Reliability Analysis of a System using Intuitionstic Fuzzy Sets

M.K.Sharma, Vintesh Sharma, Rajesh Dangwal

Abstract: ***In General fuzzy sets are used to analyze the system Reliability. Present paper attempts to review the fuzzy/possibility tools when dealing with reliability of series-parallel network systems. Various issues of reasoning-based approaches in this framework are reviewed, discussed and compared with the standard approaches of reliability. To analyze the fuzzy system reliability, the reliability of each component of the system is considered as a trapezoidal intuitionstic fuzzy number. Trapezoidal intuitionstic fuzzy number arithmetic operations are also performed to evaluate the fuzzy reliability of the system. A numerical example is also given to illustrate the method.***

Keywords: Fuzzy sets, Intuitionstic Fuzzy sets, Intuitionstic Fuzzy Numbers, Reliability, (α, β)- Cuts, Network System
AMS Subject Classification: - 60K10

I. INTRODUCTION

For the fast technology innovations, new product development is getting much complicated not only its system functions, but also on its system components. Therefore one of the important engineering tasks in design and development of a technical system is the reliability engineering. In [1] Kauffmann and Gupta pointed out that the discipline of reliability engineering encompasses a number of different activities, reliability modeling being the most important one. Network systems are one of the most complicated system products in the real world. Network systems include many different system components in order to integrate sophisticated functions under system command and control. Network system reliability [2] problem is critical and important, because not only its expression into subsystems but also success of any subsystem may collapse.

The reliability of a system is the probability that the system will perform a specified function satisfactorily during some interval of time under specified operating conditions [3]. Traditionally, the reliability of a system behaviour is fully characterized in the context of probability measures, and the outcome of the top event is certain and precise as long as the assignment of basic events are descent from reliable information. However in real life systems, the information may be inaccurate or might have linguistic representation. In such cases the estimation of precise values of probability becomes very difficult. In order to handle this situation, fuzzy approach [4-5] is used to evaluate the failure rate status.

The theory of fuzzy sets [6] (FSs), proposed by Zadeh, has gained successful applications in various fields. However, the membership function of the fuzzy set is a single value between zero and one, which combines the favoring evidence and the opposing evidence. In 1986, Atanassove introduced intuitionistic fuzzy sets [7] (IFSs) which have been found to be very useful to deal with uncertainty information. The concept of the IFSs is a generalization of that of the FSs. Gau and Buehrer [8] presented the concept of vague sets (VSs) But Burillo and Bustince [9] showed that the notion of VSs coincided with that of IFSs. Recently, IFSs have been widely studied and applied in various fields such as decision making problems, medical diagnostics, data system and pattern recognition, reliability analysis of the systems [10].

IFS are being studied and used in different fields of science. Among the research works on these sets we can mention Atanassov [7, 11]; Atanassov and Gargov [11-14]; Ban [15]; Szmidt and Kacprzk [16] proposed the definition of Intuitionstic Fuzzy numbers (IFN) and studied the perturbation of IFN and first properties of the correlation between these numbers. Mitchell [17] considered the problem of ranking of a set of IFNs to define a fuzzy rank and characteristic vagueness factor of each IFN.

In the real world problems, the collected data or system parameters are often fuzzy / imprecise because of incomplete or non-obtainable information, and the probabilistic approach to the reliability of the network system is inadequate to account for such built-in-uncertainties in data. For this reason, the concept of Intuitionstic Fuzzy set has been introduced and formulated in the context of possibility measures. Shu et al [18] used IFS for the failure analysis of the problem of printed circuit board assembly. Mahapatra et.al [19] presented the intuitionstic fuzzy numbers and used to evaluate the fuzzy reliability of the system.

This paper is organized as follows: section 2 represents the basic concept of Fuzzy sets, Intuitionstic Fuzzy sets, (α,β) −cuts. Section 3 deals with the Intuitionstic Fuzzy Numbers, their types and arithmetic operations through (α,β) −cuts. In section 4 Reliability of different networks is calculated through (α,β) −cuts. Section 5 presents the proposed algorithm to evaluate the fuzzy reliability of a series, parallel, series-parallel, parallel-series networks using arithmetic operations based on (α,β) −cuts of IFNs. In section 6 a numerical example is given to illustrate the proposed algorithm. Conclusion and interpretations are given in the last section.

II. BASIC NOTIONS AND DEFINITIONS OF INTUITIONSTIC FUZZY SETS (IFSS)

Fuzzy set theory was first introduced by Zadeh [6] in 1965.

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Let X be universe of discourse defined by $X = \{x_1, x_2 \ldots x_n\}$. The grade of membership of an element $x_i \in X$ in a fuzzy set is represented by real value between 0 and 1. It does indicate the evidence for $x_i \in X$, but does not indicate the evidence against $x_i \in X$. Atanassov [7] in 1984 presented the concept of IFS, and pointed out that this single value combines the evidence for $x_i \in X$ and the evidence against $x_i \in X$. An IFS $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}}(x)$ and a non membership function $\nu_{\tilde{A}}(x)$.

2.1 Definition of Intuitionstic Fuzzy Set: - Let E be a fixed set. An intuitionstic fuzzy set $\tilde{A}$ of E is an object having the form $\tilde{A} = \{< x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) >: x \in E\}$

Where the functions

$\mu_A: E \rightarrow [0\ 1]$ and $\nu_A: E \rightarrow [0\ 1]$ define respectively, the degree of membership and the degree of non-membership of the element $x \in E$ to the set A, which is a subset of E and for every $x \in E,\ 0 \le \mu_A(x) + \nu_A(x) \le 1$.

