Simplified Model for Representing Dynamic Textures using Markov Model

Tong Fu, Di Yin, Li Xiaoli, Chen Hui

***Abstract*—Dynamic textures are sequences of images of moving scenes that exhibit certain stationary properties in time; these include sea-waves, smoke, foliage, whirlwind etc. In previous works [1,2], dynamic textures are usually modeled as linear models, and parameters of the model are identified in the sense of maximum likelihood or minimum prediction error variance. Once its parameters are learned, a model has predictive power and can be used for extrapolating synthetic sequences. In this work we study a particular type of dynamic textures that can be represented in the form of Markov Models. An aggregation algorithm can then be adopted to reduce its complexity. The resulting low-dimensional models can capture complex visual phenomena with low computation cost.**

***Index Terms*— Dynamic Texture, Markov Model, Aggregation, Reduced order model.**

I. INTRODUCTION

The study of Dynamic Textures (DT) is a recent research topic in the field of video processing. A dynamic texture can be described as a time varying phenomenon with certain repetitiveness in both space and time. For example ripples at the surface of water or smoke or an escalator are all examples of dynamic texture. Rather than a simple extension of static textures to the time domain, a dynamic texture is a dynamic model that describes the temporal evolution of the textures. The study of dynamic texture is an active research topic with many applications such as synthesis, segmentation or characterization.

In previous works dynamic texture is usually modeled as linear dynamical systems (for example ARMA model). The parameters of the model can be identified through impulse response [13], maximum likelihood [8] or other methods. But generally, for nonlinear systems it is not easy to identify the parameters in differential equations describing the system.

In this paper we first propose a Markov model based representation of dynamic textures. This type of representation for dynamic textures is shown to be very general since it covers both linear and a large class of nonlinear systems. Unlike traditional formulations based on differential equations, the Markov model representation can keep the parameter learning algorithms simple even when the system is nonlinear. We then focus on reducing the complexity of the Markov dynamic texture model. To be more specific, The size of the state space of Markov models can be reduced through information theoretic aggregation [9].

Manuscript received on May, 2013.

Tong Fu, College of Computer Science and Engineering, Xi'an University of Technology, Xi'an, China.

Di Yin, School of Electrical Engineering, Wuhan University, Wuhan, China.

Li Xiaoli,School of Computer Science, Jiangxi Normal University, Nanchang, China.

Chen Hui, College of Information Science and Technology, Chengdu University, China

This aggregation algorithm is based on Kullback-Leibler rate metric [11, 19] which is widely applied to many areas such as machine learning, signal processing, control and estimation [9, 11, 14, 19, 20].

The remaining part of this paper is organized as follows: Section.2 provides detailed description of how to represent dynamic texture using Markov models. In Section.3 we proposed to use the Kullback-Leibler rate based aggregation method to reduce the complexity of Markov dynamic texture model. The advantages of this method and future work are summarized in Section 4.

II. DYNAMIC TEXTURE IN THE FORM OF MARKOV MODEL

A. Definition of Dynamic Texture

For a sequence of images (time-varying texture), individual images should not be treated as independent realizations from a stationary distribution, because there is an obvious temporal correlation intrinsic in the process. Dynamic texture capture this by making an assumption that individual images are realizations of the output of a dynamical system driven by an independent and identically distributed (i.i.d.) process.

The following definition of dynamic texture is given in [8], we also presents a more general nonlinear form here. Let $\{\mathrm{I}(\mathrm{t}), \mathrm{t}=1,2, \ldots \tau\}, \mathrm{I}(\mathrm{t}) \in R^m$ be a sequence of τ images. Suppose that at each time t a noisy version of image can be measured, denoted as $\mathrm{y}(\mathrm{t})=I(t)+w(t)$, where $\mathrm{w}(\mathrm{t}) \in R^m$ is an independent and identically distributed (i.i.d.) sequence drawn from a known distribution $\mathrm{P}(\mathrm{w})$. Similarly let $\mathrm{v}(\mathrm{t}) \in R^n$ and is an i.i.d. sequence with distribution $\mathrm{Q}(\mathrm{v})$. The sequence $\mathrm{I}(\mathrm{t})$ is called a dynamic texture if there exists a dynamic system driven by input $\mathrm{v}(\mathrm{t})$ and generates output y(t):

$$\begin{cases} x(t+1)=f(x(t), v(t)) \\ y(t)=\Phi(x(t))+w(t) \end{cases} \tag{1}$$

The output function $\Phi(.)$ here satisfies

$$\mathrm{I}(\mathrm{t})=\Phi(x(t)) \tag{2}$$

When the dynamic system is linear, the dynamic system simplifies to

$$\begin{cases} x(t+1)=Ax(t)+Bv(t)) \\ \quad y(t)=Cx(t)+w(t) \end{cases} \tag{3}$$

where A, B,C are matrices of appropriate dimensions.

