Abstract—In this paper, we have defined disjunctive sum and difference of two intuitionistic fuzzy soft sets and study their basic properties. The notions of \((\alpha, \beta)\)-cut soft set and \((\alpha, \beta)\)-cut strong soft set of an intuitionistic fuzzy soft set have been put forward in our work. Some related properties have been established with proof, examples and counter examples.

Index Terms—Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Soft Set, Disjunctive Sum, Difference, \((\alpha, \beta)\)-cut soft set, \((\alpha, \beta)\)-cut strong soft set.

I. INTRODUCTION

Most of the real life problems have various uncertainties. The Theory of Probability, Evidence Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. are mathematical tools to deal with such problems. In 1999, Molodtsov [2] introduced the Theory of Soft Set and established the fundamental results related to this theory. In comparison, this theory can be seen free from the inadequacy of parameterization tool. It is a general mathematical tool for dealing with problems in the fields of social science, economics, medical sciences etc. In 2003, Maji, Biswas and Roy [5] studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In recent times, researchers have contributed a lot towards fuzzification of Soft Set Theory. Combining fuzzy sets with soft sets, Maji et al. [4] introduced the notion of fuzzy soft sets. They studied some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan Laws in Fuzzy Soft Set Theory. Moreover Maji et al.[6] extended soft sets to intuitionistic fuzzy soft sets. Intuitionistic fuzzy soft set theory is a combination of soft sets and intuitionistic fuzzy sets initiated by Atanassov [3]. In [7] Neog and Sut have defined disjunctive sum and difference of two fuzzy soft sets. The notions of \(\alpha\) - cut soft set and \(\alpha\) - cut strong soft set of a fuzzy soft set have been put forward in their work. In this paper we have defined disjunctive sum and difference of two intuitionistic fuzzy soft sets. Further the notions of \((\alpha, \beta)\)-cut soft sets and \((\alpha, \beta)\)-cut strong soft set of an intuitionistic fuzzy soft set have been put forward in our work.

Some related properties have been established in our work with supporting proof, examples and counter examples.

II. PRELIMINARIES

A. Definition [2]

A pair \((F, E)\) is called a soft set (over \(U\)) if and only if \(F\) is a mapping of \(E\) into the set of all subsets of the set \(U\).

In other words, the soft set is a parameterized family of subsets of the set \(U\). Every set \(F(\varepsilon), \varepsilon \in E\), from this family may be considered as the set of \(\varepsilon\) - elements of the soft set \((F, E)\), or as the set of \(\varepsilon\) - approximate elements of the soft set.

B. Definition [3]

An intuitionistic fuzzy set \(A\) over the universe \(U\) can be defined as follows -

\[
A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\},
\]

where

\[
\mu_A(x) : U \rightarrow [0,1], \nu_A(x) : U \rightarrow [0,1]
\]

with the property

\[
0 \leq \mu_A(x) + \nu_A(x) \leq 1, \quad \forall x \in U.
\]

The values \(\mu_A(x)\) and \(\nu_A(x)\) represent the degree of membership and non-membership of \(x\) to \(A\) respectively.

\[
\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))
\]

is called the intuitionistic fuzzy index.

C. Definition [6]

Let \(U\) be an initial universe set and \(E\) be the set of parameters. Let \(IF(U)\) denote the collection of all intuitionistic fuzzy subsets of \(U\). Let \(A \subseteq E\). A pair \((F, A)\) is called an intuitionistic fuzzy soft set over \(U\) where \(F\) is a mapping given by \(F : A \rightarrow IF(U)\).

D. Definition [6]

A soft set \((F, A)\) over \(U\) is said to be null intuitionistic fuzzy soft set denoted by \(\varnothing\) if \(\forall \varepsilon \in A, F(\varepsilon)\) is the null intuitionistic fuzzy set \(\varnothing\) of \(U\) where \(\hat{0}(x) = 0, \forall x \in U\).

We would use the notation \((\varnothing, A)\) to represent the intuitionistic fuzzy soft null set with respect to the set of parameters \(A\).