When the universe of discourse E is discrete, an IFS $\tilde{A}$ can be written as

$$\tilde{A} = \sum_{i=1}^{n} [\mu_A(x),\ 1 - \nu_A\ (x)]/x, \forall\ x_i \in E$$

An IFS $\tilde{A}$ with continuous universe of discourse E can be written as

$$\tilde{A} = \int_E\ [\mu_A(x), 1 - \nu_A\ (x)]/x, \forall x_i \in E$$

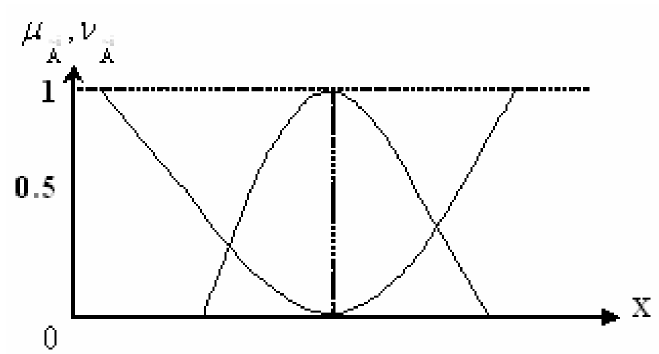

Fig. 1 Membership and non-membership functions of $\tilde{A}$

2.2 (α, β) −Level intervals or (α, β) −cuts for IFS: - A set of (α, β) −cut generated by an IFS $\tilde{A}$, where $\alpha, \beta \in [0\ 1]$ are fixed numbers such that $\alpha + \beta \le 1$ is defined as

$\tilde{A}_{\alpha,\beta} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)): x \in X, \mu_{\tilde{A}}(x) \ge \alpha, \nu_{\tilde{A}}(x) \le \beta, \alpha, \beta \in [0\ 1]\}$ We define (α, β)-level interval or (α, β) −cut, denoted by $\tilde{A}_{\alpha,\beta}$, as the crisp set of elements x which belongs to $\tilde{A}$ at least to the degree α and which belongs to $\tilde{A}$ at most to the degree β.

3. Intuitionstic Fuzzy Numbers (IFN):- An IFN $\tilde{A}$ is defined as follows:

(i) An intuitionstic fuzzy subset of the real line.

(ii) Normal i.e. there is any $x_0 \in R$ such that $\mu_A(x_0); \alpha, \beta \in [0\ 1]$

(iii) Convex for the membership function $\mu_{\tilde{A}}(x)$ i.e.

$\mu_{\tilde{A}}[\lambda x_1 + (1 - \lambda x_2] \ge \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)], \forall\ x_1, x_2 \in R, \lambda \in [0\ 1]$

(iv) Concave for the non-membership function $\nu_{\tilde{A}}(x)$ i.e.

$\nu_{\tilde{A}}[\lambda x_1 + (1 - \lambda x_2] \le \max[\nu_{\tilde{A}}(x_1), \nu_{\tilde{A}}(x_2)], \forall\ x_1, x_2 \in R, \lambda \in [0\ 1]$

Here we have two types of intuitionstic fuzzy numbers:

(i) Triangular Intuitionstic Fuzzy Numbers

(ii) Trapezoidal Intuitionstic Fuzzy Numbers In the present paper we have introduced trapezoidal intuitionstic fuzzy numbers by using (α, β) −cuts.

3.1 Trapezoidal Intuitionstic Fuzzy Numbers (TrIFN):-

Let $(a_1 \le a_2 \le a_3 \le a_4 \le a'_1 \le a_2 \le a_3 \le a'_4)$. A Trapezoidal intuitionstic Fuzzy number (TrIFN) $\tilde{A}$ in R, written as $(a_1, a_2, a_3, a_4, a'_1, a_2, a_3, a'_4)$ has the membership function and the non-membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & for\ a_1 \le x \le a_2 \\ 1, & for\ a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3}, & for\ a_3 \le x \le a_4 \\ 0, & otherwise \end{cases} \quad \text{and} \quad \nu_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, & for\ a'_1 \le x \le a_2 \\ 0, & otherwise \\ \frac{x-a_3}{a'_4-a_3}, & for\ a_3 \le x \le a'_4 \\ 1, & otherwise \end{cases}$$

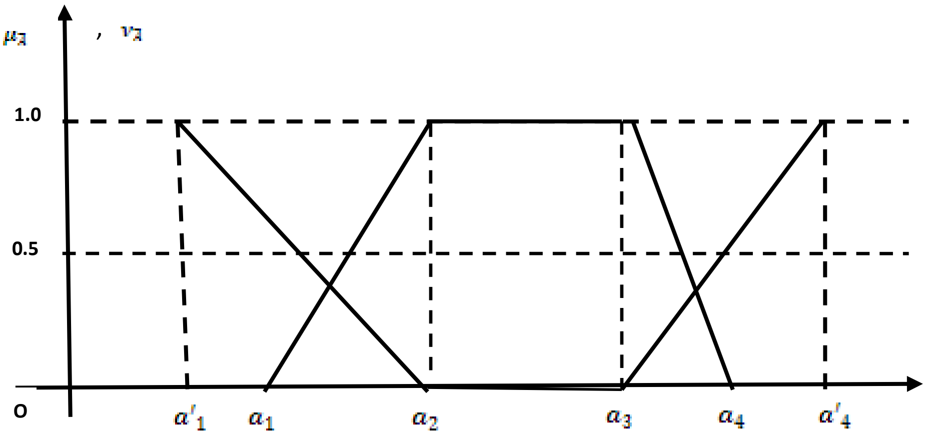

Fig. 2 Membership and non-membership functions of TrIFN

3.2. Arithmetic operations on IFNs:-

The arithmetic operations denoted generally by *, of two IFNs is a mapping of an input subset of R x R (with elements $x = (x_1, x_2)$) onto an output subset of R (with elements denoted by y). Let $A_1 and\ A_2$ be two IFNs, and $(A_1 * A_2)$ the resultant of operations then:

$$A_1 * A_2(y) = \left\{ \begin{pmatrix} y, \\ \vee_y = x_1 * x_2[A_1(x_1) \wedge A_2(x_2)], \\ \wedge_y = x_1 * x_2[A_1(x_1) \vee A_2(x_2)] \end{pmatrix} T, \forall x_1, x_2, y \in R \right\}$$