Learning or inference of dynamic textures is an important task. Taking linear dynamic texture for example, given a sequence of observed images $\mathrm{y}(1), \mathrm{y}(2) \ldots \mathrm{y}(\tau)$, the objective is to find the parameters for matrices $\mathrm{A}, \mathrm{B}, \mathrm{C}$ as well as the distribution of the driving random process $\mathrm{Q}(\mathrm{v})$.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

This learning task can be formulated as the following maximum likelihood problem [8]:

$$, \hat{B}, \hat{C}, \widehat{Q(.)} = arg\max_{A,B,C,Q(.)} \log p(y(1), y(2) \dots y(\tau))$$
(4)

Subject to the dynamic constraints of equation (3). The inference method depends crucially upon whattype of representation is chosen for the distribution $Q(.)$. Note that theabove inference problem involves multiplications of unknown variables, for example, the state variables $x(t)$ multiplied by the unknown matrix A . Therefore the problem is non-linear even though the state-space model is linear. In general, one coulduse iterative techniques that alternate between estimating (sufficient statistics of) the conditional density ofthe state and maximizing the likelihood with respectto the unknown parameters, in a fashion similar to theexpectation-maximization (EM). The convergence of such iterative techniques and other identification algorithms are outside the scope of this paper, interested readers are referred to the discussion in [1, 6, 8, 13,] and references therein.

The complexity of identifying a nonlinear system has made nonlinear dynamic texture models difficult to apply in real-world applications. When the dynamic texture model is complex (for example, the number of unknown parameters is large), the learning task also become very difficult. This is true even for linear dynamic texture models. In the next section we propose a Markov model for dynamic texture which can represent a large class of dynamic behaviors and also lead to an efficient method of model size reduction through information theoretic aggregation [9].

B. *Markov Model for Nonlinear Stochastic Systems*

A Markov Chain is a random process (usually of discrete state space) characterized by memory-less property: the next state depends only on the current state and not on the sequence of events that preceded it. This property, also named as “Markov property” is the key feature of Markov models. A typical state transition matrix for a 2-state Markov Chain is shown in equation (5).

$$P = \begin{bmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{bmatrix} \quad (5)$$

Because the Markov property limits the behavior of a Markov Chain, it cannot be used to represent all types of dynamical systems. A more general class of model is called Hidden Markov Model (HMM), where an output layer is added to generate observations, while the ‘hidden’ states observe a Markov model. The structure of an example HMM is given in Figure.1.

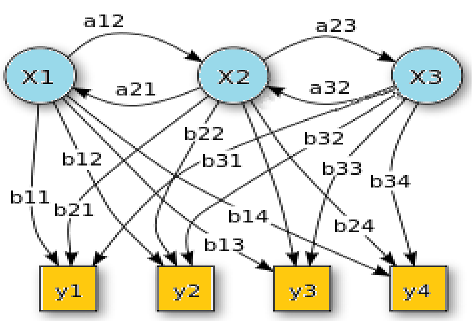

Fig.1 Typical structure of a Hidden Markov Model

The key elements of a HMM are the state transition matrix for the hidden states, denoted as H , the observation probability matrix, denoted as O (which gives the probability of observing a particular output when the true state is of a given value), and the initial distribution of hidden states, denoted as π. We use the following notation for a HMM

$$\Sigma = (H, O, \pi) \quad (6)$$

Hidden Markov model can represent a large class of dynamical systems, both linear and nonlinear. Therefore it is a good candidate for nonlinear dynamic texture model. For example, equation (1) can be equivalently represented by a Hidden Markov model: the state equation f corresponds to H, the output equation corresponds to O, and an initial state x_0 corresponds to π.

Table.1 Comparison between difference equation representation and HMM representation of dynamic textures.

	Difference Equations	HMM
State dynamics	$f(.,.)$ with noise v(t)	H
Output	$\Phi(.)$ with noise w(t)	O
Initial Condition	x_0	π

C. *Dynamic Texture as Markov Model*

Through some standard technique, we can use a Markov model to represent a HMM. This is achieved by combining hidden states and some observations to form new states for the Markov model [14]. The resulting model is equivalent to the HMM, but satisfies Markov property. This model is much larger in size because the many possible combinations of hidden state and observations.

