# MCM and CPM Combination as Compared to The Use of FDE for CPM

# A. Montazeri, J. Haddadnia

*Abstract*—This paper compares Multi-Carrier Modulation (MCM) and Continuous Phase Modulation (CPM) combination, with the use of Frequency Domain Equalization (FDE) for Continuous Phase Modulation (CPM). It is shown that these two constant envelope methods exploit the frequency diversity of multi path channel. In addition, they have 0dB Peak to Average Power Ratio (PAPR) and high power efficiency. However, MCM-CPM combination outperforms applying FDE for CPM due to superior performance of multicarrier systems in frequency selective channels by using orthogonal subcarriers.

*Index Terms*—Multi Carrier Modulation (MCM), Continuous Phase Modulation (CPM), Frequency Domain Equalization (FDE).

## I. INTRODUCTION

Wireless broadband communications systems are characterized by very dispersive channels, resulting to high delay spreads. This characteristic raises the question of anti-multipath measures with low cost. To face this phenomenon, three methods can be used: single carrier modulation with time domain equalization (SC-TDE), single carrier modulation with frequency domain equalization (SC-FDE), and multi-carrier modulation (MCM). There are currently growing needs for both higher bit rate data transmission and multiple accesses. However, the complexity and the required digital processing speed of conventional SC-TDE systems become exorbitant. In this direction, MCM is found to be the optimal technique for such systems, because of its high spectral efficiency, achieved by the orthogonal carriers and ability to resist multipath fading channels. Other advantages of MCM include simple receiver (since only one tap equalizer is required) and excellent robustness in multi path environment. The third possible solution is SC-FDE transmission, effectively equivalent to MCM, except that an inverse discrete Fourier transform is moved from the transmitter to the receiver. SC-FDE and MCM are considered as physical layer transmission techniques for wideband wireless communication systems. MCM has been adopted by IEEE 802.11 wireless local area network standard, and IEEE 802.16 wireless metropolitan area network standard. Also, SC-FDE is suggested by IEEE802.16, together with MCM [1-2].

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Alimohammad Montazeri, Electrical and Computer Engineering Department, Hakim Sabzevari University, Sabzevar, Iran.

Javad Haddadnia, Electrical and Computer Engineering Department, Hakim Sabzevari University, Sabzevar, Iran. Despite all the attractive advantages, MCM has two primary drawbacks. The first one is sensitivity to imperfect frequency synchronization. The second problem with MCM is that the signal has large amplitude fluctuations caused by the summation of the complex sinusoids. MCM's high peak to average power ratio (PAPR) requires system components with a large linear range capable of accommodating the signal due to the fact that nonlinear distortion results in a loss of subcarrier orthogonality which degrades performance. On the contrary with MCM, SC-FDE don't suffer PAPR and is more compatible than MCM with nonlinear component.

Recently, the idea of constant envelope MCM with continuous phase modulation system has been introduced. The significance of the 0dB PAPR achieved by using CPM mapping is that the signal can be amplified with power efficient nonlinear power amplifier. Its signal-space, spectrum and performance in Stanford University Interim (SUI) multipath channels were investigated, and a suboptimal phase demodulator receiver was proposed [3-5].

In addition, FDE approach has been extended to the equalization of CPM lately [6]. However, CPM is a nonlinear modulation scheme and the application of SC-FDE is not easy. To keep CPM phase continuity when appending a cyclic guard interval, tail bits insertion is considered. Besides, the CPM signal phase is changing continuously, so that a discrete Fourier transform cannot be directly applied to the received samples. Therefore, using CPM decomposition methods, an approach for applying FDE to CPM signals was investigated.

In this paper, MCM and CPM combination signal description as well as the use of FDE for CPM is explained and the performance of these two constant envelope methods with zero dB PAPR is studied. Finally, the simulation results under SUI channels are then presented, and the performance comparison between them is performed.