With

$$\mu_{(A_1 * A_2)}(y) = \vee_y = x_1 * x_2[A_1(x_1) \wedge A_2(x_2)]$$

$$and \quad \nu_{(A_1 * A_2)}(y) = \wedge_y = x_1 * x_2[A_1(x_1) \vee A_2(x_2)]$$

The arithmetic operations on IFNs can be defined by using the (α, β) −cut method. Let $\alpha, \beta \in [0, 1]$ be fixed numbers such that $\alpha + \beta \le 1$. A set of (α, β) −cut generated by an IFS A is defined by:

$\tilde{A}_{\alpha,\beta} = \{(x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}}(x) \ge \alpha, \nu_{\tilde{A}}(x) \le \beta, \alpha, \beta \in [0\ 1]\}$

The (α, β) −cut of Trapezoidal Intuitionstic Fuzzy Number is defined as usually by

$A_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)]\}$

$\alpha + \beta \le 1, \alpha, \beta \in [0, 1]$

Where

$$A_1(\alpha) = a_1 + \alpha(a_2 - a_1), \quad A_2(\alpha) = a_4 - \alpha(a_4 - a_3)$$

And $\quad A'_1(\beta) = a_2 - \beta(a_2 - a'_1), \quad A'_2(\beta) = a_3 + \beta(a'_4 - a_3)$

With the following properties:

i) $A_1(\alpha)$, $A'_2(\beta)$ are continuous, monotonic increasing functions of α, respective β.

ii) $A_2(\alpha)$, $A'_1(\beta)$ are continuous, monotonic decreasing functions of α, respective β.

III. RELIABILITY EVALUATION OF A SERIES, PARALLEL, PARALLEL-SERIES AND SERIES-PARALLEL NETWORKS

(a) **Series Networks:-**This arrangement represents a system where subsystem/components form a series network. If any of subsystem fails, the series system experiences an overall system failure.

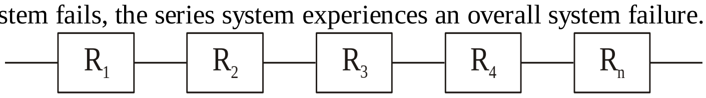

Fig. 3 Series Network

The fuzzy reliability $\tilde{R}_s = {}^{n}_{i}\otimes \tilde{R}_i$ can be evaluated by using the proposed algorithm

$$\tilde{R}_s = \{(r_{11}, r_{12} r_{13}, r_{14} ; r'_{11}, r'_{12} r'_{13}, r'_{14}) \otimes (r_{21}, r_{22} r_{23}, r_{24} ; r'_{21}, r'_{22} r'_{23}, r'_{24}) \otimes \ldots\ldots\ldots\ldots\ldots\ldots \otimes (r_{n1}, r_{n2} r_{n3}, r_{n4} ; r'_{n1}, r'_{n2} r'_{n3}, r'_{n4})\}$$

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It can be approximated to a TrIFN as

$$= (\prod_{j=1}^{n} r_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r_{j4}; \prod_{j=1}^{n} r'_{j1}, \prod_{j=1}^{n} r'_{j2}, \prod_{j=1}^{n} r'_{j3}, \prod_{j=1}^{n} r'_{j4}$$

Where $\tilde{R}_j = \left(r_{j1}, r_{j2}, r_{j3}, r_{j4}; r'_{j1}, r'_{j2}, r'_{j3}, r'_{j4}\right)$ is the intuitionstic fuzzy reliability of the j^{th} component for j=1, 2, 3, 4……..n

(b) **Parallel Networks: -** Consider a parallel network consisting of ‘n’ components as shown in figure. The fuzzy reliability $\tilde{R}_p = 1 \ominus {}^{n}_{i}\otimes(1 \ominus \tilde{R}_i)$ of the parallel system shown in figure can be evaluated by using the proposed algorithm.

$$\tilde{R}_p = 1 \ominus [(1 \ominus \left(r_{11}, r_{12} r_{13}, r_{14}; r'_{11}, r'_{12} r'_{13}, r'_{14}\right)) \otimes \dots\dots\dots\dots \otimes (1 \ominus \left(r_{n1}, r_{n2} r_{n3}, r_{n4}; r'_{n1}, r'_{n2} r'_{n3}, r'_{n4}\right))]$$

It can be approximated to a TrIFN as

$$= [1 - \prod_{j=1}^{n}(1 - r_{j1}), 1 - \prod_{j=1}^{n}(1 - r_{j2}), 1 - \prod_{j=1}^{n}(1 - r_{j3}), 1 - \prod_{j=1}^{n}(1 - r_{j4}); 1 - \prod_{j=1}^{n}(1 - r'_{j1}), 1 - \prod_{j=1}^{n}(1 - r'_{j2}), 1 - \prod_{j=1}^{n}(1 - r'_{j3}), 1 - \prod_{j=1}^{n}(1 - r'_{j4})]$$

Where $\tilde{R}_j = \left(r_{j1}, r_{j2}, r_{j3}, r_{j4}; r'_{j1}, r'_{j2}, r'_{j3}, r'_{j4}\right)$ is the intuitionstic fuzzy reliability of the j^{th} component for j=1, 2, 3, 4……..n

