Performance Analysis of Singular Value Decomposition (SVD) and Radial basis Function (RBF) Neural Networks for Epilepsy Risk Levels Classifications from EEG Signals

R.Hari Kumar, B.Vinoth Kumar, K.Karthik, Jagdsh L.K.Chand, C.Navin Kumar

Abstract—The objective of this paper is to compare the performance of Singular Value Decomposition (SVD) method and Radial Basis Function (RBF) Neural Network for optimization of fuzzy outputs in the epilepsy risk level classifications from EEG (Electroencephalogram) signals. The fuzzy pre classifier is used to classify the risk levels of epilepsy based on extracted parameters like energy, variance, peaks, sharp and spike waves, duration, events and covariance from the EEG signals of the patient. SVD and RBF neural network is exploited on the classified data to identify the optimized risk level (singleton) which characterizes the patient’s epilepsy risk level. The efficacy of the above methods is compared based on the benchmark parameters such as Performance Index (PI), and Quality Value (QV).

Index Terms—Singular Value Decomposition, Radial Basis Function Neural Network, Fuzzy Techniques, EEG Signals, Epilepsy risk levels.

I. INTRODUCTION

Epilepsy is a chronic disease characterized from recurrent seizures that cause sudden but reversible changes in the brain functions. Classification of epilepsy risk levels, according to international standard is difficult because individual laboratory findings and symptoms are often inconclusive [1]. Approximately 1% of the people in the world suffer from epilepsy. The electroencephalogram (EEG) signal is used for the purpose of epileptic detection as it is a condition related to the brain’s activity. EEG is an important clinical tool for diagnosing, monitoring and managing neurological disorders related to epilepsy [2].

This disorder is characterized by sudden recurrent and transient disturbances of mental function and or movements of body that results in excessive discharges group of brain cells [3]. The presence of epileptiform activity in the EEG confirms the diagnosis of epilepsy, which sometimes confused with other disorders producing similar seizure like activity. Between seizures, the EEG of a patient with epilepsy may be characterized by occasional epileptic form transients-spikes and sharp waves.

With real-time monitoring to detect epileptic seizures gaining wide spread recognition, the advent of computers has made it possible to effectively apply a host of methods to quantify the changes occurring based on the EEG signals [8]. One of them is a classification of risk level of epilepsy using Fuzzy techniques and Genetic algorithms [6]. This paper addresses the application and comparison of Singular Value Decomposition (SVD) and RBF neural networks as post classifier towards optimization of fuzzy outputs in the classification of epilepsy risk levels.

II. MATERIALS AND METHODS

The EEG data used in the study were acquired from ten epileptic patients who had been under the evaluation and treatment in the Neurology department of Sri Ramakrishna Hospital, Coimbatore, India. A paper record of 16 channel EEG data is acquired from a clinical EEG monitoring system through 10-20 international electrode placing method. With an EEG signal free of artifacts, a reasonably accurate detection of epilepsy is possible; however, difficulties arise with artifacts [2]. With the help of neurologist, the artifact free EEG records with distinct features were selected. These records were scanned by Umax 6696 scanner with a resolution of 600dpi.

Figure1. Fuzzy Techniques and SVD, RBF System for Epilepsy Risk level Classification

Figure 1 enumerates the overall epilepsy risk level (Fuzzy-SVD/ RBF) classifier system. The motto of this research is to classify the epilepsy risk level of a patient from EEG signal parameters. Alison A.et al4, and Hauquo Got man5 mentioned about the extraction of prominent features in EEG signals for detection and classification of epilepsy and the same is explained as follows,

1. The energy in each two-second epoch is given by

\[ E = \sum_{i=1}^{n} x_i^2 \]  

Where \( x_i \) is signal sample value and \( n \) is number of samples. The scaled energy is taken by dividing the energy term by 1000.

Manuscript received on September 2012.

Dr.R.Harikumar, Professor, ECE Department, Bannari Amman Institute of Technology, Sathyamangalam, India.

