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Parameter Estimation in Wireless Sensor Networks with Normally Distributed Sensor Gains

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Abstract— Wireless sensor networks (WSN) have attracted significant attention recently. The distributed estimation problem is an important research topic in WSNs. In the distributed estimation problem, the fusion center estimates an unknown parameter based on information gathered from sensors. Usually, it is assumed that sensors have identical gains. However, this may not be true due to manufacture errors or environmental influence. In this paper, we assume sensor gains follow normal distribution and present a maximum likelihood estimation (MLE) approach for distributed estimation in WSNs with normally distributed sensor gains. Moreover, the Cramer-Rao lower bound (CRLB) corresponding to this MLE approach is also derived. Simulation results showed that the root square mean (RMS) estimation errors given by this MLE approach were close to the CRLB if the variance of the sensor gains is small. If the variance of the sensor gains was large, the RMS estimation errors were not close to the CRLB.

Keywords— Distributed estimation, maximum likelihood estimation, Gaussian distribution, wireless sensor networks.

I. INTRODUCTION

Due to a vast number of applications, wireless sensor networks (WSNs) have gained significant attention [1]-[13]. Usually, a WSN consists of many sensors which will send gathered informaiton to a fusion center [14]. After acquiring information from sensors, the fusion center can perform many tasks such as tracking, detection and distributed estimation [14]. Distributed estimation is of particular interests because it serves as a cornerstone for many other tasks.

In the distributed estimation problem, the fusion center tries to estimate unknown parameters based on information from sensors. To estimate one unknown parameter, a maximum likelihood estimation (MLE) approach was presented in [15][16]. Then, the MLE approach was extended to consider imperfect communication channels in [17]. However, in [15]-[17], the sensor gains were assumed to be identical while in many applications, sensor gains are not identical due to manufacture errors or environmental influences.

In this paper, we assume that sensor gains follow a Gaussian distribution. This assumption is valid when the sensor gains are determined by a sum of many factors. If this is true, the sensor gains will follow a Gaussian distribution [18]. Then, the distribution information of sensor gains is incorporated into the MLE framework to address heterogeneous sensor gains.

The main contribution of this paper is a MLE approach for distributed estimation in WSNs with normally distributed sensor gains. Moreover, the Cramer-Rao lower bound (CRLB) corresponding to this MLE approach is also derived. Simulation results showed that when the variance of the Gaussian distribution is small, root square mean (RMS) estimation errors were close to the CRLB. When the variance of the Gaussian distribution was large, RMS estimation errors deviated from the CRLB.

This paper is organized in the following way. Section II presented the MLE approach for distributed estimation in WSNs with normally distributed sensor gains. Section III presents the CRLB, followed by simulation setup in Section IV. Section V provides results and analysis. Finally, Section VI delivers concluding remarks.

II. DISTRIBUTED ESTIMATION METHOD IN WIRELESS SENSOR NETWORKS WITH NORMALLY DISTRIBUTED SENSOR GAINS

A WSN consists of a fusion center and many sensors (Figure 1). After sensors measure the parameter θ, sensors will quantize the measurement according to a set of pre-determined thresholds $\vec{\gamma}_i$. Sending quantized data instead of analogy data to the fusion center can save communication bandwidth and sensor energy [14]. The fusion center can use a MLE approach to estimate the unknown parameter θ based on information from sensors. Now, this MLE approach is discussed in details.