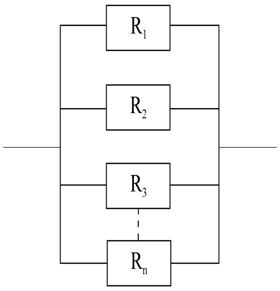

Fig. 4 Parallel networks

(c) **Parallel-series system:-**

Consider a parallel-series network consisting of ‘m’ connections connected in parallel and each connection contains ‘n’ components as shown in the figure. The fuzzy reliability is given by $\tilde{R}_{ps} = 1 \ominus {}^{m}_{i}\otimes(1 \ominus ({}^{n}_{i}\otimes \tilde{R}_{ki}))$ of the parallel-series network shown in figure. Reliability can be evaluated by the proposed algorithm, where $\tilde{R}_{ki}$ represents the reliability of the i^{th} component at the k^{th} network.

$$\tilde{R}_{ps} = 1 \ominus {}^{m}_{i}\otimes[(1 \ominus \{\left(r_{11}, r_{12} r_{13}, r_{14}; r'_{11}, r'_{12} r'_{13}, r'_{14}\right) \otimes \left(r_{21}, r_{22} r_{23}, r_{24}; r'_{21}, r'_{22} r'_{23}, r'_{24}\right) \otimes \dots\dots\dots\dots\dots \otimes \left(r_{n1}, r_{n2} r_{n3}, r_{n4}; r'_{n1}, r'_{n2} r'_{n3}, r'_{n4}\right)\}]$$

$$= 1 \ominus {}^{m}_{i}\otimes[(1 \ominus \left(r_{11}, r_{12} r_{13}, r_{14}; r'_{11}, r'_{12} r'_{13}, r'_{14}\right)) \otimes \dots\dots\dots \otimes (1 \ominus \left(r_{n1}, r_{n2} r_{n3}, r_{n4}; r'_{n1}, r'_{n2} r'_{n3}, r'_{n4}\right))]$$

$$= 1 \ominus {}^{m}_{i}\otimes[(1 \ominus \left(r_{11}, r_{12} r_{13}, r_{14}; r'_{11}, r'_{12} r'_{13}, r'_{14}\right)) \otimes \dots\dots\dots \otimes (1 \ominus \left(r_{n1}, r_{n2} r_{n3}, r_{n4}; r'_{n1}, r'_{n2} r'_{n3}, r'_{n4}\right))]$$

It can be approximated to a TrIFN as

$$R_{ps} = 1 \ominus {}^{m}_{i}\otimes[1 \ominus \left(\prod_{j=1}^{n} r_{j1}, \prod_{j=1}^{n} r_{j2}, \prod_{j=1}^{n} r_{j3}, \prod_{j=1}^{n} r_{j4}; \prod_{j=1}^{n} r'_{j1}, \prod_{j=1}^{n} r'_{j2}, \prod_{j=1}^{n} r'_{j3}, \prod_{j=1}^{n} r'_{j4}\right)]$$

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$$= 1 \ominus {}^{m}_{i}\otimes\left[\left(\prod_{j=1}^{n}(1 \ominus r_{j1}), \prod_{j=1}^{n}(1-r_{j2}), \prod_{j=1}^{n}(1-r_{j3}), \prod_{j=1}^{n}(1-r_{j4}); \prod_{j=1}^{n}(1-r'_{j1}), \prod_{j=1}^{n}(1-r'_{j2}), \prod_{j=1}^{n}(1-r'_{j3}), \prod_{j=1}^{n}(1-r'_{j4})\right)\right]$$

$$= 1 \ominus \left[\left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right); \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right), \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right)\right)\right]$$

$$= \left[\left\{1 \ominus \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r_{ji})\right)\right)\right\}; \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right)\right\}, \left\{1 - \left(\prod_{i=1}^{m}\left(\prod_{j=1}^{n}(1\ominus r'_{ji})\right)\right)\right\}\right]$$

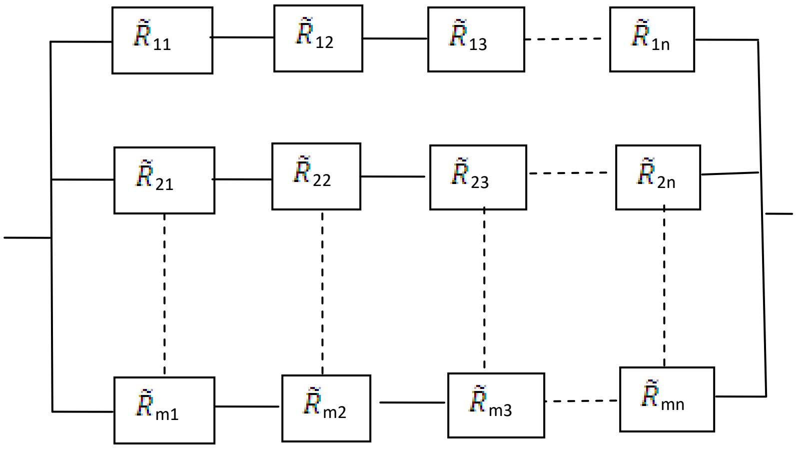

Fig. 5 Parallel-series networks

(d) **Series-parallel systems: -** Consider a series-parallel network consisting of 'n' connections connected in parallel and each connection contains 'm' components as shown in the figure. The fuzzy reliability is given by $\tilde{R}_{sp} = {}_{k=1}^{n}\otimes(1 \ominus {}_{i=1}^{m}(1-\tilde{R}_{ik}))$ of the series-parallel network shown in figure. Reliability can be evaluated by the proposed algorithm, where $\tilde{R}_{ik}$ represents the reliability of the k^th component at the i^th stage.

$$\tilde{R}_{sp} = {}_{k=1}^{n}\otimes\{1 \ominus [(1 \ominus (r_{1k}, r_{2k} r_{3k}, r_{4k}; r'_{1k}, r'_{2k}, r'_{3k}, r'_{4k})) \otimes \dots\dots\dots\dots \otimes (1 \ominus (r_{mk}, r_{mk}, r_{mk}, r_{m4}; r'_{mk}, r'_{mk}, r'_{mk}, r'_{mk}))]$$