E. Definition [6]

A soft set \((F, A)\) over \(U\) is said to be absolute intuitionistic fuzzy soft set denoted by \(\tilde{A}\) if \(\forall \varepsilon \in A, F(\varepsilon)\) is the absolute intuitionistic fuzzy set \(\tilde{A}\) of \(U\) where \(\tilde{A}(x) = 1, \forall x \in U\).

We would use the notation \((U, A)\) to represent the intuitionistic fuzzy soft absolute set with respect to the set of parameters \(A\).

F. Definition [6]

For two intuitionistic fuzzy soft sets \((F, A)\) and \((G, B)\) over \((U, E)\), we say that \((F, A)\) is an intuitionistic fuzzy soft subset of \((G, B)\), if
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(i) $A \subseteq B$.

(ii) For all $\varepsilon \in \mathcal{E}$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \subseteq (G, B)$.

G. Definition [6]

Union of two intuitionistic fuzzy soft sets $(F, A)$ and $(G, B)$ over $(U, E)$ is an intuitionistic fuzzy soft set $(H, C)$ where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \bigcap (G, B) = (H, C)$.

H. Definition [6]

Let $(F, A)$ and $(G, B)$ be two intuitionistic fuzzy soft sets over $(U, E)$. Then intersection $(F, A)$ and $(G, B)$ is an intuitionistic fuzzy soft set $(H, C)$ where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$

We write $(F, A) \bigcap (G, B) = (H, C)$.

I. Definition [6]

Let $(F, A)$ and $(G, B)$ be two fuzzy soft sets in a soft class $(U, E)$ with $A \cap B \neq \emptyset$. Then intersection of two fuzzy soft sets $(F, A)$ and $(G, B)$ in a soft class $(U, E)$ is a fuzzy soft set $(H, C)$ where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$$

We write $(F, A) \bigcap (G, B) = (H, C)$.

J. Definition [6]

The complement of an intuitionistic fuzzy soft set $(F, A)$ is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow [0, 1]^U$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$. Thus if $F(\varepsilon) = [\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)]_{x \in U}$, then

$$F^c(\varepsilon) = [\mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)]_{x \in U}$$

where $\forall \varepsilon \in A$. $F^c(\varepsilon) = (F(\varepsilon))^c = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U \}$

K. Definition [6]

If $(F, A)$ and $(G, B)$ are two intuitionistic fuzzy soft sets, then "$(F, A)$ AND $(G, B)$" is an intuitionistic fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta) \forall \alpha \in A$ and $\forall \beta \in B$, which $\cap$ is the operation intersection of two intuitionistic fuzzy sets.

L. Definition [6]

If $(F, A)$ and $(G, B)$ are two intuitionistic fuzzy soft sets, then "$(F, A)$ OR $(G, B)$" is an intuitionistic fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta) \forall \alpha \in A$ and $\forall \beta \in B$, which $\cup$ is the operation union of two intuitionistic fuzzy sets.

III. SOME NEW OPERATIONS OF INTUITIONISTIC FUZZY SOFT SETS

A. Definition (Disjunctive Sum of Intuitionistic Fuzzy Soft Sets)

Let $(F, A)$ and $(G, B)$ be two intuitionistic fuzzy soft sets over $(U, E)$. We define the disjunctive sum of $(F, A)$ and $(G, B)$ as the intuitionistic fuzzy soft set $(H, C)$ over $(U, E)$, written as $(F, A) \oplus (G, B) = (H, C)$, where $C = A \cup B \neq \emptyset$ and $\forall \varepsilon \in C$, $x \in U$, 

$$\mu_{H(\varepsilon)}(x) = \max\{\min\{\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}, \min\{\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}\}$$

$$\nu_{H(\varepsilon)}(x) = \min\{\max\{\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}, \max\{\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}\}$$

B. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$.