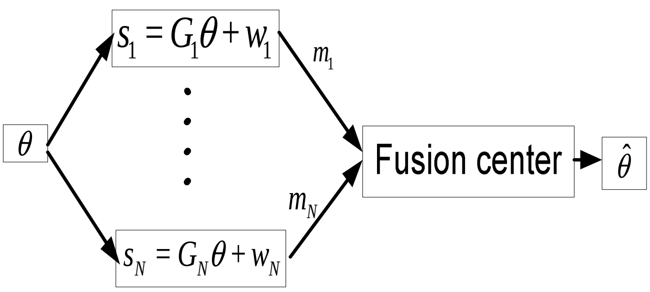

Figure 1: Distributed estimation system diagram

Following the setup in [11][14]-[16], we use N sensors to estimate the unknown parameter θ. However, sensor gains are not identical and follow the normal distribution. The gain of the sensor i is G_i, and G_i follows Normal distribution with mean u and variance σ_1^2

$$f(G_i) = \frac{1}{\sqrt{2\pi}\sigma_1} e^{-\frac{(G_i-u)^2}{2\sigma_1^2}}, \quad G_i \in (-\infty, +\infty). \tag{1}$$

The ith sensor receives the signal from θ and this signal can be denoted as a_i, which is defined as

$$a_i = G_i\theta. \tag{2}$$

Because of the presence of noises, the signal arrives at the ith sensor is s_i, which can be defined as

$$s_i = a_i + w_i. \tag{3}$$

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The noise w_i in (3) is a Gaussian noise with zero mean and variance σ_2^2.

The propbality density function (PDF) of the sum of two Normally distributed random variables also follows Normal distribution [18]. Therefore, we can have

$$f(s_i)=\frac{1}{\sqrt{2\pi}\sqrt{(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}e^{-\frac{(s_i-u)^2}{2(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}, \quad . \tag{4}$$
$$G_i\in(-\infty,+\infty)$$

After a sensor acquires the measurement, the sensor quantizes the measurement into a decision m_i according to a set of pre-determined thresholds $\vec{\gamma}_i$

$$\vec{\gamma}_i=[\gamma_{i0},\gamma_{i1},...,\gamma_{iL}]. \tag{5}$$

The quantization process can be expressed by

$$m_i=\begin{cases}0 & -\infty<s_i<\gamma_{i1}\\ 1 & \gamma_{i1}<s_i<\gamma_{i2}\\ \vdots & \vdots\\ L-2 & \gamma_{i(L-2)}<s_i<\gamma_{i(L-1)}\\ L-1 & \gamma_{i(L-1)}<s_i<\infty \end{cases}. \tag{6}$$

For a specific θ, the probability that m_i is equal to l is

$$p_{il}(\vec{\gamma}_i,\theta)=R(\gamma_{il})-R(\gamma_{i(l+1)})=\int_{\gamma_{il}}^{\gamma_{i(l+1)}}f(s_i)ds_i \tag{7}$$

In (7), $R(x)$ is defined as

$$R(x)=\int_x^{\infty}f(s_i)ds_i \tag{8}$$

Then, sensors will send the decision vector

$$\mathbf{M}=[m_1,\ m_2,\ \cdots,m_{N-1}\ m_N] \tag{9}$$

to the fusion center, and the fusion center estimates θ by finding the θ value to maximize

$$\ln p(\mathbf{M}|\theta)=\sum_{i=1}^{N}\sum_{l=0}^{L-1}\delta(m_i-l)\ln\left[p_{il}(\vec{\gamma}_i,\theta)\right] \tag{10}$$

where

$$\delta(x)=\begin{cases}1, & x=0\\ 0, & x=0\end{cases}. \tag{11}$$

The maximum likelihood estimator can be expressed as

$$\hat{\theta}=\max_{\theta}\ln p(\mathbf{M}|\theta). \tag{12}$$

If an unbiased estimate of θ exists, the CRLB can be calculated by

$$\mathbf{J}=-E\left[\nabla_\theta\nabla_\theta^T\ln p(\mathbf{M}|\theta)\right]. \tag{13}$$

