

Study the Liquid Surface Capillary Wave Profile by Optical Method

Dibyendu Chowdhury, Sunandan Bhunia, Tarun Kumar Barik

Abstract: In this article, we describe a simple optics-based non-destructive experimental technique devised with the necessary background theoretical formulation to study the liquid properties like surface tension and surface capillary wave profile like wavelength and phase velocity of pure and impure water. Experimentally, we have estimated the surface tension of pure water using this optical method is (73.6 ± 1.5) dyne/cm. We have also studied the variation of surface tension of water with adding organic and inorganic impurities using this same method. Finally we estimate the wavelength and phase velocity of capillary wave using the measured values of surface tensions of the above liquids.

Keywords- Fraunhofer diffraction, Surface tension, Capillary wave profile

I. INTRODUCTION

Diffraction and scattering of light, as is known over centuries, are therefore capable of providing a variety of information about the diffraction element. In this article, we revisit a well-known classical method of investigating liquid surface capillary waves using light diffraction. Ranjit Bahadur et al. [1] have estimated surface tensions using molecular dynamics simulation for liquid-vapor (l-v), solid-liquid (s-l), and solid-vapor (s-v) interfaces of the NaCl-water-air system. They Reported theoretical values of surface tensions for NaCl-air interface is 114 mN m^{-1} , NaCl-soln interface is 63 mN m^{-1} , soln-air interface is 82 mN m^{-1} , and for water-air interface is 66 mN m^{-1} . Again, J. Vanhanen et al. [2] have studied the surface tension of ternary solution of sodium chloride, succinic acid and water as a function of both composition and temperature by using the capillary rise technique. Also, T. Mukhim et al. [3] have measured surface tension and electrical conductance of cetylpyridinium chloride (CPC) in water in the presence of lithium chloride, sodium chloride and potassium chloride at 295 K, 298 K and 303 K, respectively. On the other hand, P. A. Kralchevsky et al. [4] present new experimental data for the dependence of the natural pH of NaL and NaMy solutions on the carboxylate concentration at fixed content of NaCl and also obtained surface-tension isotherms of NaL and NaMy in presence of NaCl and NaOH.

In this experiment, Fraunhofer diffraction is used as a way of measuring the surface tension of water. Weisbuch and Garbag first introduced this type of experimental procedure for measurement of surface tension of low viscous liquid [5]. A wave at the surface of water is a simple illustration of the concept of waves and the wave dynamics can be dominated either by gravity or by surface tension. The wave length of the surface tension waves can be measured by light diffraction. The water-air interface appears as a diffraction grating (phase grating) for the incident laser beam. The diffraction pattern on the screen is centered on the reflected beam. It is reported that the surface tension of water decreased due to adding succinic acid at very low concentrations and increased the surface tension linearly as a function of the concentration due to adding sodium chloride [2]. There are different classical methods for determination of surface tension of liquid. But the non-destructive light diffraction method is a better supplement to ordinary static methods of measuring surface tension of liquid.

II. THEORITICAL BACKGOUND

This experiment is based on the relationship between the angular frequency (ω) and the wavevector (q) [5]-[11], when water is made to oscillate with surface tension waves. Using a theory developed by Kelemens, a dispersion relation is found based on the Lagrange's equation. The potential energy of surface deformation arises from the gravity and as well as surface tension. The gravity and surface tension pieces are added and the result is integrated over a finite length. The kinetic energy is found by integrating over the entire velocity field. The flow is assumed to be irrotational, so that the velocity can be derived from a velocity potential function. The velocity potential function and the wave displacement are related at the surface, since both are a measure of the vertical component of the velocity at the surface. The total kinetic describes a harmonic oscillator from which the dispersion relation for surface tension waves is written as –

$$\omega^2 = (gq + \frac{Aq^3}{\rho}) \text{ ----- (1)}$$

In fact gravity can be neglected because the gravitational effect on the surface tension of water surface is much smaller at low frequency. In this case the wave can be considered as a surface tension wave and the dispersion relation can be approximately written as-

$$\omega^2 = \left(\frac{A}{\rho}\right)q^3 \text{ ----- (2)}$$

Now taking logarithm of equation (2) we get,

$$2 \ln \omega = \ln \left(\frac{A}{\rho}\right) + 3 \ln q$$

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Study the Liquid Surface Capillary Wave Profile by Optical Method

$$\text{or, } \ln \omega = \frac{1}{2} \ln \left(\frac{A}{\rho} \right) + \frac{3}{2} \ln q$$

$$\text{or, } \ln \omega = \frac{3}{2} \ln q + \frac{1}{2} \ln \left(\frac{A}{\rho} \right) \text{----- (3)}$$

Equation (3) is similar to straight line equation $y = mx + c$. When we plot $\ln(\omega)$ vs $\ln(q)$; we get the information for surface tension from y-intercept.