#### **II. MCM AND CPM COMBINATION**

The Combination of MCM and CPM is a modulation format that can be viewed as a mapping of the MCM signal onto the unit circle. The resulting signal has a constant envelope leading to a 0 dB PAPR. The MCM signal is transformed through continuous phase modulator to a zero PAPR signal prior to the PA and, at the receiver, the inverse transform by a phase demodulator is performed prior to MCM demodulation as shown in Fig. 1. The base band of the proposed method waveform is represented by:





$$s(t) = e^{j\phi(t)},\tag{1}$$

where the phase signal during the nth block is written as

$$\phi(t) = \theta_n + 2\pi h C_N \sum_{k=1}^N I_{n,k} q_k (t - nT_B),$$
<sup>(2)</sup>

for  $nT_{B\leq t} < (n+1)T_B$ , which is the MCM waveform plus memory term  $\theta_n$ . Here *h* refers to modulation index; *N* is the number of sub-carriers;  $\{I_{n,k}\}$  represents M-PAM data symbols;  $T_B$  is the block interval, and  $\{q_k(t)\}$  represents the set of orthogonal subcarrier waveforms. The subcarriers must also be real-valued and  $\{q_k(t)\}$  may be expressed as

$$q_{k}(t) = \begin{cases} \cos(2\pi k t/T_{B}) & ,0 \le t < T_{B}, K \le N/2 \\ \sin(2\pi (k - N/2)t/T_{B}) & ,0 \le t < T_{B}, K > N/2 \\ 0 & , otherwise \end{cases}$$
(3)

The normalizing constant is set to  $C_N = (2/N\sigma_I^2)^{-0.5}$ , where  $\sigma_I^2$  is the variance of the data symbols, and consequently the variance of the phase signal will be  $\sigma_{\phi}^2 = (2\pi h)^2$ . Assuming that the data is independent and identically distributed, it follows that  $\sigma_I^2 = (M^2 - 1)/3$ .

To reduce adjacent channel interference, MCM and CPM Combination signal is made phase-continuous with the introduction of memory. The benefit of continuous phase is a more compact signal spectrum. The phase signal, as defined by (1), has phase jumps at each signaling interval boundary without  $\{\theta_n\}$ . By including memory terms, these jumps are eliminated. The memory term  $\theta_n$ , is a function of all data symbols during and prior to the nth signalling interval.

The phase demodulator receiver is a practical implementation of MCM-CPM Combination receiver and is therefore of practical interest. However, phase demodulator receiver is not optimum necessarily. The optimum receiver is a bank of MN matched filters, one for each potentially transmitted signal. The phase demodulator receiver essentially consists of a phase demodulator followed by a conventional MCM demodulator. The discrete-time phase demodulator is distinguished in Fig. 1. The received signal is first passed through a front-end band pass filter, which limits the bandwidth of the additive noise [3-4].

As long as the duration of the guard interval is greater than or equal to the channel's maximum propagation delay, that is,  $T_g >> \tau_{\rm max}$ , and a cyclic prefix is transmitted during the guard interval, the performance of conventional MCM in a time-dispersive channel is equivalent to flat fading performance. In other words, the multi path fading performance is the same as single path fading performance. It can be said that MCM lacks frequency diversity as well. In MCM, the wideband frequency-selective fading channel is converted into N contiguous frequency non-selective fading channels. Therefore any multipath diversity inherent to the channel is not exploited by the MCM receiver. It is notable that MCM systems typically employ channel coding and frequency-domain interleaving, which offers diversity.



Figure 1. MCM-CPM Combination block diagram

Here, Taylor expansion is applied to consider MCM and CPM Combination behavior in multipath channels. The signal, with  $\theta_n = 0$ , can be written as

$$s(t) = e^{j\phi(t)} = e^{j\sigma_{\phi}m(t)} = \sum_{i=0}^{\infty} [(j\sigma_{\phi})^{i}/i!]m^{i}(t),$$
(4)

where m(t) is the normalized MCM message signal. This can be seen by viewing MCM and CPM Combination waveform by the Taylor series expansion

$$s(t) = [1 + j\sigma_{\phi}m(t) - (\sigma_{\phi}m(t))^{2}/2 - j(\sigma_{\phi}m(t))^{3}/6 + ...] , \qquad (5)$$

for  $0 \le t \le T_B$ , the higher-order terms  $m^n(t)$ , n > 1, results in a frequency spreading of the data symbols. In general, it can be said that the N data symbols that constitute the Combination of MCM and CPM signal are not simply confined to N frequency bins, as is the case with conventional MCM. The phase modulator mixes and spreads, in a nonlinear and exceedingly complicated manner, the data symbols in frequency diversity in the channel. These results indicate that the Combination of MCM and CPM receiver exploits the multipath diversity of the channel. The fact that the proposed method exploits multipath diversity is an interesting result since conventional MCM doesn't. This isn't necessarily the case, however. For small values of modulation index, where only the first two terms in (5) contribute, that is,

$$s(t) = [1 + j\sigma_{\phi}m(t)], \tag{6}$$

the signal doesn't have the frequency spreading given by the higher-order terms. In this case, the proposed method signal is essentially equivalent to a conventional MCM signal, m(t), and therefore doesn't have the ability to exploit the frequency diversity of the channel. Simply put, the Combination of MCM and CPM has frequency diversity when the modulation index is large and doesn't have frequency diversity when the modulation index is small [5].