III. Performance Evaluation-Cramer-Rao Lower Bound

If the estimation result of (12) is unbiased, the $\mathbf{J}$ matrix can be derived by an approach similar to that in [6][11][14], which is

$$\frac{\partial^2\ln p(\mathbf{M}|\theta)}{\partial P_0^2}=\sum_i\sum_l\left\{\begin{aligned}&\left[-\frac{\delta(m_i-l)}{p_{il}^2(\vec{\gamma}_i,\theta)}\left[\frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial P_0}\right]^2\right]\\ &+\frac{\delta(m_i-l)}{p_{il}(\vec{\gamma}_i,\theta)}\frac{\partial^2 p_{il}(\vec{\gamma}_i,\theta)}{\partial P_0^2}\end{aligned}\right\}. \tag{14}$$

We can use $E[\delta(m_i-l)]=p_{il}(\vec{\gamma}_i,\theta)$ to simplify (14). Then, (14) can be expressed as

$$E\left[\frac{\partial^2 p(\mathbf{M}|\theta)}{\partial P_0^2}\right]=\sum_i\sum_l-\frac{1}{p_{il}(\vec{\gamma}_i,\theta)}\left[\frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial\theta}\right]^2. \tag{15}$$

In (14), $p_{il}(\vec{\gamma}_i,\theta)$ can be expressed as

$$p_{il}(\vec{\gamma}_i,\theta)=\int_{\gamma_{il}}^{\gamma_{i(l+1)}}\frac{1}{\sqrt{2\pi}\sqrt{(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}e^{-\frac{(s_i-u)^2}{2(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}ds_i. \tag{16}$$

The derivative of $p_{il}(\vec{\gamma}_i,\theta)$ can be calculated by

$$\begin{aligned}\Delta&=\frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial\theta}\\ &=\int_{\gamma_{il}}^{\gamma_{i(l+1)}}\left[\begin{aligned}&\frac{-\theta\sigma_1^2(\theta^2\sigma_1{}^2+\sigma_2{}^2)^{-1}}{\sqrt{2\pi}\sqrt{(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}e^{-\frac{(s_i-u)^2}{2(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}\\ &+\frac{(s_i-u)^2\theta\sigma_1^2}{\sqrt{2\pi}(\theta^2\sigma_1{}^2+\sigma_2{}^2)^{\frac{5}{2}}}e^{-\frac{(s_i-u)^2}{2(\theta^2\sigma_1{}^2+\sigma_2{}^2)}}\end{aligned}\right]ds_i.\end{aligned} \tag{17}$$

Then, we can have

$$\mathbf{J}=E\left[\frac{\partial^2\ln p(\mathbf{M}|\theta)}{\partial P_0^2}\right]=\sum_i\sum_l-\frac{\Delta^2}{p_{il}}. \tag{18}$$

We can calculate other elements of $\mathbf{J}$ similarly.

IV. SIMULATION SETUP

In this paper, to simplify simulations, we used binary decisions (0s and 1s). Our MLE approach for distributed estimation can be validated by comparing the normalized estimation error squared (NEES) values given by our MLE approach with the confidence interval. To generate NEES values, we set $\sigma_2=0.2$, $u=0.1$, $\theta=5$, $\gamma=5$ and varied the σ_1 value. The NEES values were averaged over 100 runs. Table 1 shows the results of NEES. It is obvious that σ_1 values can affect RMS estimation errors and CRLB. To see the effect, we set $\sigma_2=0.2$, $u=0.1$, $\theta=5$, $\gamma=5$ and varied the σ_1 value. Figure 2 shows the RMS errors and the CRLB. Similarly, γ values can affect the RMS estimation errors and CRLB. To see the effect, we set $\sigma_1=1$, $\sigma_2=0.2$, $u=0.1$, $\theta=5$, and varied the γ value. Figure 3 shows the RMS errors and CRLB.

V. RESULTS AND ANALYSIS

The 95% confidence interval for our MLE approach corresponding to 100 runs is [0.7422 1.2956] [11]. Therefore, we can see that NEES values given by the MLE approach using $\sigma_1=1$, $\sigma_1=2$ and $\sigma_1=3$ were within the confidence interval (Table I). Moreover, the MLE approach using $\sigma_1=4$ and $\sigma_1=5$ were outside the confidence interval.

