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Thermal Study of Viscoelastic Material between Two Rotating Concentric Annuli: Application at Drilling Process

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Abstract—This article presents a numerical investigation of the thermal convection for a viscoelastic material in the annular space between two coaxial rotating cylinders. The problem is considered when the inner cylinder rotates about the common axis with constant angular velocity and the outer cylinder at the rest. The horizontal endplates are assumed adiabatic. The Carreau stress-strain constitutive equation was adopted to model the rheological material characteristics. The governing equations are numerically solved by a time-marching finite element algorithm. It is employed to compute numerical solution through a semi-implicit Taylor-Galerkin / pressure-correction scheme, based on a fractional-step formulation. The effect of rheological parameters on the heat transfer and on the flow is analysed. The results of natural, forced and mixed convections are presented and discussed.

Index Terms— Drilling process, Numerical simulation, Rotating concentric cyloinders, Thermal study, Viscoelastic material.

I. INTRODUCTION

The flow of viscoelastic materials is present in a large number of industrial processes. These include sterilization of industrial or pachaging processes of foods, pharmaceutic products, cosmetics and lubricants, the drilling process of oil wells, and the extrusion of ceramic catalyst supports, to name a few. In these processes, heat transfer between rotating concentric annulus is encountered.

An intteresting example is the drilling process of petroleum wells. To accomplish a number of functions, the drilling mud flows down through the column and then up through the annular space between the column and the rock formation. The drilling fluid must have the appropriate properties to ensure the success of a drilling operation. However, in order to assure the correct properties of the drilling fluid during the process, especially the rheological ones which are typically strong functions of temperature, the heat transfer rates occurring during the process must be accessible a priori.

Numerous articles about heat transfer during axisymmetric flows of non-Newtonian fluids can be found in the literature. Several authors [1-6] analyzed the heat transfer problem in flows of power-law fluids through tubes and proposed correlations for the Nusselt number. For vibrating tubes, numerical and experimental studies can be found in [5-6].

The numerical simulation of the flow and the heat transfer of Carreau's fluid and for fluid with variable properties enclosed between two concentric cylinders are studied [7-8]. The impact of temperature -dependent rheological properties for Ostwald-De-Waele power law [9] on the flow and on the heat transfer is analyzed numerically [10].

In the present work, heat transfer to Carreau materials in laminar flow between rotating concentric vertical cylinders is numerically analyzed by a computational code. This code, developed by using the finite element method, is validated by comparison with results reported in the literature. The numerical solutions reported here are for forced convection, mixed and natural convection flow regimes. The effects of rheological parameters on the Nusselt number are examined.

II. FORMULATION OF THE PROBLEM

The investigated geometry is shown in Fig. 1. We consider two coaxial cylinders with of a finite length H. The inner cylinder, of radius R_i, is in rotation at constant angular velocity Ω, it is maintained at a hot uniform temperature T_h. The outer cylinder of radius R_e is at rest and cold temperature T_c.

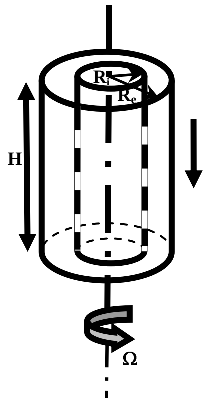

Figure 1: Geometry of the problem

The flow is assumed to be laminar, incompressible and axisymmetric. Non- Newtonian effects are considered for fluids obeying the Carreau constitutive relationship.