$F(\varepsilon_1) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\}$

$F(e_2) = \{(a, 0.7, 0.1), (b, 0.0, 0.8), (c, 0.3, 0.5)\}$

$F(e_3) = \{(a, 0.6, 0.3), (b, 0.0, 0.7), (c, 0.9, 0.0)\}$

Then

$$(F, A) \oplus (G, B) = (H, C)$$

where $C = A \cup B$ is an intuitionistic fuzzy soft set denoted

$$(H, C) = \{e_1, e_2\}$$

and

$$(H, C) = \{e_1, e_2\}$$

C. Proposition

Let $(F, A)$ and $(G, B)$ be two intuitionistic fuzzy soft sets over $(U, E)$. Then the following results hold.

(i) $(F, A) \oplus (G, B) = (G, B) \oplus (F, A)$

(ii) $(F, A) \oplus (G, B) \oplus (H, C) = (F, A) \oplus (G, B) \oplus (H, C)$

Proof

(i) $(F, A) = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \}, \forall x \in U, \forall \varepsilon \in A$

$(G, B) = \{x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) \}, \forall x \in U, \forall \varepsilon \in B$

Let $(F, A) \oplus (G, B) = (H, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C, x \in U$,

$$H(\varepsilon)(x) = \max\{\min\{\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}, \min\{\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}\}$$

Let $(G, B) \oplus (F, A) = (I, D)$, where $D = A \cap B$ and $\forall \varepsilon \in D, x \in U$,

$$H(\varepsilon)(x) = \min\{\max\{\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)\}, \max\{\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)\}\}$$

It follows that

$$H(\varepsilon) = (I, D)$$

Therefore $(F, A) \oplus (G, B) = (G, B) \oplus (F, A)$.
Proof of (ii) can be done in a similar way.

**D. Proposition**

(i) \((F, A) \cong (\varphi, A) = (F, A)\)

(ii) \((F, A) \cong (U, A) = (F, A)^C\)

**Proof**

(i) Let \(F, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall x \in U, \nu \in A\}\) and \((\varphi, A) = \{(0,1), \nu \in U, \nu \in A\}\)

Let \((F, A) \cong (\varphi, A) = (H, A)\), where \(\forall \nu \in A, \nu \in U\), we have

\[\mu_{H(x)}(x) = \max \{\min \{\mu_{F(x)}(x), \mu_{\varphi(x)}(x)\}, \min \{\nu_{F(x)}(x), \mu_{\varphi(x)}(x)\}\}\]

\[v_{H(x)}(x) = \min \{\max \{\nu_{F(x)}(x), \mu_{\varphi(x)}(x)\}, \max \{\mu_{F(x)}(x), \nu_{\varphi(x)}(x)\}\}\]

Then \((H, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall \nu \in C = A, x \in U\}\)

It follows that \((F, A) \cong (\varphi, A) = (F, A)\)

(ii) Let \((F, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall x \in U, \nu \in A\}\) and \((U, A) = \{(0,1), \nu \in U, \nu \in A\}\)

Let \((F, A) \cong (U, A) = (H, A)\), where \(\forall \nu \in A, \nu \in U\), we have

\[\mu_{H(x)}(x) = \max \{\min \{\mu_{F(x)}(x), \nu_{U(x)}(x)\}, \min \{\nu_{F(x)}(x), \mu_{U(x)}(x)\}\}\]

\[v_{H(x)}(x) = \min \{\max \{\nu_{F(x)}(x), \mu_{U(x)}(x)\}, \max \{\mu_{F(x)}(x), \nu_{U(x)}(x)\}\}\]

It follows that \((H, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall \nu \in A, x \in U\}\) and \((F, A) \cong (U, A) = (F, A)^C\)

Consequently \((F, A) \cong (U, A) = (F, A)^C\)

**B. Definition (Difference of Intuitionistic Fuzzy Soft Sets)**

Let \((F, A)\) and \((G, B)\) be two intuitionistic fuzzy soft sets over \((U, E)\). We define the difference of \((F, A)\) and \((G, B)\) as the intuitionistic fuzzy soft set \((H, C)\) over \((U, E)\), written as \((F, A) \triangle (G, B) = (H, C)\), where \(C = A \cap B = \varnothing\) and \(\forall \nu \in C\),

\[\mu_H(x) = \min \{\mu_{F(x)}(x), \nu_{G(x)}(x)\}\]

\[v_H(x) = \max \{\nu_{F(x)}(x), \mu_{G(x)}(x)\}\]

**C. Example**

Let \(U = \{a, b, c\}\) and \(E = \{e_1, e_2, e_3, e_4\}\), \(A = \{e_1, e_2, e_4\} \subseteq E\).