As for the effect of the σ_1 value on RMS estimation errors and the CRLB, we can see that when σ_1 value was small, RMS estimation errors and the CRLB were low, and the RMS estimation errors were close to the CRLB (Figure 1). When

σ_1 value was large, RMS estimation errors and CRLB were also large, and the RMS estimation errors were not close to the CRLB. The reason is that if the variance of sensor gains is too large, which means sensor gains vary dramatically from one to another, the MLE approach cannot accommodate so different sensor gains and estimation performance will suffer.

As for the effect of γ on RMS estimation errors and CRLB, we can see that when γ was from 10 to 13, the RMS estimation errors were close to the CRLB. When γ was greater than 13, RMS errors deviated from the CRLB (Figure 3). This highlights the importance of the γ value. The reason is that if large γ value is used, not enough sensors will send 1s to the fusion center, the estimation performance will suffer. If low γ value is used, too many sensors will send 1s to the fusion center, estimation performance will also suffer.

Table 1: NEES values for different σ_1 values ($\sigma_2 = 0.2$, $u = 0.1$, $\theta = 5$, $\gamma = 5$ and 100 runs)

σ_1	1	2	3	4	5
NEES	0.9654	0.9879	0.9994	1.4028	2.7526

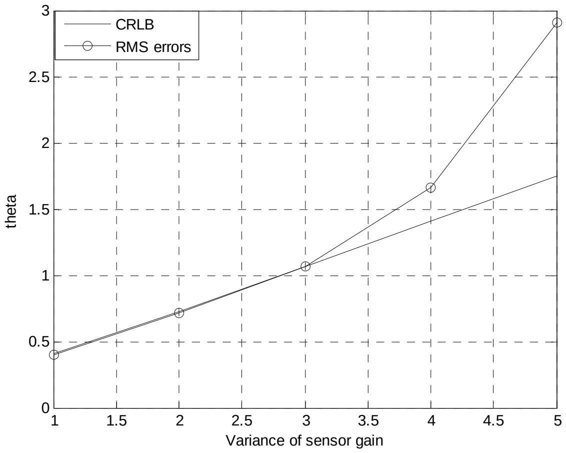

Figure 2: RMS estimation errors compared to the CRLB ($\sigma_2 = 0.2$, $u = 0.1$, $\theta = 5$, $\gamma = 5$, 100 runs and different σ_1 value)

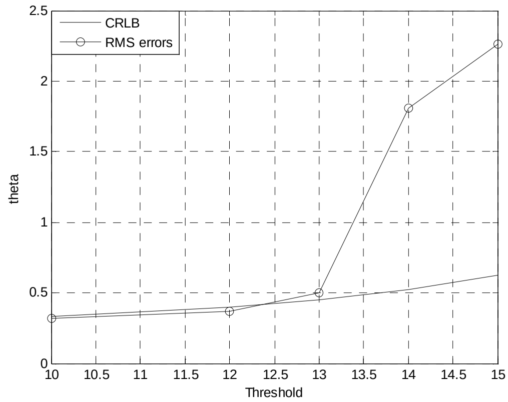

Figure 3: RMS estimation errors compared to the CRLB ($\sigma_1 = 1$, $\sigma_2 = 0.2$, $u = 0.1$, $\theta = 5$, 100 runs and different γ values)

VI. CONCLUSION

In this paper, a MLE approach for distributed estimation in WSNs with normally distributed sensor gains was presented. This approach can alleviate performance degradation cause by heterogeneous sensor gains, which follow the Gaussian distribution. Simulation results showed that the RMS errors given by this MLE approach were close to the CRLB if the variance of the sensor gain was small. In many applications, the sensor gains are influenced by many variables and the overall effect of these variables is the sum of these variables. If the sensor gains will follow the Gaussian distribution, our MLE approach can be used to alleviate performance degradation.

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