In dimensionless form, the Carreau constitutive relationship is given by the following constitutive equation:

$$\eta = \left(1 + W_e^2 \dot{\gamma}^2\right)^{\frac{n-1}{2}} \qquad (1)$$

where the flow index n and the Weissenberg number *We*, describe the rheological property of the fluid and the equations for the conservation of mass, momentum and energy equations are:

$$\frac{1}{r}\frac{\partial (rv)}{\partial r} + \frac{\partial u}{\partial z} = 0 \qquad (2)$$

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$$\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial r}+u\frac{\partial v}{\partial z}-\frac{w^2}{r}\right)= -\frac{\partial p}{\partial r}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{r}\left[\frac{\partial\left(2r\eta\frac{\partial v}{\partial r}\right)}{\partial r}\right]-\frac{\eta v}{r^2}+\frac{\partial}{\partial z}\left[\eta\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}\right)\right]\right\} \quad (3)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial r}+u\frac{\partial u}{\partial z}= -\frac{\partial p}{\partial z}+\frac{1}{\mathrm{Re}}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\eta\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}\right)\right]+\frac{\partial}{\partial z}2\eta\frac{\partial u}{\partial z}\right\}+\frac{Gr}{\mathrm{Re}^2}\theta \quad (4)$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial r}+u\frac{\partial w}{\partial z}+\frac{wv}{r}= \frac{1}{\mathrm{Re}}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\eta\frac{\partial w}{\partial r}\right)+\frac{\partial}{\partial z}\left(\eta\frac{\partial w}{\partial z}\right)-\eta\frac{w}{r^2}\right] \quad (5)$$

$$\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial r}+u\frac{\partial \theta}{\partial z}=\frac{1}{\mathrm{Re}\ \mathrm{Pr}}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right)+\frac{\partial^2\theta}{\partial z^2}\right\} \quad (6)$$

The problem is characterized by the following parameters of similarity; Prandtl number (*Pr*) Reynolds number (*Re*); Weisseneberg (*We*), Grashof number (Gr), (or Rayleigh number $Ra = Gr\,\mathrm{Pr}$).

III. NUMERICAL RESOLUTION

A time-marching finite element algorithm is employed in this investigation to compute numerical solutions through a semi-implicit Taylor-Galerkin/pressure-correction scheme [11], based on a fractional-step formulation. This involves discretisation, first in the temporal domain, adopting a Taylor series expansion in time and a pressure-correction operator-split, to build a second-order time-stepping scheme. Spatial discretisation is achieved via Galerkin approximation for the both momentum and energy equations. The used finite element basis functions are quadratic φ_j for velocities and temperature, and linear ψ_k for pressure.

IV. RESULTS AND DISCUSSION

All the calculations have been conducted for a fluid with Pr=7, corresponding to the dilute solutions of a polymer in water, the geometry of the annulus is fixed at AR=1 and K=2.

Furthermore, in order to validate the numerical code used for the present study, the steady-state solutions obtained as time-asymptotic solutions for an untilted square cavity with differentially heated sidewalls and adiabatic top and bottom walls, have been compared with the benchmark results by De Vahl Davis [12]. In particular, the average Nusselt numbers obtained at Rayleigh numbers in the range between 10^3, 10^4, 10^5, 10^6 and the maximum horizontal and vertical velocity components on the vertical and horizontal midplanes of the enclosure, have been found to be within 1%-3% of the benchmark data.

When the inner cylinder is at the rest (Re=0, case of natural convection), the increases in the flow index have almost no effect on heat transfer (Fig 2, 3).

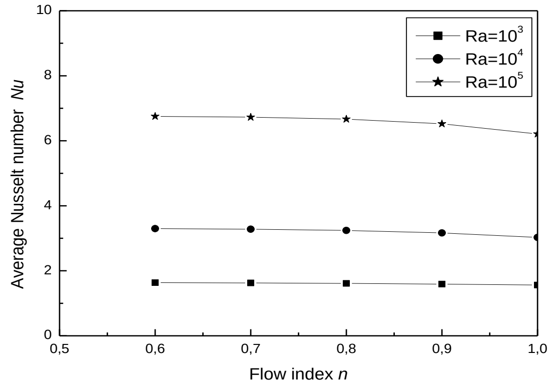


Figure 2: Average Nusselt number as function of flow index for differents Rayleigh number at *Re*=0