The wave vector, q , is given by the following relation [5]

$$q = \frac{2\pi}{\lambda} \sin\left(\frac{r}{2}\right) \left[\sin\left(\theta - \frac{r}{2}\right) + \sin\left(\theta + \frac{r}{2}\right) \right]$$

$$= \frac{2\pi}{\lambda} \sin(\theta) \sin(r)$$

Where θ is the glancing angle and r is the small angle produced at scattering point due to central spot and 1st order spot. Because of $\sin(\theta) = \theta$ and $\sin(r) = r$ at small angles, the above expression can be simplified to

$$q = \frac{2 \times \pi \times \theta \times r}{\lambda} = \frac{2\pi h d}{\lambda l^2} \text{----- (4)}$$

Note that, $r = \frac{d}{l}$, $\theta = \frac{h}{l}$ respectively.

Where d = separation between the central spot and 1st order spot, l = distance of the screen from the scattering point and h = height of the central spot from water surface level. Hence, by measuring h, d, l and knowing the wavelength of incident monochromatic light (λ), one can easily estimate the wavevector (q) for a particular angular frequency (ω) of the surface tension wave.

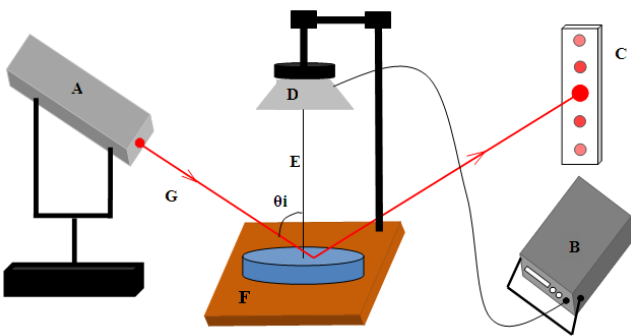


Fig.1: Schematic diagram of the experimental setup to study capillary wave on liquid; A: Laser, B: Frequency generator, C: Screen, D: Loudspeaker and E: Exciter F: Petridis G: Laser Beam

III. EXPERIMENTAL PROCEDURE

A schematic diagram of the experimental setup is shown in Fig. 1. Water is kept in a petri dish of diameter 16 cm. The depth of water in the dish is 1.0 cm. A long metallic pin is glued to the center of the diaphragm of a small loudspeaker, which is driven by a low-frequency signal generator. A sinusoidal signal of frequency (f) (400 Hz in our experiment) is used to drive the pin. This vibrating pin is the surface capillary wave exciter. The circular ripples, which are formed due to the vertically oscillating pin, act as a dynamical diffraction phase grating on the water surface (as shown in Fig. 2). Best results are obtained when the pin is just below the

liquid surface. If the pin is immersed more deeply, splashing and other associated effects can occur.

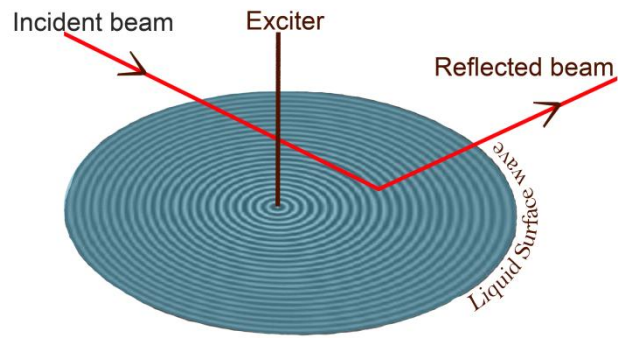


Fig.2: Schematic of surface wave profile of the dynamical phase grating after sinusoidal excitation.