\(B = \{e_1, e_2, e_3\} \subseteq E\)

\(F, A) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\}\)

\(F, e_2) = \{(a, 0.7, 0.1), (b, 0.0, 0.8), (c, 0.3, 0.5)\}\)

\(F, e_4) = \{(a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.01)\}\)

\((G, B) = \{(a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)\}\)

\((G, c_2) = \{(a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\}\)

\((G, c_3) = \{(a, 0.0, 0.6), (b, 0.0, 0.8), (c, 0.1, 0.5)\}\)

Let \((F, A) \triangle (G, B) = (H, C)\), where \(C = A \cap B = \{e_1, e_2\}\). Then

\((H, C) = \{(a, \min(0.5, 0.6), \max(0.1, 0.2)), (b, \min(0.1, 0.1), \max(0.8, 0.7)), (c, \min(0.2, 0.1), \max(0.2, 0.8))\}\)

\(H(e_2) = \{(a, \min(0.7, 0.4), \max(0.1, 0.4)), (b, \min(0.0, 0.3), \max(0.8, 0.5)), (c, \min(0.3, 0.5), \max(0.5, 0.4))\}\)

\(= \{(a, 0.5, 0.2), (b, 0.1, 0.8), (c, 0.1, 0.8)\}\)

\(H(e_2) = \{(a, 0.4, 0.4), (b, 0.0, 0.8), (c, 0.3, 0.5)\}\)

**D. Proposition**

(i) \((F, A) \cong (\varphi, A) = (F, A)\)

(ii) \((F, A) \cong (U, A) = (F, A)^C\)

**Proof**

(i) Let \((F, A) \cong (\varphi, A) = (H, A)\), where \(\forall \nu \in A, \nu \in U\), we have

\[H(x) = \min \{\mu_{F(x)}(x), \nu_{G(x)}(x)\}\]

\[v_H(x) = \max \{\nu_{F(x)}(x), \mu_{G(x)}(x)\}\]

Therefore \((H, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall \nu \in A, x \in U\}\)

It follows that \((F, A) \cong (\varphi, A) = (F, A)\)

(ii) Let \((F, A) \cong (U, A) = (H, A)\), where \(\forall \nu \in A, x \in U\)

\[H(x) = \min \{\mu_{F(x)}(x), \nu_{U(x)}(x)\}\]

\[v_H(x) = \max \{\nu_{F(x)}(x), \mu_{U(x)}(x)\}\]

Therefore \((H, A) = \{(x, \mu_{F(x)}(x), \nu_{F(x)}(x)) \mid \forall \nu \in A, x \in U\}\)

It follows that \((F, A) \cong (U, A) = (F, A)^C\)
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$$v_{H(e)}(x) = \max \left\{ v_{F(e)}(x), \mu_{U(e)}(x) \right\}$$
$$= \max \left\{ v_{F(e)}(x), 1 \right\}$$
$$= 1$$
Therefore $$\langle H, A \rangle = \{(0,1), \forall e \in A, x \in U\}$$

It follows that $$\langle F, A \rangle \subseteq (H, A) = (\varnothing, A)$$

**E. Definition (\(\alpha, \beta\) – Cut Soft Set of an Intuitionistic Fuzzy Soft Set)**

Let $$\langle F, A \rangle$$ be an intuitionistic fuzzy soft set over \((U, E)\). We define the \((\alpha, \beta)\) – cut soft set of the intuitionistic fuzzy soft set \((F, A)\), denoted by \((F, A)_{(\alpha, \beta)}\), as the soft set \((F_{(\alpha, \beta)} A), \forall e \in A, \,$$

$$F_{(\alpha, \beta)}(e)$$

$$= \{x : \mu_{F(e)}(x) \geq \alpha, v_{F(e)}(x) < \beta; x \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1\}$$