B.Vinnoth Kumar, Assistant Professor (Sr.Gr),EEE Department, Bannari Amman Institute of Technology, Sathyamangalam, India.

K.Karthik, Jagdsh L.K.Chand, C.Navin Kumar, UG Students (ECE) Department of ECE, Bannari Amman Institute of Technology, Sathyamangalam, India.
2. The total number of positive and negative peaks exceeding a threshold is found.

3. Spikes are detected when the zero crossing duration of predominantly high amplitude peaks in the EEG waveform lies between 20 and 70 ms and sharp waves are detected when the duration lies between 70 and 200ms.

4. The total numbers of spike and sharp waves in an epoch are recorded as events.

5. The variance is computed as \( \sigma \) given by

\[
\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}
\]

Where \( \mu = \frac{\sum_{i=1}^{n} x_i}{n} \) is the average amplitude of the epoch.

6. The average duration is given by

\[
D = \frac{\sum_{i=1}^{p} t_i}{p}
\]

Where \( t_i \) is one peak to peak duration and \( p \) is the number of such durations.

7. Covariance of Duration. The variation of the average duration is defined by

\[
CD = \frac{\sum_{i=1}^{p} (D - t_i)^2}{pD^2}
\]

**A. Fuzzy Membership Functions**

The energy is compared with the other six input features to give six outputs. Each input feature is classified into five fuzzy linguistic levels viz., very low, low, medium, high and very high [3], [19]. The triangular membership functions (a simple one) are used for the linguistic levels of energy, peaks, variance events, spike and sharp waves, average duration and covariance of duration. The output risk level is classified into five linguistic levels namely normal, low, medium, high and very high. Rules are framed in the format.

**IF Energy is low AND Variance is low THEN Output Risk Level is low**

**B. Estimation of Risk Level in Fuzzy Outputs**

The output of a fuzzy logic represents a wide space of risk levels. This is because there are sixteen different channels for input to the system at three epochs. This gives a total of forty-eight input output pairs. Since we deal with known cases of epileptic patients, it is necessary to find the exact level of risk the patient. A specific coding method processes the output fuzzy values as individual code. Since working on definite alphabet is easier than processing numbers with large decimal accuracy, we encode the outputs as a string of alphabets. The alphabetical representation of the five classifications of the outputs is as shown below.

Normal U, Low W, Medium X, High Y, Very High Z

A sample output of the fuzzy system with actual patient readings is shown in figure 2 for eight channels over three epochs. It can be seen that the Channel 1 shows medium risk levels while channel 8 shows very high risk levels. Also, the risk level classification varies between adjacent epochs. The Performance of the Fuzzy method is defined as follows [5]

\[
PI = \frac{PC - MC - FA \times 100}{PC}
\]

Where PC – Perfect Classification, MC – Missed Classification, FA – False Alarm, PI= [(0.5-0.2-0.1)/0.5] *100 =40%. These perfect classification represents when the physicians and fuzzy classifier agrees with the epilepsy risk level. Missed classification represents a true negative of fuzzy classifier in reference to the physician and shows High level as Low level. The performance for Fuzzy classifier is as low as 40%. Singular Value Decomposition (SVD) optimization technique (post classifier) [6] is utilized to optimize risk level and is given below.

<table>
<thead>
<tr>
<th>Epoch 1</th>
<th>Epoch 2</th>
<th>Epoch 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>YYYYXX</td>
<td>ZYYWXX</td>
<td>YYYYXYZ</td>
</tr>
<tr>
<td>YYYYXY</td>
<td>ZZYYZZ</td>
<td>YYYYXYZ</td>
</tr>
<tr>
<td>YYYYYY</td>
<td>ZZZYZZ</td>
<td>YYYYYY</td>
</tr>
<tr>
<td>YYYYYY</td>
<td>YYYYXX</td>
<td>YYYYXX</td>
</tr>
<tr>
<td>YYYYYY</td>
<td>YYYYYY</td>
<td>YYYYYY</td>
</tr>
<tr>
<td>ZZZYZZ</td>
<td>ZZYYZZ</td>
<td>ZZYYZZ</td>
</tr>
</tbody>
</table>

