from [18], section 8.35, page 890).

Let \(x = y_0\) in (6), we have

\[
g(x) = \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \frac{\lambda_i (j - 1, \nu_y) - \Gamma(m)}{x} = \mu \lambda_i (j - 1, \nu_y) - \Gamma(m) \tag{7}
\]

Then

\[
\frac{dg(x)}{dx} = - \frac{1}{x^2} \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \frac{\lambda_i (j - 1, \nu_y)}{x} < 0 \forall x > 0
\]

Moreover, from (7) \(\lim_{x \to 0} g(x) = +\infty\) and \(\lim_{x \to \gamma^*} g(x) = -\Gamma(m) < 0\). Thus, it can be concluded that there is a unique \(\gamma_0\) for which \(g(\gamma_0) = 0\), which satisfies (6). Substituting (2) into (3), the spectrum efficiency using OPRA policy, \(\mathcal{C}_{\text{OPRA}} / B\), is given by

\[
\mathcal{C}_{\text{OPRA}} / B = \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \mu \frac{\lambda_i}{y_0} \left[ 1 - \frac{1}{\lambda_i} \right] \int_{0}^{\gamma^*} \log_2 \left( \frac{y}{y_0} \right) y^{j-1} e^{\frac{-\eta y}{\lambda_i}} dy \tag{8}
\]

where \(\gamma^*\) is the average received SNR.

Making change of variables in the integral of (8), where \(t = \frac{y}{\gamma^*}\) and \(dt = \frac{dy}{\gamma^*}\)

\[
\mathcal{C}_{\text{OPRA}} / B = \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \mu \frac{\log_2 \zeta \frac{\gamma^*}{y_0}}{\Gamma(j)} \zeta_{j,y} \tag{9}
\]

Eq. (9) gives the spectrum efficiency, \(\mathcal{C}_{\text{OPRA}} / B\) [bits/Hz], under OPRA policy for spatially correlated MIMO-OFDM systems employing OSFBC over multipath frequency-selective fading channels. Substituting (2) into (5), the expression for outage probability is given as

\[
P_{\text{out}} = 1 - \int_{y_0}^{\gamma^*} \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \frac{\lambda_i (j - 1, \nu_y)}{y_0} \exp \left( -\frac{\eta y}{\lambda_i} \right) dy \tag{10}
\]

\[
= 1 - \left( \int_{y_0}^{\gamma^*} \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \frac{\lambda_i (j - 1, \nu_y)}{y_0} \exp \left( -\frac{\eta y}{\lambda_i} \right) dy \right) \tag{11}
\]

Making change of variables in the integral of (11) where \(t = \frac{y}{\gamma^*}\) and \(dt = \frac{dy}{\gamma^*}\),

\[
P_{\text{out}} = 1 - \sum_{i=1}^z \sum_{j=1}^z \mu_{ij} \Gamma(j) \left[ \lambda_i (j, \nu_y) / \lambda_i \right] \tag{12}
\]

where \(\lambda_i (x, \alpha) = \int_0^\alpha t^{x-1} e^{-t} dt\) is the incomplete gamma function (from [18], section 8.35, page 890).
Optimal Rate Adaptation (ORA) with constant transmit power policy

Adapting the code rate to channel conditions with a constant transmit power, the channel capacity, \( \langle C \rangle_{\text{ora}} \) [bits/s] is given as [13]

\[
\langle C \rangle_{\text{ora}} = B \int \log_2 \left( 1 + \gamma f_0(y) \right) dy.
\]  

(13)

Substituting (2) into (13), the spectrum efficiency using ORA policy, \( \langle C \rangle_{\text{ora}} / B \), is given by

\[
\langle C \rangle_{\text{ora}} = B \int \log_2 \left( 1 + \gamma \sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \frac{\mu_{ij} \gamma^{j-1}}{(j-1)! \Gamma(j)} \right) \frac{\eta^i \gamma^j}{\lambda_i} \exp \left( -\frac{\eta \gamma}{\lambda_i} \right) d\gamma.
\]  

(14)

Simplifying and rearranging (14), we have

\[
\frac{\langle C \rangle_{\text{ora}}}{B} = \sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \frac{\mu_{ij} \gamma^{j-1}}{(j-1)! \Gamma(j)} \log_2 \left( \frac{\eta^i \gamma^j}{\lambda_i} \right) \exp \left( -\frac{\eta \gamma}{\lambda_i} \right)
\]  

(15)

where \( \zeta = \frac{\eta}{\gamma \lambda_i} \).