**F. Example**

Let $$U = \{a, b, c\}$$ and $$E = \{e_1, e_2, e_3, e_4\}$$, \(A = \{e_2, e_3, e_4\} \subseteq E\). Let us consider an intuitionistic fuzzy soft set \((F, A)\) as \((F, A) = \{F(e_2) = \{(a,0.3,0.2) \mid (b,0.1,0.8) \mid (c,0.4,0.5)\},\ F(e_3) = \{(a,0.7,0.2) \mid (b,0.4,0.3) \mid (c,0.5,0.1)\},\ F(e_4) = \{(a,0.6,0.2) \mid (b,0.3,0.5) \mid (c,0.3,0.6)\}\}\). Let \(\alpha = 0.3, \beta = 0.5, \alpha, \beta \in [0,1]\). Then \((F, A)_{(0,3,0,5)} = \{F_{(0,3,0,5)} A)\), \(F_{(0,3,0,5)}(e_2) = \{a, c\}\), \(F_{(0,3,0,5)}(e_3) = \{a, b, c\}\), \(F_{(0,3,0,5)}(e_4) = \{a, b\}\)

**G. Definition (\(\alpha, \beta\) – Cut Strong Soft Set of an Intuitionistic Fuzzy Soft Set)**

Let \((F, A)\) be an intuitionistic fuzzy soft set over \((U, E)\). We define the \((\alpha, \beta)\) – cut strong soft set \((F_{(\alpha, \beta)} A)\) of the intuitionistic fuzzy soft set \((F, A)\), denoted by \((F, A)_{(\alpha, \beta)}^s\) as the soft set, where \(\forall e \in A, \,$$

$$F_{(\alpha, \beta)}^s(e)$$

$$= \{x : \mu_{F(e)}(x) > \alpha, v_{F(e)}(x) < \beta; x \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1\}$$

**H. Example**

Let $$U = \{a, b, c\}$$ and $$E = \{e_1, e_2, e_3, e_4\}$$, \(A = \{e_2, e_3, e_4\} \subseteq E\). Let us consider an intuitionistic fuzzy soft set \((F, A)\) as \(F(e_2) = \{(a,0.3,0.2) \mid (b,0.1,0.8) \mid (c,0.4,0.5)\},\ F(e_3) = \{(a,0.7,0.2) \mid (b,0.4,0.3) \mid (c,0.5,0.1)\},\ F(e_4) = \{(a,0.6,0.2) \mid (b,0.3,0.5) \mid (c,0.3,0.6)\}\). Let \(\alpha = 0.3, \beta = 0.5, \alpha, \beta \in [0,1]\). Then \((F, A)_{(0,3,0,5)} = \{F_{(0,3,0,5)}(e_2) = \{a, c\}\), \(F_{(0,3,0,5)}(e_3) = \{a, b, c\}\), \(F_{(0,3,0,5)}(e_4) = \{a, b\}\)

**I. Proposition**

Let \((F, A), (G, B)\) be two intuitionistic fuzzy soft sets over \((U, E)\). Then the following results hold for all \(\alpha, \beta \in [0,1]\).

(i) \((F, A) \subseteq (G, B) \) \(\Rightarrow (F, A)_{(\alpha, \beta)} \subseteq (G, B)_{(\alpha, \beta)} \)

(ii) \((F, A) \cap (G, B)_{(\alpha, \beta)^+} = (F, A)_{(\alpha, \beta)} \cap (G, B)_{(\alpha, \beta)^+} \)

Proof

(i) \((F, A) \subseteq (G, B) \) \(\Rightarrow (F, A)_{(\alpha, \beta)} \subseteq (G, B)_{(\alpha, \beta)} \)