Figure 2. Fuzzy Logic Output

### III. SINGULAR VALUE DECOMPOSITION FOR OPTIMIZATION OF FUZZY OUTPUTS

Our objective is to merge the epilepsy risk level representation, with approximate reasoning capabilities, and symbolic decision trees while preserving advantages of both: uncertainty handling and gradual processing of the former with the comprehensibility, popularity, and ease of application of the later.

The singular value decomposition is a well-known approach that may be used for such tasks as dimensionality reduction, and determining the modes of a complex linear dynamical system [7]. SVD of a matrix has one or more columns that are identical, or that several groups of columns that are same which is useful in signal processing problems and applications. A SVD of an \( m \times n \) matrix \( A=[a_1, a_2, \ldots, a_n] \) is the composition of \( A \) into the product of three matrices as follows

\[
A=U \Sigma V^T = \Sigma \ P \; \mathbf{u}_k \; \mathbf{v}_k^T
\]

Where \( p=\min(m,n) \), \( U=[u_1, u_2, \ldots, u_n] \) is an \( m \times n \) orthogonal normal matrix, \( V=[v_1, v_2, \ldots, v_n] \) is an \( n \times m \) ortho normal matrix, and \( \Sigma \) is an \( m \times n \) matrix with elements \( \sigma_{k} \) along the diagonal and zeros everywhere else. Matrix \( U \) is called left singular matrix, \( V \) is called right singular matrix, and \( \Sigma \) is the singular value matrix [7]. If the singular values are ordered so that \( \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_p \), and if the matrix \( A \) has a rank \( r<p \), then the last \( p-r \) singular values are equal to zero, and SVD becomes \( A= \Sigma \ P \; \mathbf{u}_k \; \mathbf{v}_k^T \).

SVD procedure takes vectors in one space and transforms them into another space. Advantages in using SVD to combine two different uncertainty representations into a metric as total uncertainty. SVD decomposes uncertainty measures (possibility, belief, probability etc.), combined as a collection of vectors of different units, into a principle space. We need this feature since our uncertainty measures cannot be added directly, they contain different units (epilepsy risk level codes). SVD has been

\[
\text{PC} = \frac{PI \times PC}{100}
\]
applied successfully in many other technical disciplines as a tool to reduce coupled non-linear behavior to uncoupled collections of linear behavior.

The fuzzy outputs are (16x3 matrix) considered as matrix A and SVD is taken for that matrix. The highest Eigen value is considered as the pattern of the known patient's epilepsy risk level. A group of ten patients are analyzed in this study. The obtained singleton results are immensely helpful in devising the therapeutic procedure of the epileptic patients.

IV. ROLE OF NEURAL NETWORKS IN THE OPTIMIZATION OF FUZZY OUTPUTS

Unlike traditional classifiers, ANN models can examine numerous competing hypotheses simultaneously using massive interconnections among many simple processing elements. In addition, ANNs perform extremely well under noise and distortion [15]. Since, ANN is data driven self-adaptive methods in that they adjust themselves to the data without any explicit specification of functional or distribution form for the underlying model. They are universal functional approximators in that neural networks can approximate any function with arbitrary accuracy [16]. Finally, neural networks are able to estimate the posterior probabilities, which provide the basis for establishing optimization rule and performing statistical analysis [17]. Although many types of neural networks can be used for classification purposes, our focus nonetheless is on Radial Basis Function (RBF) and Multilayer Perceptron (MLP) which are the most widely studied and used Neural networks. Most of the issues discussed in the paper can also apply to other neural network model also [18].

The Radial Basis Function (RBF) neural network is widely used for function approximation, pattern classification and recognition due to its structural simplicity, universal approximators, and faster learning abilities due to locally tuned neurons [8].