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## III. THE USE OF FDE FOR CPM

The CPM complex envelope with normalized amplitude is

$$S(t) = e^{j\phi(t;\chi)} \tag{7}$$

where

$$\phi(t;\chi) = 2\pi h \sum_{n=0}^{N-1} x_n q(t-nT)$$
(8)

is the excess phase, h is the modulation index,  $\chi$  is a length-N information sequence, where  $x_n$  represents M-PAM data symbols, M is the alphabet size, and T is the symbol duration. Here MSK, a binary full response (L=1) CPM with modulation index h = 1/2 and phase shaping function q (t) = t/2T, when  $0 \le t < T$  and 1/2 when  $t \ge T$ , is considered.

Applying FDE for CPM requires a discrete representation of the CPM signal. The direct approach invokes a match-filter bank, where the outputs of the match filters provide the discrete CPM signal. By using Laurent's decomposition method, the number of the match filters can be reduced. From Laurent's decomposition, the transmitted CPM signal, s (t), is a linear combination of partial-response PAM signals, cp (t). FDE can be applied to the partial-response PAM waveforms directly, thus reducing the number of states required in the Viterbi Algorithm. However, Laurent's decomposition needs a noise-whitening filter to decorrelate the colored noise introduced by the partial-response waveforms.

With Laurent's decomposition, the receiver has 2L-1 filters matched to the cp (t) pulses. The number of matched filters can be reduced to  $K \le 2L$ -1 when a good approximation to the CPM signal can be obtained with K of the {cp(t)} pulses. Often, the pulse  $c_0$  (t) contains most of the signal energy, so to illustrate the idea of applying SC-FDE, consider the special case, approximate with K = 1. In this case, when CPM signal is transmitted over the multipath channel, the overall channel can be modeled as a concatenation of channel and Laurent filter. Finally, a simple differential decoder can be applied to yield the optimal ML decoding performance [6]. The receiver structure for h = 1/2 binary CPM is shown in Fig. 2.

Similar to the guard interval used in MCM and SC-FDE for linear modulation schemes, a cyclic guard interval is appended to the transmitted CPM signal, such that cyclic guard interval length equals or exceeds the maximum expected channel length. Here, a cyclic prefix is assumed. Addition of the cyclic prefix (CP) is not straightforward with CPM, because phase continuity must always be maintained. To ensure phase continuity when a cyclic prefix is used, it is essential that the path also returns to the zero state when n = N-G. A CPM modulator can be represented as a continuous phase encoder (CPE) followed by a memory less phase modulator, where the CPE determines the trellis structure of the CPM modulator. Using  $\ell_t$  tail symbols to return the encoder to the zero state at epoch n = N - G, the trellis path can be returned to the zero state by flushing the state memory of the CPE. The length  $\ell_t$  depends on the tilted-phase trellis structure, and is equal to the maximum number of inputs needed to return the path to the zero state from any other trellis state.



Figure 2. The use of FDE for CPM block diagram with h=1/2.

## IV. SIMULATION RESULTS AND COMPARISION ANALYSES

In this section, the performance of MCM-CPM combination and the performance of Applying FDE for CPM in multipath channels are presented. Bit error rate (BER) of these methods is evaluated using computer simulation. In this study, the channel is assumed to be known perfectly at the receiver. The parameters of the representative systems used for this study are demonstrated in Table 1. These parameters are derived from IEEE802.16 standard [7]. In this paper SUI channels are determined [8]. The parametric view of the SUI channels is summarized in the Table 2. For each simulation trial, the set of L path gains are generated randomly. Each gain is complex valued, with zero mean and variance. Both the real and imaginary parts of the path gains are Gaussian distributed, thus the envelope is Rayleigh distributed. Also, the channels are normalized. The received signal over multipath channel can take the following form for both understudy systems:

$$r(t) = \int_0^{\tau_{\max}} h(t,\tau) s(t-\tau) d\tau + n(t),$$
(9)

where  $h(t,\tau)$  is the channel impulse response having a maximum propagation delay  $\tau_{max}$  and n(t) is complex Gaussian noise. Channel is assumed to be wide sense stationary uncorrelated scattering (WSSUS), and comprises of L discrete paths. For the proposed system, a cyclic prefix guard interval is transmitted. At the receiver, r (t) is sampled, the guard time samples are discarded and the block time samples are processed. Then frequency domain equalizer is applied.

