Reliability and Availability Analysis of Database System with Standby Unit Provided by the System Provider

Sukhvir Singh, Rahul Rishi, Gulshan Taneja, Amit Manocha

Abstract—The present paper deals with the study of a database system having Primary database and hot standby database unit which is provided by the system provider itself. There is an agreement with the system provider that on the failure of the hot standby unit, another similar unit is immediately provided by him. The primary unit is a production unit and synchronized with hot standby unit through online transfer of archive redo logs. Data being saved in the primary unit gets simultaneously stored in the hot standby unit. When the primary database unit fails, the hot standby database unit becomes the production database and primary database unit goes under repair.

The system is analyzed by making use of semi-Markov processes and regenerative point technique. Expression for Mean Time to System Failure, Mean Time to Failure of Primary Database Unit and Availability of Primary Unit are obtained. Graphical study has also been done.

Keywords-

I. INTRODUCTION

Reliability is defined as the probability of failure-free system operation in a specified environment for a specified period of time. It is important to assess the reliability of the critical and non-critical systems and availability of the system. Reliability models are a powerful tool for predicting, controlling, and assessing software reliability. A lot of work on standby systems has been done by various researchers including [1-6] in the field of reliability modeling. But the reliability modeling on primary and standby databases is yet to be reported in the literature of reliability. Our aim is to fill in such a gap. Thus, in the present paper, a system is analyzed when we have primary database and hot standby database provided by the service provider. Initially primary is operative and others are standby. Data is stored in the primary database and its redo log files are created at the primary site. These redo log files are archived to the redo log files at the standby sites and then are stored in the standby databases. As the primary database fails one of the hot standby databases becomes operative and at least one hot standby system is always available.

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Sukhvir Singh, Department of Computer Science & Engg., NCCE, Israna, Haryana, India.
Dr. Rahul Rishi, Department of Computer Science & Engg., UIET, M.D.U. Rohtak, Haryana, India.
Dr. Gulshan Taneja, Department of Mathematics, MDU, Rohtak, Haryana, India.
Dr. Amit Manocha, Department of Mathematics, TITS, Bhiwani, Haryana, India.
The transition probabilities are as follows:
\[ dQ_{01} = p_{10} e^{-\lambda t} dt \]
\[ dQ_{02} = p_{20} e^{-\lambda t} dt \]
\[ dQ_{10} = p_{15} e^{-\lambda t} dt + Q_1(0) \]
\[ dQ_{11} = h_1(t) dt \]
\[ dQ_{12} = e^{-\lambda t} g_2(t) dt \]
\[ dQ_{20} = e^{-\lambda t} g_1(t) dt \]
\[ dQ_{21} = e^{-\lambda t} G(t) dt \]
\[ dQ_{22} = \lambda e^{-\lambda t} G(t) dt \]

The non-zero elements \( p_{ij} \) of the generator matrix are given by:
\[ p_{01} = p_1 \]
\[ p_{02} = p_2 \]
\[ p_{10} = p_3 \]
\[ p_{11} = 1 \]
\[ p_{12} = 0 \]
\[ p_{21} = 1 \]
\[ p_{22} = 0 \]

The mean Sojourn Time (\( \mu \)) in the regenerative state 'i' is given by
\[ \mu_i = \frac{1}{\lambda_i} \]

The mean entrance time into state 'i' is mathematically stated as:
\[ m_i = \int_0^\infty t dQ_{ij} (t) = -q_{ij}^{-1} (0) \]

The unconditional time taken by system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically stated as:
\[ m_{ij} = \int_0^\infty t dQ_{ij} (t) = -q_{ij}^{-1} (0) \]

Thus
\[ m_{10} = m_{01} + m_{02} = m_0 = \mu_0 \]
\[ m_{11} = \mu_1 \]
\[ m_{12} = \mu_2 \]
\[ m_{12} + m_{21} = \mu_1 \]
\[ m_{10} + m_{15} = m_5 = g_1(0) = k_1 \]