For example, given a HMM with four possible hidden states a, b, c, d and three possible observation x, y, z. Then the size of the state transition matrix for the hidden states is 4×4 and the size of the observation matrix is 4×3. To obtain a Markov model, we can re-define new states as follows

$$s_1 = (a, x), s_2 = (a, y), s_3 = (a, z)$$
$$s_4 = (b, x), s_5 = (b, y), s_6 = (b, z)$$
$$s_7 = (c, x), s_8 = (c, y), s_9 = (c, z)$$
$$s_{10} = (d, x), s_{11} = (d, y), s_{12} = (d, z)$$

Starting from any of these 12 states, the probability of transition to all 12 states can be computed from the observation matrix and hidden state transition matrix of the HMM. This Markov model has a transition matrix of size 12×12. Using this method we can transform any HMM representation of dynamic texture into a larger Markov model.

III. SIMPLIFYING DYNAMIC TEXTURE BY MARKOV AGGREGATION

The Markov model obtained in previous section usually has large dimension. It is desirable if the size of this model can be reduced since it will lead to a reduction of both the memory needed for model parameter storage and the computation cost when using this model.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

In this section we propose to use the Kullback-Leibler rate based method in [9] to reduce the Markov dynamic texture model.

A. Kullback-LeiblerRate Metric between Markov Models

Assume the Markov model obtained from previous section is a first-order homogeneous Markov chain, defined on finite dimensional state space $N = \{1,2, \dots n\}$. The (i, j)-th entry of the $n \times n$ transition probability matrix P is denoted as

$$P_{i,j} = Prob(X(t+1) = j | X(t) = i), \text{ i,j} \in N \quad (7)$$

Here X(t) denotes the state value of the Markov chain at time step t. Let π be the stationary distribution and its ith entry be π_i(it is assumed that this Markov chain is stationary, meaning that π is unique). The following formula gives the Kullback-Leibler divergence rate between two stationary Markov chains P and Q with stationary distribution πand θ, respectively.

$$R(P||Q) = \sum_{i,j \in N} \pi_i P_{i,j} \log \frac{P_{i,j}}{Q_{i,j}} \quad (8)$$

To avoid divided by zero, it is required that P is absolutely continuous with respect to Q, which means $Q_{i,j} = 0$ implies $P_{i,j} = 0$.

B. Model Reduction for Markov Dynamic Texture

Let (π, P) be a stationary Markov dynamic texture model that needs to be reduced. The state space reduction is achieved through aggregation, meaning we put together states of (π, P) that is "close" to each other and treat them as a new state. The resulting Markov chain, denoted as (θ, Q), have fewer states than the original one, however it should be not very different from the original model (π, P) when measured through the Kullback-Leibler rate (8). Suppose (θ, Q) has m states with $m < n$, and denote the state space for this new Markov chains as $M = \{1,2, \dots m\}$, the optimization problem that generates parameters for (θ, Q) is posed as:

$$\min_{\omega . Q} R^{\omega}(P||Q) \text{s.t. } \sum_{l \in M} Q_{kl} = 1, k \in M, \quad Q_{kl} \geq 0, \ k, l \in M \quad (9)$$

Here $R^{\omega}(P||Q)$ is the Kullback-Leibler divergence rate between$P$ and the π-lifted version of Q, denoted as $\widehat{Q}$. This lifting is necessary because it makes the size of the state space of $\widehat{Q}$ to be n, and based on definition (8) two Markov chains with state space of different sizes cannot be compared. Only after lifting the comparison using Kullback-Leibler divergence rate make sense.

The optimal solution for optimization problem (9) is not easy to solve directly. In [9] the authors took a two-step approach for the $m = 2$ case, and then the more general problem (9) can be solved recursively applying this bi-section procedure. We brief the two-step approach in the next sub-section just to make this paper self-contained. For more details the interested readers are referred to [9].

C. Solution of Bi-section Problem based on Spectral Partition

The bi-section version of problem (9) can be solved in two-steps: firstly, a partition function $\omega: N \to M$ is computed to decide which state of P belongs to which partition; then under the given partition function, the optimal parameters of Q is computed so that the Kullback-Leibler divergence rate is minimized. This can be summarized by the following two problems:

$$\text{P1: } \min_{\omega : N \to M} R^{\omega}(P||Q) \quad (10)$$

Suppose the solution of P1 is $\widehat{\omega}$, then the second problem is

$$\text{P2: } \min_{Q} R^{\omega}(P||Q), \text{ with } \omega = \widehat{\omega} \quad (11)$$

It has been proved that the solution of problem P2 can be written in closed form with respect to πand $\widehat{\omega}$. The problem P1 can be approximated by the solution of an eigenvalue problem:

$$\left(\frac{\Pi P + P' \Pi}{2}\right) v = \lambda \Pi v \quad (12)$$

This eigenvalue problem can be solved relatively easily and the sign structure of the resulting eigen-vector v gives the optimal partition function $\widehat{\omega}$.