A. Architecture of an RBF Neural Network

The architecture of an RBF neural network is shown in fig. 3. It consists of one input layer, one hidden layer and one output layer. Each input neuron is corresponds to an element of an input vector and is fully connected to the hidden layer neurons and the bias neuron. Again, each of the hidden neuron and the bias neuron also fully connected to the output neurons. The output of a hidden layer neuron is usually generated by a Gaussian function as follows [12],

\[ \phi_i(X) = \begin{cases} \exp\left(-\frac{\|X - t_i\|^2}{2\sigma_i^2}\right) & ; i = 1,2,\ldots,n \\ 1 & ; i = 0(\text{bias neuron}) \end{cases} \]  

Where X is an input vector and ti, σi are the center and the width of the respective field of the i\textsuperscript{th} neuron of the hidden layer respectively. The number of neurons in the output layer is equal to the possible classes of the given problem. Each output layer neuron computes a linear weighted sum of the outputs of the hidden layer neurons as follows[10]:

\[ z_j = \sum_{i=0}^{n} \phi_i(X)w_{ij}; j = 1,2,\ldots,c, \]  

Where wij is the weight between ith hidden layer neuron and jth output layer neuron.
Performance Analysis of Singular Value Decomposition (SVD) and Radial basis Function (RBF) Neural Networks for Epilepsy Risk Levels Classifications from EEG Signals

Table 1. Estimation of MSE in Various RBF Network Architectures

<table>
<thead>
<tr>
<th>Methods</th>
<th>Weighted delay (s)</th>
<th>False-alarm rate/set</th>
<th>Performance Index %</th>
<th>Quality value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy logic</td>
<td>4</td>
<td>0.2</td>
<td>40</td>
<td>6.25</td>
</tr>
<tr>
<td>SVD Method</td>
<td>1.9832</td>
<td>0.0292</td>
<td>95.88</td>
<td>21.99</td>
</tr>
<tr>
<td>RBF Network</td>
<td>1.978</td>
<td>0.0108</td>
<td>98.92</td>
<td>23.98</td>
</tr>
</tbody>
</table>

V. RESULTS AND DISCUSSIONS

To study the relative performance of these Fuzzy techniques, SVD, and RBF Neural networks, we measure two parameters, namely the Performance Index and the Quality Value. These parameters are calculated for each set of ten patients and compared.

A. Performance Index

A sample of Performance Index for a known epilepsy data set at average value is shown in table 2.

Table 2. Performance Index

<table>
<thead>
<tr>
<th>Methods</th>
<th>Perfect Classification</th>
<th>Missed Classification</th>
<th>False Alarm</th>
<th>Performance Index %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy logic</td>
<td>50</td>
<td>20</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>SVD Method</td>
<td>96.04</td>
<td>1.04</td>
<td>2.92</td>
<td>95.88</td>
</tr>
<tr>
<td>RBF Network</td>
<td>98.92</td>
<td>-</td>
<td>1.08</td>
<td>98.92%</td>
</tr>
</tbody>
</table>

It is evident that the RBF Neural network optimization gives a better performance than the fuzzy techniques and SVD due to its lower false alarms and no missed classifications. But at the same time SVD post classifier is also performed well than the basic fuzzy classifier in terms of Performance Index. When compare to missed classification and false alarm rate SVD is inferior to RBF neural network however, the computation complexity for achieving higher performance in the RBF neural network is very high rather than SVD method. Depends upon applications any one of the post classifiers can be chosen.