Fig. 3 illustrates MCM and CPM combination BER over SUI 1-6 channel models for M=4 and  $2\pi$ h=1. This figure compares the SUI 1-6 channels with the AWGN and Rayleigh channels results. As shown in this figure, the performance of the SUI 1-6 channels using MMSE equalizer outperforms the Rayleigh channel performance. For example, in Fig. 3, performance at BER=10<sup>-4</sup>, over SUI6 is 15dB better than single path Rayleigh channel.



The results presented in Fig. 3 show that multi-path diversity is exploited by the phase demodulator receiver as expected from the aforementioned analysis. The multipath diversity depends not only on the number of independent paths but also on the way in which the power is distributed over the paths. It is worth noting that the frequency non-selective channel models considered have L = 1 path of which 100% of the channel gain depends, and thus these channels have no multipath diversity. This is the reason that multipath channels outperform Rayleigh channel.

In Fig. 4, applying FDE for CPM is simulated over SUI multipath as well as Rayleigh channels. For this simulation, the reduced complexity receiver, shown in Fig. 2, is applied. As it can be seen in Fig. 4, the performance in multipath channels outperforms Rayleigh channel performance. In that case both under study systems exploit channel frequency diversity.

As stated before, MCM-CPM combination has frequency diversity when the modulation index is large and doesn't have frequency diversity when the modulation index is small. This property is demonstrated in Fig. 5. As shown in this figure, MCM-CPM with a small modulation index lacks frequency diversity. Notice that for  $2\pi h=0.1$  the single-path and multi-path performance is essentially the same. By contrast, for the large modulation index e.g.  $2\pi h=1.1$ , the multi-path performance is significantly better than the single-path performance. Considering the power amplifier nonlinearities, in Fig. 6, the performance of MCM-CPM is compared with the conventional 16PSK-MCM over SUI4 channel. In this case  $2\pi h=1$  and M=16. In addition, the solid state power amplifier (SSPA) model is employed at 0dB input power back-off (IBO) level. Here, the advantage of the MCM-CPM combination is operating with IBO = 0dB. As shown in Fig. 6, over the region  $0dB \leq E_b/N_0 \leq 10dB$ , the MCM system performs better than the MCM-CPM combination. Under this 10dB threshold, nonlinear and non-Gaussian noise is injected into the MCM demodulator (following the phase demodulator) and causes performance degradation. As a result, this figure reveals that although MCM-CPM exploits the frequency diversity inherent to the channel, however, it exhibits a poor performance at low SNR due to the threshold effect.

In addition, Fig. 7 illustrates the performance of MCM-CPM combination, with applying FDE for CPM, over SUI3. The results over other SUI channels are comparable. As it can be seen, the superiority performance of MCM-CPM combination with respect to applying FDE for CPM is obvious as the performance of MCM-CPM combination is about 7dB better than applying FDE for CPM. This superiority is permanent even by increasing SNR (Signal to noise ratio). These results show that by solving PAPR problem and lack of channel frequency diversity of conventional MCM in MCM-CPM combination, the inherent superiority of multicarrier methods in frequency selective channels with respect to single carrier methods is dominated. As a result, MCM-CPM performance outperforms applying FDE for CPM. These results can be justified as multi-carrier techniques cause better performance by converting wideband frequency selective fading channel into N contiguous frequency non-selective fading channels. However, under 10dB SNRs, the performances of these two understudy systems are approximately close to each other. The reason for the similarity of the MCM-CPM combination and applying FDE for CPM performance under 10dB threshold is inherent degradation of phase demodulator receiver. Under this 10dB threshold, nonlinear and non Gaussian noise is injected into the MCM demodulator (following the phase demodulator) and causes performance degradation.