It can be approximated to a TrIFN as

$= {}_{k=1}^{n}\otimes\, [1-\prod_{j=1}^{n}(1-r_{jk}), 1-\prod_{j=1}^{n}(1-r_{jk}), 1-\prod_{j=1}^{n}(1-r_{jk}), 1-\prod_{j=1}^{n}(1-r_{jk}) ; 1-\prod_{j=1}^{n}(1-r'_{jk}), 1-\prod_{j=1}^{n}(1-r'_{jk}), 1-\prod_{j=1}^{n}(1-r'_{jk}), 1-\prod_{j=1}^{n}(1-r'_{jk})]$

$= [\{\prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r_{jk}))\}; \{\prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r'_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r'_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r'_{jk})), \prod_{k=1}^{n}(1-\prod_{j=1}^{n}(1-r'_{jk}))\}]$

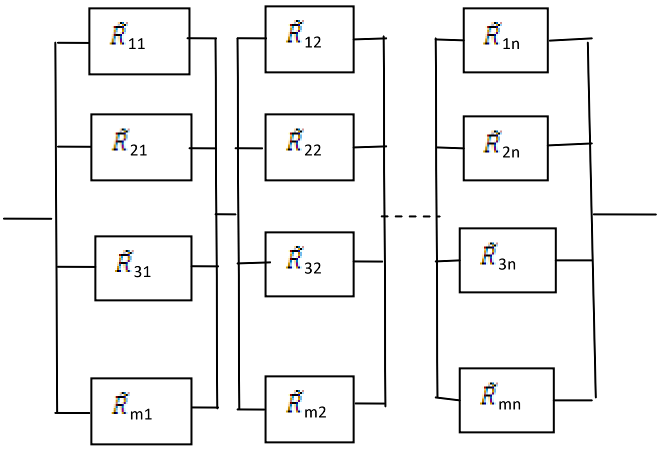

Fig. 6 Series-Parallel networks

5. Proposed Algorithm for Calculating the Reliability of the Network Systems: - In this section an algorithm has been proposed to perform arithmetic operations among intuitionstic fuzzy numbers through (α, β)-cuts, where the reliability of different components has been taken in the form of IFNs.

If $\tilde{A}$ is an IFN, then (α, β)-level intervals or (α, β)-cut is given by

$$\tilde{A}_{\alpha,\beta} = \begin{cases} [A_1(\alpha), A_2(\alpha)] \text{ for degree of acceptance } \alpha \in [0,1] \\ [A'_1(\beta), A'_2(\beta)] \text{ for degree of rejection } \beta \in [0,1] \end{cases} \text{ with } \alpha + \beta \le 1,$$

Here $\frac{dA_1(\alpha)}{d\alpha} > 0, \frac{dA_2(\alpha)}{d\alpha} < 0 \;\forall\, \alpha \in (0,1), A_1(1) \le A_2(1)$

and $\frac{dA'_1(\beta)}{d\beta} < 0, \frac{dA'_2(\beta)}{d\beta} > 0 \;\forall\, \beta \in (0,1), A'_1(0) \le A'_2(0)$

It is expressed as $\tilde{A}_{\alpha,\beta} = \{[A_1(\alpha), A_2(\alpha)], [A'_1(\beta), A'_2(\beta)], \alpha+\beta \le 1, \alpha, \beta \in [0,1]\}$

Step I: -First we will construct intuitionstic fuzzy numbers for all the components of the network.

Step II: - In this step we will evaluate the (α, β)-cut for each intuitionstic fuzzy number as in step I.

Step III: - If $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b_2, b_3, b'_4)$ are two TrIFN, then $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is also TrIFN $\tilde{A} \oplus \tilde{B} = (a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4; a'_1+b'_1, a_2+b_2, a_3+b_3, a'_4+b'_4)$.

With the transformation z = x+y, we can find the membership function of acceptance (membership) IFS $\tilde{C} = \tilde{A} \oplus \tilde{B}$ by the $\alpha - cut$ method.

α −cut for membership function of $\tilde{A}$ is $[a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)] \;\forall\, \alpha \in [0,1]$

i.e. $x \in [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$

α −cut for membership function of $\tilde{B}$ is $[b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)] \;\forall\, \alpha \in [0,1]$

i.e. y$\in [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]$

So, z (= x+y) $\in \left[a_1 + b_1 + \alpha\left((a_2 - a_1) + (b_2 - b_1)\right), a_4 + b_4 - \alpha\left((a_4 - a_3) + (b_4 - b_3)\right)\right]$

So, we have the membership (acceptance) function $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is

$$\mu_{\tilde{C}}(z) = \begin{cases} \frac{z - a_1 - b_1}{(a_2 - a_1) + (b_2 - b_1)}, & \text{for } a_1 + b_1 \le z \le a_2 + b_2 \\ 1, & \text{for } a_2 + b_2 \le z \le a_3 + b_3 \\ \frac{a_4 + b_4 - z}{(a_4 - a_3) + (b_4 + b_3)}, & \text{for } a_3 + b_3 \le z \le a_4 + b_4 \\ 0, & \text{otherwise} \end{cases}$$

This is the addition rule for membership function.