Channel Inversion with Fixed Rate (CIFR) policy

In this policy, the transmitter adapts its power to maintain a constant SNR at the receiver (i.e., inverts the channel fading). This technique uses fixed-rate modulation and a fixed code design since the channel after channel inversion appears as a time-invariant AWGN channel. As a result, CIFR policy is the least complex technique to implement, assuming good channel estimates are available at the transmitter and receiver. The channel capacity with CIFR policy, \( \langle C \rangle_{\text{cifr}} \) [bps] is given as [13]

\[
\langle C \rangle_{\text{cifr}} = B \log_2 \left( 1 + \frac{1}{J_0(y) \gamma} \right) \left( 1 - P_{\text{out}} \right)
\]  

(16)

Substituting (2) into (16), the capacity per unit bandwidth for total channel inversion with spatial correlation on OSFBC-OFDM system is given by

\[
\frac{C_{\text{cifr}}}{B} = \log_2 \left[ 1 + \left( \sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \frac{\mu_{ij} \gamma^{j-1}}{(j-1)! \Gamma(j)} \right) \frac{\eta^i \gamma^j}{\lambda_i} \exp \left( -\frac{\eta \gamma}{\lambda_i} \right) \right]
\]  

(17)

Making change of variables in the integral of (17), where \( t = \frac{\gamma}{\eta} \) and \( dt = \frac{\gamma}{\eta} dy \),

\[
\frac{C_{\text{cifr}}}{B} = \log_2 \left[ 1 + \frac{\Gamma(j)}{\sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \mu_{ij} \eta \gamma} \right]
\]  

(18)

Truncated channel Inversion with Fixed Rate (TIFR) policy

Channel inversion with fixed rate suffers capacity penalty for deep channel fades since a large amount of the transmitted power is needed to compensate for the channel fades. Another approach is to use a modified inversion policy which inverts the channel fading only above a fixed cutoff fade depth \( \gamma_0 \). The capacity with this TIFR policy, \( \langle C \rangle_{\text{tifr}} \) [bps] is given as [13]

\[
\langle C \rangle_{\text{tifr}} = B \log_2 \left( 1 + \frac{1}{J_0(y) \gamma} \right) \left( 1 - P_{\text{out}} \right)
\]  

(19)

where \( P_{\text{out}} \) is given by (12). The cutoff of level, \( \gamma_0 \), can be selected to achieve a specified outage probability, or alternatively, to maximize \( \langle C \rangle_{\text{tifr}} \). Substituting (2) into (19), the spectrum efficiency, \( \langle C \rangle_{\text{tifr}} / B \), is given as

\[
\frac{C_{\text{tifr}}}{B} = \log_2 \left[ 1 + \left( \sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \frac{\mu_{ij} \gamma^{j-1}}{(j-1)! \Gamma(j)} \right) \frac{\eta^i \gamma^j}{\lambda_i} \exp \left( -\frac{\eta \gamma}{\lambda_i} \right) \right] \left( 1 - P_{\text{out}} \right)
\]  

(20)

Making change of variables in the integral of (20) where \( t = \frac{\gamma}{\eta} \) and \( dt = \frac{\gamma}{\eta} dy \),

\[
\frac{C_{\text{tifr}}}{B} = \log_2 \left[ 1 + \left( \sum_{i=1}^{W_1} \sum_{j=1}^{W_2} \frac{\mu_{ij} \gamma^{j-1}}{(j-1)! \Gamma(j)} \right) \frac{\eta^i \gamma^j}{\lambda_i} \exp \left( -\frac{\eta \gamma}{\lambda_i} \right) \right] \left( 1 - P_{\text{out}} \right)
\]  

(21)

where \( P_{\text{out}} \) is given by (12) and \( \mu = \frac{\gamma_0}{\eta \lambda_i} \).

III. NUMERICAL RESULTS

Fig. 1 shows the channel capacity per unit bandwidth curves for spatial correlation on MIMO OSFBC-OFDM system for correlation coefficient \( \rho = 0 \) and \( \rho = 0.75 \), under optimal power and rate adaptation policy. These curves are obtained using the closed form expression, (9). When the correlation coefficient increased from \( \rho = 0 \) to \( \rho = 0.75 \), in 2 X 2 (\( \gamma \) X \( \eta \)) case capacity is improved by 0.2976 bps/Hz, and for 3 X 3 case capacity is improved by 0.9862 bps/Hz.

Fig. 2 shows the channel capacity per unit bandwidth curves for spatial correlation on OSFBC-OFDM system for different correlation coefficient under ORA policy. These curves are obtained using closed form expression (15). When the correlation coefficient increased from \( \rho = 0 \) to \( \rho = 0.75 \), in 2 X 2 (\( \gamma \) X \( \eta \)) case capacity is improved by 0.0923 bps/Hz, and for 3 X 3 (\( \gamma \) X \( \eta \)) case capacity is improved by 0.1342 bps/Hz.
Fig. 3 shows the channel capacity per unit bandwidth curves for spatial correlation on OSFBC-OFDM system for different correlation coefficients under CIFR policy. These curves are obtained using the closed form expression, (18). When the correlation coefficient increased from $\rho = 0$ to $\rho = 0.75$, in $2 \times 2$ ($n_\tau \times n_\gamma$) case capacity is improved by 0.1916 bps/Hz, and for $3 \times 3$ ($n_\tau \times n_\gamma$) case capacity is improved by 0.2970 bps/Hz.