(ii) \((F, A) \cap (G, B)_{(\alpha, \beta)^+} = (F, A)_{(\alpha, \beta)} \cap (G, B)_{(\alpha, \beta)^+} \)\)

The reverse inclusion here is not valid which is clear from the following example –

**J. Example**

Let $$U = \{a, b, c\}$$ and $$E = \{e_1, e_2, e_3, e_4\}$$, \(A = \{e_1, e_2, e_4\} \subseteq E\). Let us consider an intuitionistic fuzzy soft set \((F, A)\) as \(F(e_1) = \{(a,0.1,0.8) \mid (b,0.2,0.7) \mid (c,0.4,0.5)\},\ F(e_2) = \{(a,0.7,0.5) \mid (b,0.5,0.1)\},\ F(e_4) = \{(a,0.6,0.2) \mid (b,0.5,0.1)\}\). Let \(\alpha = 0.3, \beta = 0.5, \alpha, \beta \in [0,1]\). Then \((F, A)_{(0,3,0,5)} = \{F_{(0,3,0,5)}(e_1) = \{b, c\}\), \(F_{(0,3,0,5)}(e_2) = \{a, c\}\), \(F_{(0,3,0,5)}(e_4) = \{a, c\}\)

Here

$$\langle F, A \rangle_{(0,3,0,6)} \subseteq \langle G, B \rangle_{(0,3,0,6)}$$

It is clear that \((F, A)_{(0,3,0,6)} \subseteq \langle G, B \rangle_{(0,3,0,6)}\) but \((F, A) \not\subseteq (G, B)\) as \(\mu_{F(e_2)}(a) = 0.6\) and \(\mu_{G(e_2)}(a) = 0.3\), \(v_{F(e_2)}(a) = 0.3\) and \(v_{G(e_2)}(a) = 0.4\)

Thus \(\mu_{F(e_2)}(a) > \mu_{G(e_2)}(a), v_{F(e_2)}(a) < v_{G(e_2)}(a)\)

\(\langle F, A \rangle \sim \langle G, B \rangle\) \(\neq\) \(\langle H, C \rangle\). Then \(C = A \cup B\) and \(\forall e \in C, \)

\(F(e)\), if \(e \in A - B\)

\(G(e)\), if \(e \in B - A\)

\(F(e) \cup G(e)\), if \(e \in A \cap B\)
Now \((F,A)\cup (G,B)\) is \((H,C)\), where \(C = A \cup B\) and \(\forall \in C\).

Let \(x \in H(\alpha,\beta)\) for some \(\in C\). Then

\[
\mu_{H(\alpha,\beta)}(x) = \begin{cases}
\mu_{F(\alpha)}(x), & \text{if } x \in A - B \\
\mu_{G(\alpha)}(x), & \text{if } x \in B - A \\
\max\{\mu_{F(\alpha)}(x), \mu_{G(\alpha)}(x)\}, & \text{if } x \in A \cap B
\end{cases}
\]

\[
\nu_{H(\alpha,\beta)}(x) = \begin{cases}
\nu_{F(\alpha)}(x), & \text{if } x \in A - B \\
\nu_{G(\alpha)}(x), & \text{if } x \in B - A \\
\min\{\nu_{F(\alpha)}(x), \nu_{G(\alpha)}(x)\}, & \text{if } x \in A \cap B
\end{cases}
\]

Thus \((H,C)\) is \((F,A)\cup (G,B)\).

For the converse part,

\[
(F,A)\cup (G,B) = (F,A)\cup (G,B) = (I,C)
\]

where \(C = A \cup B\) and \(\forall \in C\).