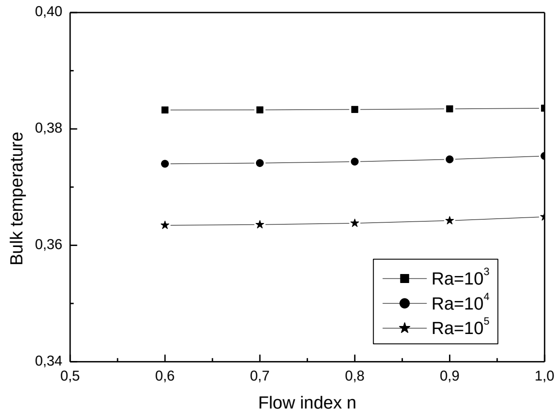
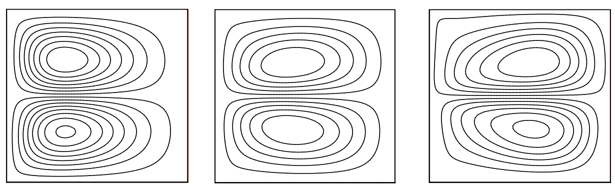


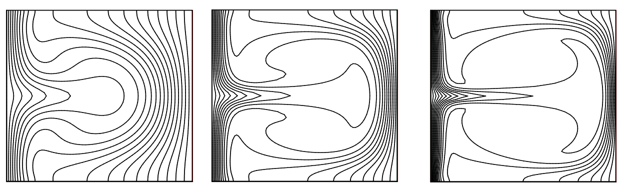
Figure 3: Bulk temperature as function of flow index for differents Rayleigh number at Re=0

For forced convection (Ra=0 and Reynolds $\neq 0$), Figure 4 shows the streamlines and the isotherms at n=0.8 and three values of Reynolds number. We can observe a pair of counter-rotating cells. The isotherms, in that case, are distorted in the vicinity of the recirculating flow zone.

In the mixed convection (Ra $\neq 0$ and Re $\neq 0$) and for n=0.8, we notice in figure 5 that for low Reynolds number (Re≤70), the heat transfer regime is quasi conductif. Beyond this value, the Nusselt number increases with Reynolds number.



(a) Flow patterns



(b) Isotherms

Re = 100 Re = 200 Re = 400

Figure 4: Flow patterns and isotherms for different Reynolds numbers at flow index n=0.8 and Ra=0

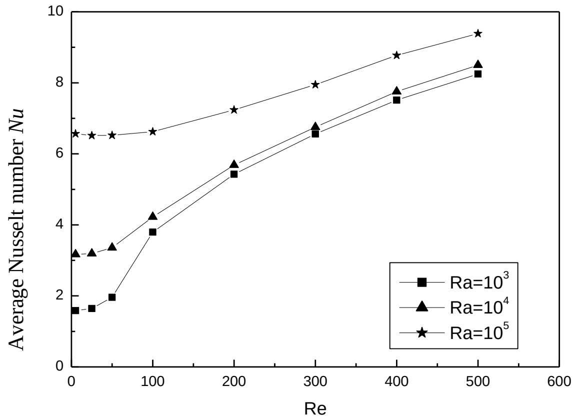


Figure 5: Average Nusselt number as function of Reynolds number for differents Rayleigh number at n=0.8

V. CONCLUSION

This paper presented a study of the heat transfer problem for the flow of Carreau materials confined in a differentially heated annular cylindrical space with rotating inner cylinder.

The governing equations were solved numerically via a finite element technique. Results were presented in the form of flow patterns, isotherms, bulk temperature profile and Nusselt number as a function of rheological parameters.

It was noted that the results show that the non-Newtonian effects are important on the structure of the flow and on the heat transfer. This fact can be explored to understand the rheological behaviour of the fluid intervening in most engineering applications.

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