$T_B$			Block Interval	114 <i>µs</i>		
$T_g$			Guard Interval	$32 \mu s$		
$T_F$			Frame Interval	146 µs		
J			Oversamplig Factor	8		
$F_{sa} = JN_B / T_B$			Sampling Frequency	14Mega (samp./sec)		
$N_B$	$N_{g}$	$N_F$	Num. of Carriers	200	56	256
$BW = N / T_B$			Bandwidth	1.75 <i>MHz</i>		
$\eta_t$			Transmission efficiency	114/146 = %78		
$1/T_B$			Subcarrier Spacing	8750 <i>Hz</i>		

TABLE I. SYSTEM AND SIGNAL PARAMETERS

TABLE II.SUI CHANNELS PARAMETERS

	Delay	L (N	Delay			
Model	Gain	Tap1	Tap2	Tap3	$(\tau_{rms})$	
CI.	П 1	0 <i>µs</i>	0.4 <i>µs</i>	0.8µs	0.111.00	
3011		0dB	-15dB	-20dB	0.111µs	
SI	П 2	0µs	0.5 <i>µs</i>	1µs	0.202µs	
30	01.2	0dB	-12dB	-15dB		
SI	П 3	0µs	0.5 <i>µs</i>	1µs	0.264µs	
50	515	0dB	-5dB	-10dB		
SI	ПЛ	0µs	$2\mu s$	$4\mu s$	1.257µs	
50	JI 4	0dB	-4dB	-8dB		
SI	П.5	0µs	$5\mu s$	10 <i>µs</i>	2.842.00	
5015		0dB	-5dB	-10dB	2.042µs	
SUI 6		0µs	14 µs	20 <i>µs</i>	5.240µs	

## V. CONCLUSION

In this paper, the performance of applying FDE for CPM and MCM-CPM combination for SUI multi path channels is compared. System description and block diagram of both techniques are presented. The results obtained show that MCM-CPM exploits the frequency diversity of the multi path channel. In addition, Taking into account the IBO, MCM-CPM is shown to outperform MCM at high bit energy to-noise density ratios (Eb/N0). However, at low SNR the MCM-CPM phase demodulator receiver suffers from a threshold effect. The comparison of MCM-CPM combination and applying FDE for CPM over SUI3 is also presented. Performance of MCM-CPM is about 7dB better than CPM-FDE, and this superiority is permanent even by However, under 10dB SNR, the

increasing SNR. However, a performances of these two systems are approximately comparable. The reason is that the nonlinear and non Gaussian

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noise is injected into the MCM demodulator (following the phase demodulator) and causes performance degradation under this 10dB threshold.

### REFERENCES

- Z. Wang, Xiaoli Ma, and G. B. Giannakis, "OFDM or Single-Carrier Block Transmissions?" IEEE Transaction on Communications, VOL.52, NO.3, March 2004.
- Falconer, D.; Ariyavisitakul, S.L.; Benyamin-Seeyar, A.; Eidson, B.; "Frequency domain equalization for single-carrier broadband wireless systems" IEEE Communications Magazine, Volume 40, Issue 4, Page(s):58 – 66, April 2002.
- [3] Thompson, S.C.; Ahmed, A.U.; Proakis, J.G.; Zeidler, J.R.; Geile, M.J.;" Constant Envelope OFDM", IEEE Transactions on Communications, Volume 56, Issue8, Page(s):1300–1312, August 2008.
- [4] Tsai, Y.; Zhang, G.; Pan, J., L.; "Orthogonal frequency division multiplexing with phase modulation and constant envelope design", IEEE Military Communications Conference 2005, Page(s):2658 – 2664, 17-20 October 2005.
- [5] Vakily, V. T., Montazeri, Am., "OFDM-CPM BER Performance in SUI Multi path channels". IEEE International Conference on Circuits & Systems for Communications 2008, Page(s):877-881, 26-28 May 2008.
- [6] Tan, J.; Stuber, G., L.; "Frequency-domain equalization for continuous phase modulation", IEEE Transactions on Wireless Communications, Volume 4, Issue 5, Page(s):2479 – 2490, September 2005.
- [7] IEEE 802.162004," IEEE Standard for Local and Metropolitan Area Networks -Part16: Air Interface for Fixed Broadband Wireless Access Systems", 1October, 2004.
- [8] V. Erceg, K.V.S. Hari, M.S. Smith, D.S. Baum et al, "Channel Models for Fixed Wireless Applications", IEEE 802.16.3 Task Group Contributions, February 2003.



Figure 3. MCM and CPM combination Performance in SUI Channels



Figure 5. MCM and CPM combination Performance Simulation (Modulation index effect)



Figure 6. Performance Simulation (MCM-CPM combination and 16PSK-MCM)



Figure 7. MCM-CPM combination and applying SC-FDE for CPM BER Performance comparison over SUI3



Figure 4. CPM with SC-FDE Performance Simulation



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