Fig. 4 shows the channel capacity per unit bandwidth curves for spatial correlation on MIMO OSFBC-OFDM system for different correlation coefficients under TIFR policy. These curves are obtained using the closed form expression, (21).
When the correlation coefficient increased from $\rho = 0$ to $\rho = 0.75$, in $2 \times 2$ case, the capacity of OFDM system is improved by $0.2192 \text{ bps/Hz}$, and for $3 \times 3$ case, capacity is improved by $0.2801 \text{ bps/Hz}$. Fig. 5 and Fig. 6 show the calculated channel capacity per unit bandwidth as a function of $\gamma$ for the different adaptation policies for correlation coefficient $\rho = 0$ and $\rho = 0.75$, for spatial correlation on MIMO OFDM system. These curves are obtained using the closed form expressions (10), (16), (19) and (22). From Fig. 5 and 6, it can be observed that OPRA policy yields a significant increase in capacity as compared to ORA and CIFR policies. The spectral efficiency curve obtained using TIFR policy lies in between the curves obtained for OPRA policy and ORA policy. In the multiuser case spectral efficiency for correlated case is higher than that for uncorrelated case.

IV. CONCLUSIONS

Closed form expressions for the spectral efficiency for the three adaptation policies are derived for spatially correlated OSFBC-OFDM system. Capacity improves with an increase in the correlation coefficient and an increase in the number of antennas. Channel capacity with OPRA policy, $C_{\text{opra}}$, outperforms the channel capacities with other policies, $C_{\text{ora}}$, $C_{\text{cifr}}$, and $C_{\text{tifr}}$. Capacity improvement shown by OPRA policy is relatively higher when compared to the other policies. ORA policy shows the least spectrum efficiency as compared to other policies. Thus, OPRA policy is best suited for all three adaptation policies. Spatial correlation is beneficial for the multiuser MIMO systems.

REFERENCES


AUTHORS PROFILE

Siva Rama krishna received his B.TECH. degree in Electronics and Communication Engineering from Mother Teresa Institute of Science & Technology, JNTUH, University in India in 2009, M.TECH. degree in Communication Systems from S.R.M. University, Chennai in 2011. He is currently working as an Assistant Professor in the Department of Electronics and Communication Engineering at SVIST Engineering College, Tiruvuru, India. Since 2011, his research interests include wireless communications, error control coding, diversity combining and MIMO.

Vidyacharan Bhaskar received the B.Sc. degree in Mathematics from D.G. Vaishnav College, Chennai, India in 1992, M.E. degree in Electrical & Communication Engineering from the Indian Institute of Science, Bangalore in 1997, and the M.S.E. and Ph.D. degrees in Electrical Engineering from the University of Alabama in Huntsville in 2000 and 2002 respectively. During 2002–2003, he was a post-doc fellow with the Communications Research Group at the University of Toronto. From September 2003 to December 2006, he was an Associate Professor in the Département Génie des systèmes d’information et de Télécommunication at the Université de Technologie de Troyes, France. Since January 2007, he is a Professor in the Department of Electronics and Communication Engineering at S.R.M. University, Kattankulathur, India. His research interests include wireless communications, signal processing, error control coding and queuing theory. He has published 48 international journal papers, presented 18 conference papers in various international conferences, published a book on “Adaptive Rate Coding for A-CDMA Systems” in January 2011, and co-authored a book on MATLAB in 1998. Besides, he has 13 other refereed journals currently under review. He is also an active reviewer of refereed journals such as the IEEE Transactions on Communications, IEEE Transactions on Wireless Communications, IEEE Communication Letters, IEEE Transactions on Vehicular Technology, Wireless Personal Communications, International Journal of Network and Computer Applications, International Journal of Applied Mathematical Modeling, International Journal of Computer Communications, and International Journal of Computers and Electrical Engineering. Dr. Bhaskar’s name was recently nominated for inclusion in the upcoming 2011 edition of Who’s Who in the World, which is scheduled for publication in November 2010. Dr. Bhaskar’s name was also recently nominated for inclusion in International Publishing House, Cambridge, UK as one of the “Top 2000 Intellectuals of the 21st Century”, scheduled for publication in late 2011.

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Email: sivaram_chavalam@yahoo.co.in

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