Let \(x \in I(\alpha,\beta)\) for some \(\in C\). Then

\[
\mu_{I(\alpha,\beta)}(x) = \begin{cases}
\mu_{F(\alpha)}(x), & \text{if } x \in A - B \\
\mu_{G(\alpha)}(x), & \text{if } x \in B - A \\
\max\{\mu_{F(\alpha)}(x), \mu_{G(\alpha)}(x)\}, & \text{if } x \in A \cap B
\end{cases}
\]

\[
\nu_{I(\alpha,\beta)}(x) = \begin{cases}
\nu_{F(\alpha)}(x), & \text{if } x \in A - B \\
\nu_{G(\alpha)}(x), & \text{if } x \in B - A \\
\min\{\nu_{F(\alpha)}(x), \nu_{G(\alpha)}(x)\}, & \text{if } x \in A \cap B
\end{cases}
\]

Thus \((F,A)\cup (G,B)\) is \((F,A)\).
It can also be verified that \((F, A)^{c} (\beta, \alpha) + \subseteq (F, A)^{c} (\alpha, \beta)\) and the result follows immediately.

**K. Proposition**

Let \((F, A)\) be an intuitionistic fuzzy soft set over \((U, E)\) and \(\alpha, \beta, \gamma, \delta \in [0,1]\). Then the following results hold.

(i) \((F, A)(\alpha, \beta)+ \subseteq (F, A)(\alpha, \beta)\)

(ii) \(\alpha \leq \gamma, \beta \geq \delta \Rightarrow (F, A)(\alpha, \beta) \subseteq (F, A)(\gamma, \delta)\), \(\quad (F, A)(\alpha, \beta)+ \subseteq (F, A)(\gamma, \delta)+\)

**Proof**

(i) Let \((F, A)\) be an intuitionistic fuzzy soft set over \((U, E)\).

Then \(\forall \in \in A,\)

\[
F_{(\alpha, \beta)}(\in) = \left\{ \left. x : \mu_{F(\in)}(x) > 0, \nu_{F(\in)}(x) < 0 \right| \in \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1 \right\}
\]

\[
\subseteq \left\{ \left. x : \mu_{F(\in)}(x) \geq 0, \nu_{F(\in)}(x) \leq 0 \right| \in \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1 \right\}
\]

\[
= F_{(\alpha, \beta)}(\in)
\]

Therefore \((F, A)(\alpha, \beta)+ \subseteq (F, A)(\alpha, \beta)\)

(ii) Let \((F, A)\) be an intuitionistic fuzzy soft set over \((U, E)\) and \(\alpha \leq \gamma, \beta \geq \delta\)

Then \(\forall \in \in A,\)

\[
F_{(\alpha, \beta)}(\in) = \left\{ \left. x : \mu_{F(\in)}(x) \geq 0, \nu_{F(\in)}(x) \leq 0 \right| \in \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1 \right\}
\]

\[
\subseteq \left\{ \left. x : \mu_{F(\in)}(x) \geq 0, \nu_{F(\in)}(x) \leq 0 \right| \in \in U, \gamma, \delta \in [0,1], \gamma + \delta \leq 1 \right\}
\]

\[
= F_{(\gamma, \delta)}(\in)
\]

Therefore \((F, A)(\gamma, \delta) \subseteq (F, A)(\alpha, \beta)\)

Also \(\forall \in \in A,\)

\[
F_{(\gamma, \delta)}(\in) = \left\{ \left. x : \mu_{F(\in)}(x) > 0, \nu_{F(\in)}(x) < 0 \right| \in \in U, \alpha, \beta \in [0,1], \alpha + \beta \leq 1 \right\}
\]

\[
\subseteq \left\{ \left. x : \mu_{F(\in)}(x) > 0, \nu_{F(\in)}(x) < 0 \right| \in \in U, \gamma, \delta \in [0,1], \gamma + \delta \leq 1 \right\}
\]

\[
= F_{(\gamma, \delta)}(\in)
\]

Therefore \((F, A)(\gamma, \delta)+ \subseteq (F, A)(\alpha, \beta)+\)

**IV. CONCLUSION**

In our work, we have put forward some new concepts such as disjunctive sum, difference, \((\alpha, \beta)\) - cut soft set and \((\alpha, \beta)\) - cut strong soft set of an intuitionistic fuzzy soft set. Some related properties have been established with examples and counter examples. It is hoped that our work will enhance this study in intuitionistic fuzzy soft sets.

**REFERENCES**