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For non-membership function, β-cut of $\tilde{A}$ is $[a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]\forall\beta \in [0,1]$ i.e.
$x \in [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]$
β-cut of $\tilde{B}$ is $[b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]\forall\beta \in [0,1]$
i.e. $y \in [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]$
So, z (= x+y) $\in \left[a_2 + b_2 - \beta\left((a_2 - a'_1) + (b_2 - b'_1)\right), (a_3 + b_3) - \beta\left((a'_4 - a_3) + (b'_4 - b_3)\right)\right]$
So, we have the non-membership (rejection) function of $\tilde{C} = \tilde{A} \oplus \tilde{B}$ is

$$\nu_{\tilde{C}}(z) = \begin{cases} \dfrac{a_2 + b_2 - z}{(a_2 - a'_1) + (b_2 - b'_1)}, & for\ a'_1 + b'_1 \le z \le a_2 + b_2 \\ 0, & for\ a_2 + b_2 \le z \le a_3 + b_3 \\ \dfrac{z - a_3 - b_3}{(a'_4 - a_3) + (b'_4 - b_3)}, & for\ a_3 + b_3 \le z \le a'_4 + b'_4 \\ 1, & otherwise \end{cases}$$

This is the rule for non-membership function.
Thus we have $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a_2 + b_2, a_3 + b_3, a'_4 + b'_4)$.
Step IV: - If $\tilde{A} = (a_1, a_2, a_3, a_4; a'_1, a_2, a_3, a'_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4; b'_1, b_2, b_3, b'_4)$ are two TrIFN, then $\tilde{P} = \tilde{A} \otimes \tilde{B}$ is approximated
TrIFN $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; a'_1 b'_1, a_2 b_2, a_3 b_3, a'_4 b'_4)$.
With the transformation z = xXy, we can find the membership function of acceptance (membership) IFS $\tilde{P} = \tilde{A} \otimes \tilde{B}$ by the $\alpha - cut$ method.
α −cut for membership function of $\tilde{A}$ is $\mu_{\tilde{A}}(x) \ge \alpha \Rightarrow [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]\ \forall\ \alpha \in [0,1]$
i.e. $x \in [a_1 + \alpha(a_2 - a_1), a_4 - \alpha(a_4 - a_3)]$
α −cut for membership function of $\tilde{B}$ is $\mu_{\tilde{B}}(x) \ge \alpha \Rightarrow [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]\ \forall\ \alpha \in [0,1]$
i.e. y$\in [b_1 + \alpha(b_2 - b_1), b_4 - \alpha(b_4 - b_3)]$
So, z (= xXy) $\in \left[\left(a_1 + \alpha(a_2 - a_1)\right)\left(b_1 + \alpha(b_2 - b_1)\right), \left(a_4 + \alpha(a_4 - a_3)\right)\left(b_4 - \alpha(b_4 - b_3)\right)\right]$
So, we have the membership (acceptance) function $\tilde{P} = \tilde{A} \otimes \tilde{B}$ is

$$\mu_{\tilde{P}}(z) = \begin{cases} \dfrac{-\mathrm{B}_1 + \sqrt{\mathrm{B}_1^2 - 4\mathrm{A}_1(a_1 b_1 - z)}}{2A_1}, & for\ a_1 b_1 \le z \le a_2 b_2 \\ 1, & for\ a_2 b_2 \le z \le a_3 b_3 \\ \dfrac{\mathrm{B}_2 - \sqrt{\mathrm{B}_2^2 - 4\mathrm{A}_2(a_4 b_4 - z)}}{2A_2}, & for\ a_3 b_3 \le z \le a_4 b_4 \\ 0, & otherwise \end{cases}$$

Where, $A_1 = (a_2 - a_1)(b_2 - b_1)$, $B_1 = b_1(a_2 - a_1) + a_1(b_2 - b_1)$, $A_2 = (a_4 - a_3)(b_4 - b_3)$, $B_2 = -(b_4(a_4 - a_3) + a_4(b_4 - b_3))$
For non-membership function, β-cut of $\tilde{A}$ is $\nu_{\tilde{A}}(x) \le \beta \Rightarrow [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]\forall\beta \in [0,1]$ i.e.
$x \in [a_2 - \beta(a_2 - a'_1), a_3 + \beta(a'_4 - a_3)]$
β-cut of $\tilde{B}$ is $\nu_{\tilde{A}}(x) \le \beta \Rightarrow [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]\forall\beta \in [0,1]$
i.e. $y \in [b_2 - \beta(b_2 - b'_1), b_3 + \beta(b'_4 - b_3)]$
So, z (= xXy) $\in \left[\left(a_2 - \beta(a_2 - a'_1)\right)\left(b_2 - \beta(b_2 - b'_1)\right), \left(a_3 + \beta(a'_4 - a_3)\right), \left(b_3 + \beta(b'_4 - b_3)\right)\right]$
So, we have the non-membership (rejection) function of $\tilde{z} = \tilde{A} \otimes \tilde{B}$ is

$$\nu_{\tilde{P}}(z) = \begin{cases} 1 - \dfrac{-\mathrm{B'}_1 + \sqrt{\mathrm{B'}_1^2 - 4\mathrm{A'}_1(a'_1 b'_1 - z)}}{2A'_1}, & for\ a'_1 b'_1 \le z \le a_2 b_2 \\ 0, & for\ a_2 b_2 \le z \le a_3 b_3 \\ 1 - \dfrac{\mathrm{B'}_2 - \sqrt{\mathrm{B'}_2^2 - 4\mathrm{A'}_2(a'_4 b'_4 - z)}}{2A'_2}, & for\ a_3 b_3 \le z \le a'_4 b'_4 \\ 1, & otherwise \end{cases}$$

Where,$A'_1 = (a_2 - a'_1)(b_2 - b'_1)$, $B'_1 = b'_1(a_2 - a'_1) + a'_1(b_2 - b'_1)$, $A'_2 = (a'_4 - a_3)(b'_4 - b_3)$ and
$B'_2 = -(b'_4(a'_4 - a_3) + a'_4(b'_4 - b_3))$
This is the rule for non-membership function.
Thus we have $\tilde{P} = \tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4; a'_1 b'_1, a_2 b_2, a_3 b_3, a'_4 b'_4)$.
Step V: - Operations defined in step III and IV will be used to evaluate the reliability for the whole network.