IV. DISCUSSION

In this paper we focus on the complexity reduction of dynamic textures. Our major contribution is the Markov model based dynamic texture representation, which can cover a large class of linear and nonlinear dynamic texture models. Due to the relatively large state space size of the proposed model, we also adopt the information theoretic aggregation method in [9] to do state space reduction for the Markov dynamic texture models. Previous works on differential equation based dynamic texture models are mostly limited to linear systems because the difficulty associated with nonlinear system identification. Our proposed approach can describe a variety of nonlinearities in both state evolution and observation map, and the rich literature on Markov chains provides a large pool of tools for parameter learning. In the future we will study the optimal procedure of obtaining the Markov model from images.

REFERENCES

[1] LennartLjung "System Identification: Theory for the User", Pearson Education,1998.
[2] Dmitry Chetverikov and Renaud Péteri, "A Brief Survey of Dynamic Texture Description and Recognition", Computer Recognition Systems Advances in Soft Computing Volume 30, 2005, pp. 17-26.
[3] Renaud Peteri and Dmitry Chetverikov, "Dynamic Texture Recognition Using Normal Flow and Texture Regularity", In Proceedings of Iberian Conference on Pattern Recognition and Image Analysis, 2005, pp.223--230
[4] Sun Yu, Prashant G. Mehta, "Bode-Like Fundamental Performance Limitations in Control of Nonlinear Systems". IEEE Transaction on Automatic Control, vol. 55, pp.1390-1405, 2010.
[5] Martin Szummer and Rosalind W. Picard, "Temporal Texture Modeling", In IEEE International Conference on Image Processing, 1996, pp.823—826
[6] Franco Woolfe and Andrew Fitzgibbon, "Shift-Invariant dynamic texture recognition", Proceeding of ECCV 2006, Pages 549-562
[7] Sun Yu, Prashant G. Mehta, "Fundamental performance limitations with Kullback-Leibler control cost," IEEE Conference on Decision & Control, pp.7063-7068, 2010.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

[8] Gianfranco Dorettoet.al.,"Dynamic Textures", International Journal of Computer Vision, 2001, pp.439-446
[9] Kun Deng, Y. Sun, P. Mehta and S. Meyn, "An information-theoretic framework to aggregate a Markov chain," In Proceedings of American Control Conference, pp.731-736, 2009.
[10] Guoying Zhao and MattiPietikäinen," Dynamic Texture Recognition Using Volume Local Binary Patterns", Proceedings of ECCV 2006 Workshop on Dynamical Vision, 2006, pp.165-177.
[11] Sun Yu, Prashant G. Mehta, "The Kullback-Leibler Rate Pseudo-Metric for Comparing Dynamical Systems". IEEE Transaction on Automatic Control, vol.55, pp.1585-1598, 2010.
[12] Guoying Zhao and Pietikainen, M., "Dynamic Texture Recognition Using Local Binary Patterns with an Application to Facial Expressions", IEEE Transactions on Pattern Analysis and Machine Intelligence, 29(6), 2007, pp.915-928
[13] Fujita and S. Nayar. "Recognition of Dynamic Textures using Impulse Responses of State Variables", In Proceedings of the Third International Workshop on Texture Analysis and Synthesis, 2003
[14] Y. Sun and P. G. Mehta, "Fundamental performance limitations via entropy estimates with hidden Markov models," IEEE Conference on Decision & Control, pp. 3982–3988, 2007.
[15] Ravichandran, A, Chaudhry, R.and Vidal, R. "Categorizing Dynamic Textures Using a Bag of Dynamical Systems", IEEE Transactions on Pattern Analysis and Machine Intelligence, 35(2),2013, pp.342 – 353
[16] Sun Yu, Prashant G. Mehta, "The Kullback-Leibler rate metric for comparing dynamical systems", in Procs. of Conference on Decision & Control, pp.8363-8368, 2009.
[17] Ghanem, B.; Ahuja, N., "Phase Based Modelling of Dynamic Textures," Computer Vision, 2007. ICCV 2007. IEEE 11th International Conference on, pp.14-21, 2007
[18] Ravichandran, A.; Chaudhry, R.; Vidal, R., "View-invariant dynamic texture recognition using a bag of dynamical systems," IEEE Conference on Computer Vision and Pattern Recognition, pp.1651-1657, 2009
[19] Rached, Z.; Alajaji, F.; Campbell, L.L., "The Kullback-Leibler divergence rate between Markov sources," IEEE Transactions on Information Theory, vol.50, no.5, pp.917-921, 2004
[20] Mager DE, et. al., "Kullback-Leibler clustering of continuous wavelet transform measures of heart rate variability", Biomed SciInstrum, 2004, pp.337-42.

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication