Simulink / MATLAB Dynamic Induction Motor Model for Use as A Teaching and Research Tool

Aleck W. Leedy

Abstract—A dynamic model of the induction motor developed using Simulink / MATLAB that is beneficial for use as a teaching tool in electric machines and power electronics courses, or that can be used as a research tool in the laboratory is presented. The model can be used to study the dynamic behavior of the induction motor, or can be used in various motor-drive topologies with minor modifications. The motor model presented is based on the *T-type d-q model of the induction motor. A block model approach* is used in the construction of the motor model that will allow users of the model to resolve reference frame theory issues. The model developed is intuitive, easy to use, and allows all motor parameters to be easily accessed for monitoring and comparison purposes. The model presented is capable of being used with various inverter topologies and PWM schemes with minor modifications.

Index Terms - Dynamic Model, Inverter, Induction Motor.

I. INTRODUCTION

Dynamic modeling and simulation of induction motor drives is of great importance to both industry and academia due to the prevalence of these types of drives in various industrial settings. The induction motor has seen increased use in industry in its evolution from being a constant speed motor to being a variable speed machine with the advancement of power electronics [1]. Three-phase induction machines and voltage source inverters are now mass produced, have low costs, and are readily available; making them the top choice of industry in many applications. Three-phase squirrel-cage induction motors of various sizes are commonly used as the driving units for fans, pumps, and compressors [2]. Motor drives can now be found in transit systems, onboard U.S. Navy ships and submarines [3], in some mining operations [4], and onboard commercial jet airplanes [5].

Dynamic simulations play an important role in the pre-testing of motor drive systems [6]. Pre-testing is conducted by engineers in industry as well as by researchers in academia. Pre-testing using dynamic simulations can help researchers determine the experimental setup that will be used for a given set of experimental tests. Electric machine simulation began more than 80 years ago at M.I.T. [7].

Simulation techniques have been ongoing and continue to advance with technology since the inception of simulation studies at M.I.T. The transient behavior of an electric machine is of particular importance when the drive system is to be Many different methods and control controlled [8]. algorithms are available in the literature for controlling the three-phase induction motor [9]. A dynamic model of a machine leads to insight into the electrical transients [10].

Simulink allows electrical engineers to model dynamical systems with ease using a block diagram approach that can be constructed fast and efficiently. There are many Simulink

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induction motor models in the literature [11]. However, most of them do not give details as to how the model equations and subsystems within the model are derived.

The use of PSPICE and MATLAB/Simulink software packages are critical as both teaching and research tools in the areas of electric machines and power electronics due to the complexity of the circuits involved and the waveforms generated by the circuits. It has been shown that PSPICE and MATLAB/Simulink can be used effectively to teach power electronics to large groups with various educational backgrounds [12]. The use of **PSPICE** and MATLAB/Simulink can also provide a broad platform for students to extend their basic skills.

The purpose of this paper is to present a dynamic model of the induction motor developed using Simulink / MATLAB that can be used to study the transient behavior of a motor-drive system such as the one shown in Figure 1. In this figure, a DC voltage source is connected to a three-phase inverter driving a three-phase induction motor with a load attached. In Figure 1, V_i is the inverter DC input voltage and I_i is the inverter DC input current. The goal of this paper is to develop a model that is intuitive, easy to use, and that will allow access to all motor parameters for monitoring and comparison purposes.

A model of the induction motor and other electric machine models are available in the Simulink Power System Blockset.



Figure 1. Motor-Drive System Model.

However, these models provide access to a limited number of machine parameters. For researchers and teachers with limited budgets, building a dynamic model of an induction motor or other electric machine using basic Simulink blocks is a cost effective approach to studying the dynamic behavior of an electric machine. The Simulink Power System Blockset is an extra toolbox that does not come with the student version or full version of MATLAB / Simulink. Toolboxes that are not included as standard with MATLAB / Simulink are an added cost that professors at teaching focused universities may not be able to afford.

The cost savings is not the only advantage of building your own dynamic model of a particular electric machine. Graduate and undergraduate students will learn more if they have to build a model from equations instead of just "dragging and dropping" a Simulink motor part into the Simulink work space.



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By using the differential equations that describe the operation of a particular motor-drive system, students have to develop the mathematical model of the system before working on the Simulink model.

II. INDUCTION MOTOR DYNAMIC MODEL

All analysis and simulation in this paper are based on the d-q or dynamic equivalent circuit of the induction motor represented in the rotating reference frame [13, 14] shown in Figure 2. It should be noted that all quantities in Figure 2 have been referred to the stator. For the interested reader, a paper that includes the steady-state T-type equivalent circuit model of the induction motor can be found in [15].

The differential equations produced from analysis of the circuits in Figure 2 are as follows:

$$v_{ds} = R_s i_{ds} + \frac{d\lambda_{ds}}{dt} - \omega_e \lambda_{qs}, \qquad (1)$$

$$v_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega_e \lambda_{ds}, \qquad (2)$$

$$v_{dr} = 0 = R_r i_{dr} + \frac{d\lambda_{dr}}{dt} - (\omega_e - \omega_r)\lambda_{qr}, \qquad (3)$$

and

$$v_{qr} = 0 = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} + (\omega_e - \omega_r)\lambda_{dr}$$
(4)

where *d* is the direct axis, *q* is the quadrature axis, v_{ds} is the *d*-axis stator voltage, v_{qs} is the q-axis stator voltage, v_{dr} is *d*-axis rotor voltage, v_{qr} is *q*-axis rotor voltage, i_{ds} is the *d*-axis stator current, i_{qs} is the *q*-axis stator current, i_{dr} is *d*-axis rotor current, i_{qr} is *q*-axis rotor current, R_s is the stator resistance, R_r is the rotor resistance, ω_e is the angular velocity of the

reference frame, ω_r is the angular velocity of the rotor, and λ_{ds} , λ_{ds} , λ_{ds} , and λ_{ds} are flux linkages. It is assumed that the induction motor analyzed is a squirrel cage machine, leading to the rotor voltage in (3) and (4) being zero. The flux linkages in (1-4) can be written as:

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \,, \tag{5}$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \,, \tag{6}$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \,, \tag{7}$$

and





q-axis equivalent circuit

Figure 2. d-q equivalent circuit of the Induction Motor. $\lambda_{qr} = L_r i_{qr} + L_m i_{qs}$ (8) where L_r is the rotor self inductance, L_s is the stator self inductance, L_m is the magnetizing inductance, L_{lr} is the rotor leakage inductance, and L_{ls} is the stator leakage inductance. The self inductances in (5-8) can be expressed as:

$$L_s = L_m + L_{ls} \tag{9}$$

and

$$L_r = L_m + L_{lr} av{10}$$

The currents can be written as:

$$i_{ds} = \frac{\lambda_{ds} - L_m i_{dr}}{L_s} , \qquad (11)$$

$$i_{qs} = \frac{\lambda_{qs} - L_m i_{qr}}{L_s} \quad , \tag{12}$$

$$i_{dr} = \frac{\lambda_{dr} - L_m i_{ds}}{L_r},\tag{13}$$

and

$$i_{qr} = \frac{\lambda_{qr} - L_m i_{qs}}{L_r} \,. \tag{14}$$

After making substitutions, the currents can be expressed in terms of flux linkages as:

$$i_{ds} = \frac{L_r}{L_r L_s - L_m^2} \lambda_{ds} - \frac{L_m}{L_r L_s - L_m^2} \lambda_{dr}, \qquad (15)$$

$$i_{qs} = \frac{L_r}{L_r L_s - L_m^2} \lambda_{qs} - \frac{L_m}{L_r L_s - L_m^2} \lambda_{qr},$$
 (16)

$$i_{dr} = \frac{L_s}{L_r L_s - L_m^2} \lambda_{dr} - \frac{L_m}{L_r L_s - L_m^2} \lambda_{ds}, \qquad (17)$$

and

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$$i_{qr} = \frac{L_s}{L_r L_s - L_m^2} \lambda_{qr} - \frac{L_m}{L_r L_s - L_m^2} \lambda_{qs} \,. \tag{18}$$

The electromagnetic torque of the machine can be written as:

$$T_{e} = \frac{3}{2} \frac{P}{2} L_{m} \Big[i_{qs} i_{dr} - i_{ds} i_{qr} \Big]$$
(19)

where P is the number of poles and T_e is the electromagnetic torque. Neglecting mechanical damping, the torque and rotor speed are related by:

$$\frac{d\omega_r}{dt} = \frac{P}{2J} \left(T_e - T_L \right) \tag{20}$$

where T_L is the load torque and J is the inertia of the rotor and connected load. The angle θ_e , is calculated directly by integrating the frequency of the input voltages as:

$$\theta_e = \int_0^t \omega_e \, dt + \theta_e(0) \tag{21}$$

where $\theta_e(0)$ is the initial rotor position.

Three-phase voltages can be converted to the two-phase



Published By: Blue Eyes Intelligence Engineering & Sciences Publication stationary frame using the following relationship:

$$\begin{bmatrix} v_{qs}^{s} \\ v_{ds}^{s} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} v_{an} \\ v_{bn} \\ v_{cn} \end{bmatrix}$$
(22)

where the superscript s in (22) refers to the stationary frame. The voltages can be converted from the two-phase stationary frame to the synchronously rotating frame using the following:

$$v_{qs} = v_{qs}^s \cos \theta_e - v_{ds}^s \sin \theta_e , \qquad (23)$$

and

$$v_{ds} = v_{qs}^s \sin \theta_e + v_{ds}^s \cos \theta_e \,. \tag{24}$$

The current variables can be found as:

$$i_{qs}^{s} = i_{qs} \cos \theta_e + i_{ds} \sin \theta_e \,, \tag{25}$$

$$i_{ds}^{s} = -i_{qs} \sin \theta_{e} + i_{ds} \cos \theta_{e} , \qquad (26)$$

and

$$\begin{bmatrix} i_{a} \\ i_{b} \\ i_{c} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}^{i_{qs}}_{i_{ds}}.$$
 (27)

III. MATLAB / SIMULINK MODEL

The approach used to build the Simulink model in this paper is a block type or modular approach. The approach is based on the idea presented in [16]. However, the goal of this paper is to present the model in a clear and simplified manner that is geared toward undergraduate electric machines and power electronics students. This paper also examines different types of simulations and analyses not covered in

[16].

To begin the construction of the Simulink model of the induction motor, (11-14) can be substituted into (1-4), producing the following differential equations written in terms of flux linkages:

$$\frac{d\lambda_{ds}}{dt} = v_{ds} - \frac{R_s L_r}{L_r L_s - L_m^2} \lambda_{ds} + \frac{R_s L_m}{L_r L_s - L_m^2} \lambda_{dr} + \omega_e \lambda_{qs},$$
(28)

$$\frac{d\lambda_{qs}}{dt} = v_{qs} - \frac{R_s L_r}{L_r L_s - L_m^2} \lambda_{qs} + \frac{R_s L_m}{L_r L_s - L_m^2} \lambda_{qr} - \omega_e \lambda_{ds},$$
(29)

$$\frac{d\lambda_{dr}}{dt} = \frac{-R_r L_s}{L_r L_s - L_m^2} \lambda_{dr} + \frac{R_r L_m}{L_r L_s - L_m^2} \lambda_{ds} + (\omega_e - \omega_r) \lambda_{qr}, (30)$$

and

$$\frac{d\lambda_{qr}}{dt} = \frac{-R_r L_s}{L_r L_s - L_m^2} \lambda_{qr} + \frac{R_r L_m}{L_r L_s - L_m^2} \lambda_{qs} - (\omega_e - \omega_r) \lambda_{dr} .$$
(31)

The Simulink model can now be constructed by creating Simulink subsystems using (5-8), (9, 10), and (15-31). Each Simulink subsystem solves one of the model equations in an easy to understand manner. The subsystem is constructed using basic Simulink blocks such as the integrator, gain, sum, etc. The basic Simulink blocks are standard on all versions of MATLAB; including the student version. Using basic Simulink blocks avoids having to purchase extra toolboxes such as the SimPowerSystems toolbox that requires extra expenses that would be added to research or educational budgets.

The main "upper level" system of the dynamic model of the induction motor implemented in Simulink is shown in Figure 3. A double click on any of the blocks shown in Figure 3 within the Simulink workspace will pull up a subsystem within each individual block shown in Figure 3. The details of the construction of the model that follows will be described as if the reader is looking at the model in Figure 3 while it is opened in the Simulink workspace. This approach is easier to follow if the interested reader would like to construct the model for individual use.

A double click on the abc-Synchronous block shown in Figure 3 will pull up the subsystem shown in Figure 4. The subsystem in Figure 4 implements (23) and (24). A double click on the Unit Vectors block in Figure 4 will pull up the subsystem shown in Figure 5. The subsystem in Figure 5 implements (22). The KS matrix shown in the matrix gain block in Figure 5 is the transform matrix that converts the three-phase voltages from the *abc* frame to the two-phase stationary frame. This matrix can be defined in the MATLAB editor.

A double click on the Induction Motor dq Model block shown in Figure 3 will pull up the subsystem shown in Figure 6. A double click on the lamdaDS block shown in Figure 6 will pull up the subsystem shown in Figure 7. The subsystem in Figure 7 implements (28). Implementation of (29-31) in Simulink can be achieved in the same manner as in Figure 7. Double clicking on the lamdaQS, lamdaDR, and lamdaQR blocks in Figure 6 will pull up the subsystems that implement (29-31). A double click on the ids1 block in Figure 6 will pull up the subsystem shown in Figure 8. The subsystem in Figure 8 implements (15). Implementation of (16-18) in Simulink can be achieved in the same manner as in Figure 8.

Double clicking on the iqs1, idr1, and iqr1 blocks in Figure 6 will pull up the subsystems that implement (16-18). A

double click on the Te1 block in Figure 6 will pull up the subsystem shown in Figure 9. The subsystem in Figure 9 implements (19). A double click on the wr1 block in Figure 6 will pull up the subsystem shown in Figure 10. The subsystem in Figure 10 implements (20).

A double click on the Synchronous-abc block shown in Figure 3 will pull up the subsystem shown in Figure 11. The subsystem in Figure 11 implements (25) and (26). This subsystem also converts current variables in the stationary frame to the abc frame. Double clicking on the Unit Vectors block in Figure 11 will pull up the subsystem shown in Figure 12.

The subsystem in Figure 12 implements (27). The Kabc matrix shown in the matrix gain block in Figure 12 is the transform matrix from two-phase stationary frame currents to the three-phase *abc* frame. This matrix can be defined in the MATLAB editor. A double click on the thetaE block in Figure 3 will pull up the subsystem shown in Figure 13. The subsystem in Figure 13 implements (21). A double click on the Unit Vectors block in Figure 3 will pull up the subsystem

shown in Figure 14. This subsystem implements the sine and cosine functions that are used

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within the other blocks shown in Figure 3.

The trigonometric function block must be used in the implementation of (23-26) for the sine and cosine functions. After all of the induction motor model equations have been implemented in Simulink, all of the subsystems must be tied together to produce the main "upper level" system shown in Figure 3 that will complete the implementation of the Simulink induction motor model. The Simulink model presented can also be thought of as being built in layers. The Simulink model shown in Figure 3 can be simulated in Simulink or from an *m* file in MATLAB. The *m* file can be saved, and the Simulink model of the motor can be simulated in MATLAB/Simulink by running the *m* file.



Figure 3. Simulink Induction Motor "Upper Level" System Model.



Figure 4. Subsystem that implements (23) and (24) in Simulink.



Figure 5. Subsystem that implements (22) in Simulink.

IV. SIMULATION RESULTS

The system shown in Figure 1 was simulated using the Simulink induction motor model shown in Figure 3. The inverter shown in Fig. 1 was operating as a six-step voltage source inverter with 180° conduction and a DC input voltage to the inverter of V_i =460V during the simulation. A plot of the phase *a* line-to-neutral voltage at the input terminals of the induction motor during the simulation is shown in Figure 15. The parameters of the motor shown in Figure 1 were:

 R_1 =0.087 Ω , R_2 =0.228 Ω , L_m =34.7mH, L_{ls} =0.8mH, L_{lr} =0.8mH, J=1.662kg m², P=4, and HP=50. The load applied to the motor during simulations was a pulsed torque load with the following characteristics: T_L =100N-m, T=10s, and D=4/5. To verify the induction motor Simulink model results, a PSPICE model of the system shown in Figure 1 was simulated. During PSPICE simulation tests, motor parameters, load conditions, etc. were the same as the Simulink model.



Figure 6. Induction Motor dq Model Subsystem.







Figure 8. Subsystem that implements (15) in Simulink.

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Figure 9. Subsystem that implements (19) in Simulink.



Figure 10. Subsystem that implements (20) in Simulink.



Figure 11. Current Variables Subsystem.



Figure 12. Subsystem that implements (27) in Simulink.



Figure 13. Subsystem that implements (21) in Simulink.



Figure 14. Subsystem that implements the sine and cosine functions.

Figure 16 shows the load torque and the angular velocity of the rotor using the Simulink model. Figure 17 shows the electromagnetic torque and the phase a stator current produced using the Simulink model. The PSPICE simulation results for the load torque and the angular velocity of the rotor are shown in Figure 18. Figure 19 shows the PSPICE simulation results for the electromagnetic torque and the phase a stator current. It can be observed from Figure 16-19 that the Simulink model results and PSPICE model results closely match each other. This verifies that the Simulink

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induction motor model shown in Figure 3 is a valid model that can be used for various induction motor studies. The model shown in Figure 3 was simulated again with all parameters the same except that the load torque was changed to T_L =200N-m, T=10s, and D=4/5. The motor is operating at 94% of full load with 200N-m of load torque applied. The results are shown in Figure 20 and Figure 21.

V. CONCLUSION

A dynamic model of the induction motor developed using Simulink / MATLAB that is beneficial for use in undergraduate electric machines and power electronics courses, or that can be used as a research tool in the laboratory has been presented. A block model approach was used in the construction of the motor model that allows students to resolve reference frame theory issues. The model developed is intuitive, easy to use, and allows all motor parameters to be easily accessed for monitoring and comparison purposes. The model can be used to study the dynamic behavior of the induction motor, or can be used in various motor-drive topologies with minor modifications. New subsystems can be added to the model presented to implement various types of control schemes. The model presented is also capable of being used with various inverter topologies and PWM schemes with minor modifications.



Figure 15. Six-Step Phase *a* Line-to-Neutral Voltage Waveform.



Figure 16. Simulink Load Torque and Rotor Angular Velocity Plots.



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Figure 17. Simulink Electromagnetic Torque and Stator Current Plots.



Figure 18. PSPICE Load Torque and Rotor Angular Velocity Plots.



Figure 19. PSPICE Electromagnetic Torque and Stator Current Plots.







Figure 21. Simulink Electromagnetic Torque and Stator Current Plots at 94% of Full Load.

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