A 3 mW He-Ne laser of wavelength $\lambda = 6328 \text{ \AA}$ and beam diameter 1.6 mm is directed to fall on the water surface (as shown in Fig. 1), where the surface capillary waves are formed. The angle of illumination, the angle at which the laser beam falls on the liquid surface, θ_i , in our experiment is 76° . It is known that at frequencies of the order of 400 Hz the decay length of surface capillary waves is much greater than the size of the illuminated spot [10]-[11]. Due to Fraunhofer diffraction of light by the phase grating produced by the surface capillary waves, a pattern is observed on a screen placed at a reasonably large distance, l (4.2 m in our case) from the point of diffraction. The intensity of each diffraction spot is measured by a photodiode detector. The linear relation between the photodiode current and the intensity of diffracted light was checked. To measure fringe width (d) of diffraction pattern, we use a LDR to detect the highest intensity point of each order pattern. This LDR is attached with a vertical scale of vernier constant 0.001 cm. The experimental arrangement for this purpose is shown in Fig. 3.

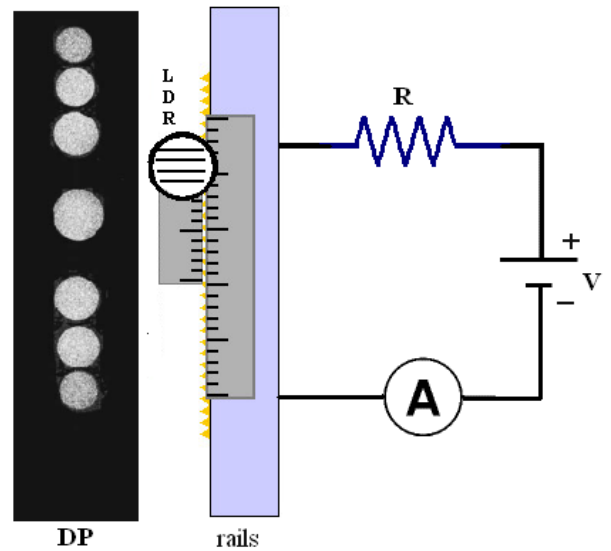


Fig.3: Experimental set-up for measurement of fringe width (d) using LDR which is attached with a vertical scale of vernier constant 0.001 cm.

IV. INTENSITY DISTRIBUTION OF DIFFRACTION SPOTS

When monochromatic light (He-Ne laser light) of wavelength λ is incident on a circular surface wave of frequency ω and wave vector q , the phase modulation produced by the surface wave is given as

$$\Phi \approx \frac{2\pi}{\lambda} \left[a \cos \theta_i \right] \sin \left(\omega t - \frac{qd}{\cos \theta_i} \right) \quad (5)$$

where the factor of $\cos \theta_i$ appears due to oblique incidence of the incident monochromatic light [9,10]. The field strength E of the diffraction pattern can be estimated from the Fourier transform of the aperture function (in this case the surface wave produced by a single exciter). The intensity of the diffraction patterns, obtained from EE^* , is given by [9, 10]

$$I \approx \sum_n J_n^2 \left(\frac{4\pi a \cos \theta_i}{\lambda} \right) \delta \left(\frac{d}{\lambda} - \frac{n}{\Lambda \cos \theta_i} \right) \quad (6)$$

where l is the horizontal distance between the location of the laser spot on the liquid surface and the screen, $\Lambda = (2\pi/q)$ is the wavelength of the surface wave and a is the amplitude of the surface wave. Again d is the coordinate which measures the distance of the diffraction spots from a reference point (central spot) on the observation plane. J_n is the Bessel function of order n , and $\delta(\cdot)$ denotes the Dirac delta function. The intensity distribution on the diffraction pattern vanishes at points where J_n is zero satisfying equation (6). The quantity involving the Bessel function in this equation gives the magnitude of the intensity of diffraction spots for different orders n , while the positions of the spots are given by the delta function.

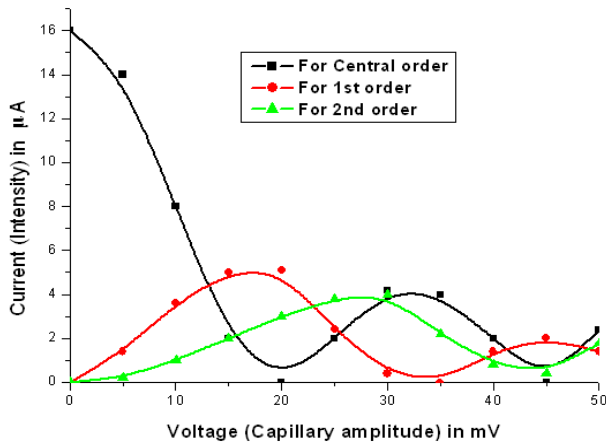


Fig.4: Theoretical plot of intensity versus the amplitude for zeroth-order (black line), first-order (red line), and second-order (green line) diffraction spots using Eqn.(6). Experimentally measured intensities for these spots are shown by the black-square, red-circle, and green triangle, respectively.

The black, red, and green lines in Fig. 4 represent the theoretical plots of intensity of the zeroth, first and second-order diffraction spots as a function of capillary wave amplitude using equation (6) (with the help of Mathematica

software). To compare these theoretical plots with experimentally measured data, we have measured the current (*i.e.* intensity) through the LDR of each order diffraction spot for different applied excitation voltage (*i.e.* amplitude) of the function generator. Experimentally measured current for different applied voltage for these spots are shown in Fig. 4 by black-square, red-circle and green triangle, respectively. The linear relation between the LDR current and the intensity of diffracted light was checked.

V. MEASUREMENT OF SURFACE TENSION OF LIQUIDS

One should be very careful to repeat the same experiment about the followings. It is important to clean the outer surfaces of the exciter in contact with the liquids and must be use clean liquids. When anyone wants to measure the separation distance (d) between the central spot to the 1st order spot, it is important to use precise scale like vernier scale of vernier constant of the order of 0.001 cm, because a small error in measurement of d can affect the result. It should be better to setup the experimental arrangement on a vibration-less table. Ordinary table is not suitable for this experiment. As the

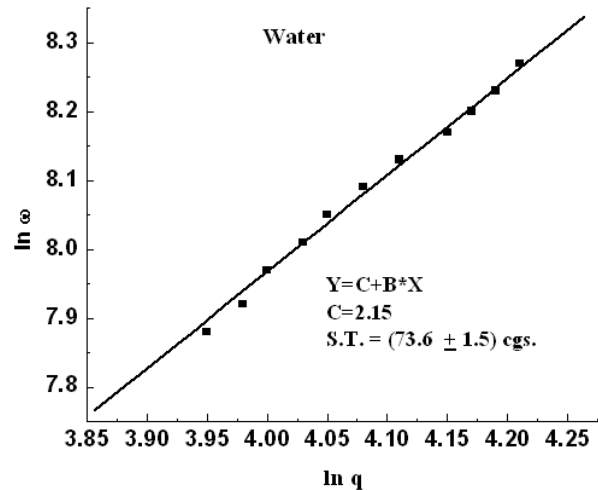


Fig.5: A log-log plot of the capillary wave frequency (ω) vs. wavevector (q) for pure water.

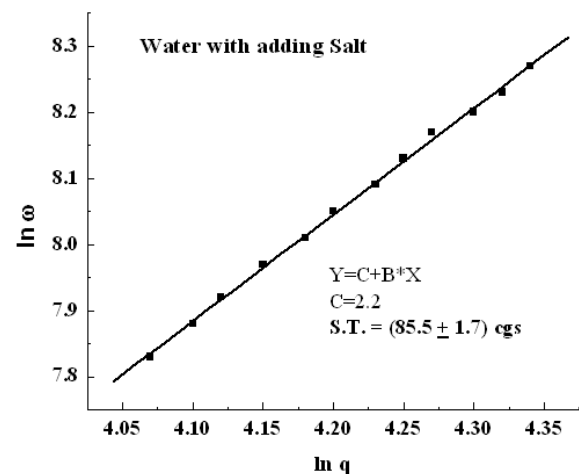


Fig.6: A log-log plot of the capillary wave frequency (ω) vs. wavevector (q) for 0.1 M NaCl-water solution.

Study the Liquid Surface Capillary Wave Profile by Optical Method

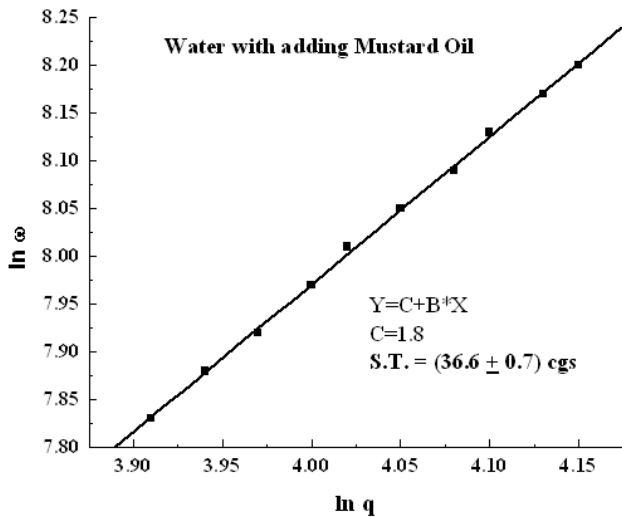


Fig.7: A log-log plot of the capillary wave frequency (ω) vs. wavevector (q).

Table-I: Comparison of surface tension at 27⁰ C.

Liquid	Measured surface tension (dyne/cm)	Reported surface tension (dyne/cm)
Water	73.6 ± 1.5	70-74 [13]-[16]
0.1 M NaCl-water solution	85.5 ± 1.7	80-85 [13], [17]-[19]
Water with adding Mustard oil	36.6 ± 0.7	34 [20]

vibration-less table is not available to us and hence, we setup the experiment at ground floor. In this experiment, we have calculated the surface tension for pure water (73.6 ± 1.5) dyne/cm, water with NaCl impurity (85.5 ± 1.7) dyne/cm and water with mustard oil impurity (36.6 ± 0.7) dyne/cm. Our calculated values are strongly supported by the reported theoretical or experimental values (see Table-I) [13]-[20]. When we add mustard oil in pure water, the presence of oil molecules at the surface, lower the strength of intermolecular bonding and hence decrease surface tension. Again, oil is nonpolar, so hydrogen bonds do not form between the molecules, so the surface tension is less. On the other hand, when we add NaCl into water, it increases the bonding strength between the water molecules and hence increases the surface tension of water.

VI. MEASUREMENT OF CAPILLARY WAVE PROFILE

To study the capillary wave properties (*i.e.* wavelength, frequency, phase velocity etc.), we have used the above mentioned data. We have calculated the wavelength of capillary wave at room temperature (27⁰C) for a fixed applied frequency ($f=400$ Hz) from the measured wavevector (q) using equation

$$\Lambda = \frac{2\pi}{q}$$

We have verified this calculated wavelength also from the Derac delta function of equation (6) considering $d/\lambda = n/A \cos \theta$. The calculated wavelengths of different liquids are shown in Table-II. Again using the measured values of surface tension (A) of liquids, we can estimate the phase velocity (V_{ph}) considering the relation-

$$v_{ph} = \sqrt{\frac{2 \times \pi \times A}{\Lambda \times \rho}}$$

where ρ is the density of liquid at room temperature. In Table-II, we have shown the calculated phase velocities of different liquids.

Table –II: The surface wave profiles of capillary waves at room temperature (27⁰ C) and frequency 400 Hz.

Liquids	Density (ρ) in gm/cm ³	Capillary Wavelength (Λ) in cm	Phase Velocity of Capillary wave (V_{ph}) in cm/sec
Water	0.997	0.126	60.7
0.1 M NaCl-water solution	1.04	0.128	63.5
Water with adding Mustard oil	0.997	0.107	46.4

VII. CONCLUSIONS

We have measured the liquid property (surface tension) of pure and impure water by optical method which strongly supports the reported data. Using these measured values of surface tensions, we also study capillary wave properties like wavelengths, phase velocities etc. of different liquids. In this experiment, we have created vibrations on the liquid surface with the exciter which is connected with a loud speaker (electrically driven) but one can also create vibrations by thermal method. Following this simple experimental set up, the other liquid properties like viscosity of liquid can also be measured by measuring the damping coefficient as indicated in ref. [11]. Finally, we conclude that this low cost experimental set up can helpful to study the both liquid properties and capillary wave profile.

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