VI. MEAN TIME TO SYSTEM FAILURE

Let \( \phi_i(t) \) be the c.d.f. of the first passage time from regenerative state 'i' to a failed state. To determine the mean time to system failure (MTSF), regarding the failed state as absorbing state and employing the arguments used for regenerative processes, we have the following recursive relations for \( \phi_i(t) \):
\[ \phi_{01}(t) = Q_{01}(0) + Q_{02}(s) \phi_{02}(t) + Q_{21}(t) \phi_1(t) \]
\[ \phi_{02}(t) = Q_{02}(0) + Q_{20}(s) \phi_{01}(t) + Q_{12}(t) \phi_1(t) \]
\[ \phi_{03}(t) = Q_{03}(0) + Q_{21}(s) \phi_{02}(t) + Q_{13}(t) \phi_1(t) \]
Now taking L.S.T. of the above equations and solving them for \( \phi_0^*(s) \), we have
\[ \phi_0^*(s) = \frac{N(s)}{D(s)} \]

where
\[ D(s) = 1 - Q_{02}^*(s) \frac{Q_{10}^*(s) - Q_{02}^*(s) Q_{20}^*(s)}{Q_{10}^*(s) Q_{31}^*(s)} \]

Now the Mean Time to System Failure (MTSF) when the system starts from state '0' is
\[ \text{MTSF} = \lim_{s \to 0} \frac{1}{s} \frac{D(s) - N(s)}{s} = N_1 \]

Where
\[ N_1 = \mu_0 + \mu_1 + \mu_2 + \mu_3 \]

IV. MEAN TIME TO FAILURE OF PRIMARY DATABASE UNIT

Using the arguments of the theory of regenerative processes, we have the following recursive relation for \( P_i(t) \):
\[ P_0(t) = Q_{01}(t) + Q_{02}(t) + Q_{03}(t) \]

Now taking L.S.T. of above equation we have
\[ P_0^*(s) = Q_{01}^*(s) + Q_{02}^*(s) + Q_{03}^*(s) \]
\[ \text{Mean Time to Failure of Primary storage device when the system starts from stage '0'} \]
\[ \text{MTTF} = \lim_{s \to 0} \frac{1}{s} \frac{1 - Q_{02}^*(s)}{N_1} = \frac{N_1}{D(s)} \]

Where
\[ N_1 = \mu_0 + \mu_1 + \mu_2 + \mu_3 \]

V. AVAILABILITY WHEN THE PRIMARY UNIT IS OPERATIVE

Using the probabilistic arguments, we have the following recursive relations for \( P_i(t) \):
\[ P_0(t) = q_{01}(t) + q_{02}(0) \cap P_1(t) + q_{03}(t) \cap P_3(t) \]
\[ P_1(t) = q_{10}(t) \cap P_0(t) + q_{15}(t) \cap P_2(t) \]
\[ P_2(t) = q_{21}(t) \cap P_1(t) \]
\[ P_3(t) = q_{31}(t) \cap P_2(t) \]

Taking Laplace transforms of the above equations and solving them for \( P_0^*(s) \), we get
\[ P_0^*(s) = N_2(s) \]

Where
\[ N_2(s) = \frac{N_2(s)}{D_1(s)} \]

Where
In steady state, the availability of Primary Unit is given by

\[ AP_0 = \lim_{s \to \infty} s\mathcal{A}P_0^* (s) = \frac{N_2}{D_1} \]

Where

\[ N_2 = [1 - p(\frac{4}{15})_1 q(\frac{4}{15})_1 q(\frac{4}{15})_2] \mu_0 + p(\frac{4}{15})_1 \mu_5 \]

and

\[ D_1 = [1 - p(\frac{4}{15})_1 q(\frac{4}{15})_1 q(\frac{4}{15})_2 - q(\frac{4}{15})_1 q(\frac{4}{15})_2] \mu_0 + p(\frac{4}{15})_1 \mu_2 + p(\frac{4}{15})_1 \mu_3] + [1 + p(\frac{4}{15})_1 K_1] \]

VI. GRAPHICAL ANALYSIS

The graphical analysis of the MTSF with respect to rate of updating the redo log files automatically and behavior of Availability with respect to updating redo log files automatically.

![MTSF Vs Rate of updating redo log files (archiving or manually) for different rates of updating redo log files (archiving manually) γ1](image)

In the figure 2 graphs is plotted between MTSF and rate of updating the redo log files automatically (γ1) for different rates of updating redo log files manually (γ2).

It can be interpreted from the graph that MTSF decreases with the increase in the values of γ1 and MTSF has higher values for lower values of γ1. It is also observed from the graph that MTSF has higher values for lower values of γ2 (rate of updating redo log files manually).

![Availability of Primary database (AP0) vs rate of updating redo log files (archiving))](image)

The figure 3 shows the behavior of Availability of primary database (AP0) with respect to rate of updating the redo log files (archiving) automatically (γ1). From the graph it is clear that the availability increases with the increase in the rate of updating redo log files (archiving) automatically.

REFERENCES