IV. NUMERICAL COMPUTATIONS

A heavy current special machine demands continuous DC power supply during a particular period model [20] has been taken to illustrate our algorithm as shown in figure. The required power can be made available through convertor. In order to ensure uninterrupted power supply, two convertors are used, so that even if one fails, the other convertor provides the necessary current. The two convertors receive their power supplies from a sub-station which is connected to the main grid. We shall

assume that the two convertors are basic components in addition to the grid and the sub-station. The machine becomes non-operative when there is no supply from the main grid, or when there if failure in the substation, or when both convertors fail to operate. The fault tree for the system is shown in figure 7.
Each event in this diagram is considered as the TrIFN.
$\tilde{R}_1$ = represents the reliability of the grid failure (F_1)
$\tilde{R}_2$ = represents the reliability of the sub-station failure(F_2)
$\tilde{R}_3$ = represents the reliability of the switch of the DC supply to machine
$\tilde{R}_4$ = represents the reliability of the convertor I fails(F_4)
$\tilde{R}_5$ = represents the reliability of the convertor II fails(F_5)
$\tilde{R}_6$ = represents the reliability of power supply to the convertor
$\tilde{R}_7$ = represents the reliability of both convertors fail
$\tilde{R}_8$ = represents the reliability of DC power supply to the machine

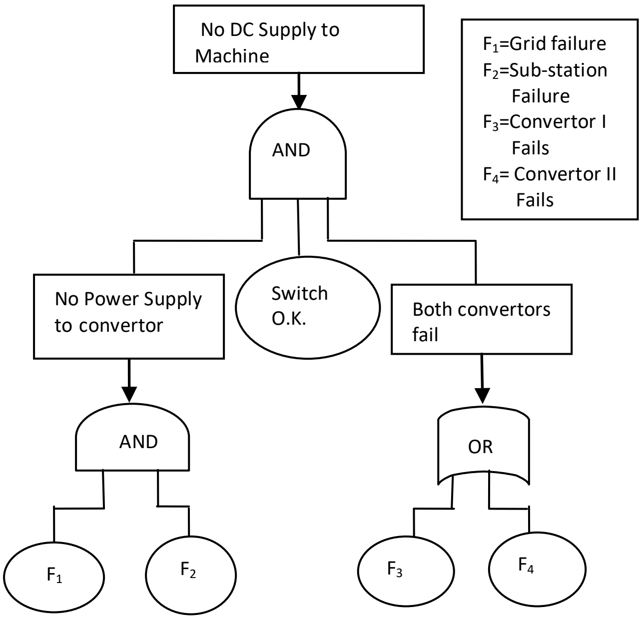

Fig. 7 Block Diagram for DC Power Supply

All the imprecise components reliability $\tilde{R}_j$ are represented by TrIFN $(r_{j1}, r_{j2}, r_{j3}, r_{j4}; r'_{j1}, r'_{j2}, r'_{j3}, r'_{j4})$ $for\ j = 1 \dots\dots 5$. Let us evaluate the reliability of the desired system (No DC supply to machine)
The reliability value, for the occurrence of the event no power supply to the convertor, $\tilde{R}_6$:
$\tilde{R}_6 = \tilde{R}_1 \otimes \tilde{R}_2 \cong (r_{11}r_{21}, r_{12}r_{22}, r_{13}r_{23}, r_{14}r_{24}; r'_{11}r'_{21}, r'_{12}r'_{22}, r'_{13}r'_{23}, r'_{14}r'_{24}$
It is an Approximated TrIFN.
Similarly, the reliability value, for the occurrence of event both convertors fail, $\tilde{R}_7$:
$\tilde{R}_7 = 1 \ominus (1 \ominus \tilde{R}_4)(1 \ominus \tilde{R}_5)$
It is approximated to a TrIFN as follows:

$$= [1 - \prod_{j=1}^{n}(1 - r_{j1}), 1 - \prod_{j=1}^{n}(1 - r_{j2}), 1 - \prod_{j=1}^{n}(1 - r_{j3}), 1 - \prod_{j=1}^{n}(1 - r_{j4}); 1 - \prod_{j=1}^{n}(1 - r'_{j1}), 1 - \prod_{j=1}^{n}(1 - r'_{j2}), 1 - \prod_{j=1}^{n}(1 - r'_{j3}), 1 - \prod_{j=1}^{n}(1 - r'_{j4})]$$

By substituting the two above calculations and the given data value, we get the reliability value for the occurrence of the top event, no DC supply to machine, $\tilde{R}_8$:
$\tilde{R}_8 = \tilde{R}_1 \otimes \tilde{R}_2 \otimes \tilde{R}_3 \otimes (1 \ominus (1 \ominus \tilde{R}_4)(1 \ominus \tilde{R}_5))$
It is approximated to a TrIFN as follows

$$= (r_{11}r_{21}r_{31}r_{41}[1-\prod_{j=1}^{5}(1-r_{j1})], r_{11}r_{22}r_{32}r_{42}[1-\prod_{j=1}^{5}(1-r_{j2})], r_{13}r_{23}r_{33}r_{43}[1-\prod_{j=1}^{5}(1-r_{j3})], r_{14}r_{24}r_{34}r_{44}[1-\prod_{j=1}^{5}(1-r_{j4})]; r'_{11}r'_{21}r'_{31}r'_{41}[1-\prod_{j=1}^{5}(1-r'_{j1})], r'_{11}r'_{22}r'_{32}r'_{42}[1-\prod_{j=1}^{5}(1-r'_{j2})], r'_{13}r'_{23}r'_{33}r'_{43}[1-\prod_{j=1}^{5}(1-r'_{j3})], r'_{14}r'_{24}r'_{34}r'_{44}[1-\prod_{j=1}^{5}(1-r'_{j4})])$$

Let the reliability of events are

$\tilde{R}_1 = (0.70,0.75,0.80,0.85;0.60,0.75,0.80,0.90)$
$\tilde{R}_2 = (0.75,0.80,0.85,0.90;0.65,0.72,0.85,0.95)$
$\tilde{R}_3 = (0.80,0.85,0.92,0.95;0.75,0.82,0.89,0.96)$
$\tilde{R}_4 = (0.76,0.84,0.89,0.92;0.71,0.78,0.85,0.92)$
$\tilde{R}_5 = (0.64,0.69,0.78,0.87;0.62,0.75,0.83,0.94)$

So results for $\tilde{R}_6$ and $\tilde{R}_7$ by using the calculations are as follows
$\tilde{R}_6 = (0.61,0.69,0.795,0.874;0.5175,0.6924,0.8412,0.9214)$
$\tilde{R}_7 = (0.8925,0.9425,0.9528,0.9871;0.862,0.9125,0.9725,0.9945)$

By substituting the above two calculated values according to the data value, the reliability of the top event, no DC supply to the machine, $\tilde{R}_8$ is $\tilde{R}_8 = (0.5125,0.5942,0.6895,0.7498;0.4621,0.5942,0.6725,0.8245)$

V. CONCLUSION

In this present paper we have proposed an algorithm, a definition of IFN according to the approach of a fuzzy number. Arithmetic operations of proposed TrIFN are evaluated based on intuitionstic fuzzy (α,β)-cut method. Here, a method to analyze the network system reliability which is based on intuitionstic fuzzy set theory has been presented, where the components of the network system are trapezoidal intuitionstic fuzzy numbers. (α,β)-cut of these TrIFNs are evaluated. Arithmetic operations over these evaluated trapezoidal intuitionstic fuzzy numbers through (α,β)-cut are used to analyze the fuzzy reliability of the series, parallel, series-parallel and parallel-series network systems. The major advantage of using intuitionstic fuzzy sets over the fuzzy sets is that intuitionstic fuzzy sets separate the acceptance and rejection evidence for the membership of a connection in the network.

REFERENCES:-

1. A. Kaufmann and M.M.Gupta, “Fuzzy mathematical models in Engineering and Management sciences (North-Holland Amsterdam, (1988).
2. M.K.Sharma and D. Pandey, “Profust and Posfust Reliability a Network System” Journal of Mountain
3. Research, Vol. 2, 2007, 97-112.
4. K.Y. Cai, “System failure and fuzzy methodology: an introductory overview” Fuzzy sets and systems,
5. 83(1996), 113-133.
6. M.K.Sharma and D. Pandey, “Reliability analysis of multistate fault tree model” Mathematics Today, Vol.
7. 25(2009) 7-21.
8. M.K.Sharma and D.Pandey, “Vague Set Theoretic Approach to Fault Tree Analysis” Journal of International
9. Academy of Physical Sciences, Vol. 14 No. 1(2010), 1-14.
10. Zadeh L. A. “Fuzzy sets”, Information and Control, Vol 8, No. 3, pp. 338-353, 1965.
11. Atanassov, K. “Intuitionistic fuzzy sets”. Fuzzy Sets and Systems, Vol 20, No. 1, pp.87-96, 1986.
12. Gau W. L, Buehrer D. J. “Vague sets”, IEEE Transactions on Systems, Man, and Cybernetics Vol 23, pp. 610-614, 1993.
13. P. Burillo, H. Bustince and V. Mohedano, Some definition of intuitionistic fuzzy number, Fuzzy based expert
14. systems, fuzzy Bulgarian enthusiasts, September 28-30, 1994, Sofia, Bulgaria.
15. Szmidt, E, Kacprzyk, J. “Intuitionistic fuzzy sets in decision making”, Notes IFS, Vol 2, No. 1, pp. 15-32, 1996.
16. Atanassov, K. Intuitionistic Fuzzy Sets: Theory and Applications, Physica-Verlag, Heidelberg, New York, 1999.
17. K.T. Atanassov, G. Gargov, Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31 (3) (1989) 343–349.
18. K.T. Atanassov, More on intuitionistic fuzzy sets, *Fuzzy* Sets and Systems 33 (1) (1989) 37-46.
19. K.T. Atanassov and G. Gargov, Elements of intuitionistic fuzzy logic, Part I, Fuzzy Sets and Systems, 95 (1) (1998) 39–52.
20. A.I. Ban, Nearest Interval Approximation of an Intuitionistic Fuzzy Number, Computational Intelligence,
21. Theory and Applications (Bernd Reusch (Ed.)), Springer-Verlag, Berlin, Heidelberg 2006 (p-229-240).
22. Szmidt, E, Kacprzyk, J. “Distances between intuitiionistic fuzzy sets”, Fuzzy Sets and Systems, Vol 114, pp. 505-518, 2000.

23. H.B. Mitchell, Ranking-Intuitionistic Fuzzy Numbers, International Journal of Uncertainty, Fuzziness and
24. Knowledge-Based Systems, 12 (3) (2004), 377-386.
25. M.H. Shu, C.H. Cheng and J.R. Chang, Using intuitionistic fuzzy sets for fault-tree analysis on printed circuit
26. Board assembly, Microelectronics and Reliability, 46 (12) (2006), 2139-2148.
27. G.S. Mahapatra, and T.K. Roy, Reliability Evaluation using Triangular Intuitionistic Fuzzy Numbers
28. Arithmetic Operations, Proceedings of World Academy of Science, Engineering and Technology, Malaysia, 38 (2009), 587-585.
29. L.S.Srinath, "Reliability Engineering." East-West Press Private Limited, New Delhi, Fourth Edition, 1985.