B. Quality Value

The quality value is determined by three factors. Classification rate, Classification delay, and False Alarm rate. The Quality Value $Q_V$ is defined as [5]

$$Q_V = \frac{C}{R_{fa} + 0.2(T_{dly} * P_{det} + 6 * P_{miss})}$$

(10)

Where, $C$ is the scaling constant, $R_{fa}$ is the number of false alarm per set, $T_{dly}$ is the average delay of the on set classification in seconds, $P_{det}$ is the percentage of perfect classification and $P_{miss}$ is the percentage of perfect risk level missed. Table 3 shows the Comparison of the fuzzy, SVD, and RBF neural network optimization techniques. It is also observed that RBF Neural network index and quality values than its counterparts.

Table 3. Results of Classifiers taken as Average of all ten Patients

<table>
<thead>
<tr>
<th>Architecture</th>
<th>Train MSE Index</th>
<th>Test MSE Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-16-1</td>
<td>3.3E-08</td>
<td>3.3E-08</td>
</tr>
<tr>
<td>2-8-2</td>
<td>4.21E-07</td>
<td>4.21E-07</td>
</tr>
<tr>
<td>4-4-4</td>
<td>3.4E-07</td>
<td>3.4E-07</td>
</tr>
<tr>
<td>8-2-8</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>16-1-16</td>
<td>0</td>
<td>2.94E-04</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

In this paper a generic classification of the epilepsy risk level of epileptic patients from EEG signals was considered. Since, the fuzzy outputs are highly nonlinear in nature with dynamic probability functions. SVD and RBF neural networks were chosen to optimize the risk level by incorporating the low false alarm and near nil missed classifications. RBF neural network performs better than SVD and Fuzzy Techniques with high PI, Quality value and with moderate time delay. Further research is in the direction to compare the SVD with Support Vector Machine (SVM) model to solve this open end problem.

VII. ACKNOWLEDGEMENTS

The authors thank the Management and the Principal of Bannari Amman Institute of Technology, Sathyamangalam for providing excellent computing facilities and encouragement. This research is also funded by AICTE RPS.: FNo 8023/BOR/RID/RPS-41/2009-10, dated 10th Dec 2010.

REFERENCES


**Dr. R. Harikumar** received his B.E (ECE) Degree from REC (NIT) Trichy 1988. He obtained his M.E (Applied Electronics) degree from College of Engineering, Guindy, Anna University Chennai in 1990. He was awarded Ph.D. in I &C Engg from Anna university Chennai in 2009. He has 22 years of teaching experience at college level. He worked as faculty at Department of ECE, PSNA College of Engineering & Technology, Dindigul. He was Assistant Professor in IT at PSG College of Technology, Coimbatore. He also worked as Assistant Professor in ECE at Amrita Institute of Technology, Coimbatore. Currently he is working as Professor ECE at Bannari Amman Institute of Technology, Sathyamangalam. He has published forty three papers in International and National Journals and also published around seventy papers in International and National Conferences conducted both in India and abroad. His area of interest is Bio signal Processing, Soft computing, VLSI Design and Communication Engineering. He is guiding nine PhD theses in these areas. He is life member of IETE, ISTE and member IEEE.

**B. Vinoth Kumar** received his B.E (EEE) degree from Bannari Amman Institute of Technology, Sathyamangalam, Bharathiar University, Coimbatore in 2002. He obtained his M.E (Applied Electronics) degree from Government College of Technology, Coimbatore in 2005. He has six years of teaching experience at college level. Currently he is working as Assistant Professor (Senior Grade) in Department of EEE at Bannari Amman Institute of Technology, Sathyamangalam. His area of interest is Medical Image Processing, Soft computing and Embedded systems.

**K.Karthik** pursuing his B.E (ECE) degree from Bannari Amman Institute of Technology, Sathyamangalam. His area of interest is bio signal Processing, Soft computing, and Image processing.

**C. Navin Kumar** pursuing his B E (ECE) degree from Bannari Amman Institute of Technology, Sathyamangalam. His area of interest is bio signal Processing, Telemedicine and Image processing.

**Jagdish L. K. Chand** pursuing his B E (ECE) degree from Bannari Amman Institute of Technology, Sathyamangalam. His area of interest is bio signal Processing, Embedded system and Image processing.