

# A New Similarity Measure for Fuzzy Sets with the Extended Definition of Complementation

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**Abstract**— Similarity measure for Fuzzy sets is one of the researched topics of Fuzzy set theory. Till now there have been several methods to measure similarity between two Fuzzy sets. The existing methods are based on traditional Zadehian Theory of Fuzzy sets where it is believed that there is no difference between Fuzzy membership function and Fuzzy membership value for the complement of a Fuzzy set which is already proved to be wrong. In this article, effort has been made to put forward a new similarity measure with the help of extended definition of complementation of Fuzzy sets using reference function. Our proposed method is based on the fact that Fuzzy membership function and Fuzzy membership value for the complement of a Fuzzy set are two different things. With this new approach an attempt has been made to design a new Similarity measure for Fuzzy sets so that it becomes free from any further controversy.

**Index Terms**— Complement of a Fuzzy set, Fuzzy membership function, Fuzzy membership value, Fuzzy reference function, Fuzzy set, Similarity measure.

## I. INTRODUCTION

Similarity measure is an important concept of Fuzzy set theory. In the literature, there are several well known Similarity measures for Fuzzy sets. Since Zadeh [1] introduced Fuzzy sets in 1965, many approaches and theories treating imprecision and uncertainty have been proposed. Some of these theories, such as intuitionistic Fuzzy sets (IFS), interval-valued Fuzzy sets (IVFS), and interval-valued intuitionistic Fuzzy sets (IVIFS), are extensions of Fuzzy set theory initiated by Zadeh. Many contributions have been already made to Similarity measures of Fuzzy sets. Szmidi and Kacprzyk[5] proposed a similarity measure for intuitionistic Fuzzy sets. Ju and Wang [6] proposed a similarity measure for interval-valued Fuzzy sets. These already proposed Similarity measures are based on Zadehian definition of Fuzzy set. In Zadehian theory of Fuzzy set, it has been believed that the classical set theoretic axioms of exclusion and contradiction are not satisfied for Fuzzy sets. Regarding this, Baruah [2,3] proposed that two functions, namely Fuzzy membership function and Fuzzy reference function are necessary to represent a Fuzzy set.

Therefore, Baruah [2, 3] reintroduced the notion of complement of a Fuzzy set in a way that the set theoretic axioms of exclusion and contradiction can be seen valid for Fuzzy sets also. In recent years researchers have contributed a lot towards Fuzzy set theory. Neog and Sut [4] have generalized the concept of complement of a Fuzzy set introduced by Baruah[2,3], when the Fuzzy reference function is not zero and defined arbitrary Fuzzy union and intersection extending the definitions of Fuzzy sets given by Baruah [2, 3].

Therefore, the existing similarity measures based on Zadehian concept which itself is controversial cannot yield a suitable result. Hence the existing Similarity measures need to be changed accordingly. In this paper, we put forward a new similarity measure for Fuzzy sets using the extended definition of complementation based on reference function[2,3,4] so that it becomes free from any doubt. Also, the application of the proposed measure has been demonstrated with the help of evaluation of some collected data.

The overall organization of this paper is as follows. In section 2 we discuss the new and extended definitions of Fuzzy set. In section 3 we propose a new similarity measure with the extended definition of complementation of Fuzzy sets. In section 4 we apply the proposed Similarity measure on some collected dataset. Some conclusions are given in secti 5.

## II. NEW EXTENDED DEFINITION OF FUZZY SET

In the Fuzzy set theory introduced by Zadeh, it has been believed that for a Fuzzy set A and its complement  $A^c$ , neither  $A \cap A^c$  is null set nor  $A \cup A^c$  is the universal set. Regarding this, Baruah[2,3] has proposed that in the Zadehian definition of the complement of a Fuzzy set, Fuzzy membership function and Fuzzy membership value had been taken to be the same, which led to the conclusion that the Fuzzy sets do not follow the set theoretic axioms of exclusion and contradiction. Therefore, Baruah has put forward an extended definition of Fuzzy set and redefined the complement of a Fuzzy set. According to Baruah, to define a Fuzzy set two functions namely- Fuzzy membership function and -Fuzzy reference function are necessary. Fuzzy membership value is the difference between Fuzzy membership function and Fuzzy reference function. Fuzzy membership function and Fuzzy membership value are two different things.

### A. Baruah's definition of Fuzzy set

Baruah put forward an extended definition of Fuzzy sets in the following manner –

Let  $\mu_1(x)$  and  $\mu_2(x)$  be two functions, where  $0 \leq \mu_2(x) \leq \mu_1(x) \leq 1$ . For a Fuzzy number denoted by  $\{x, \mu_1(x), \mu_2(x) ; x \in U\}$  we would call  $\mu_1(x)$ , the Fuzzy membership function and  $\mu_2(x)$ , a reference function such that  $\{\mu_1(x) - \mu_2(x)\}$  is the Fuzzy membership value for any x.

According to Baruah, in the definition of complement of a Fuzzy set, the Fuzzy membership value and the Fuzzy membership function have to be different in the sense that for a usual Fuzzy set the membership value and the membership function are of course equivalent.

### B. Extended Definition of Union and Intersection of Fuzzy Sets

With the help of the extended definition, Baruah put forward the notion of union and

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intersection of two Fuzzy sets in the following manner –

Let  $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$  and  $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$  be two Fuzzy sets defined over the same universe  $U$ , where  $\mu_1, \mu_2$  and  $\mu_3, \mu_4$  are membership and reference functions of  $A$  and  $B$  respectively.

Now on the basis of Baruah's extended definition of Fuzzy set, we can represent these two Fuzzy sets  $A$  and  $B$  in the number line in Fig. 1 and Fig. 2 respectively.



Fig 1. Representation of Fuzzy set A in number line.

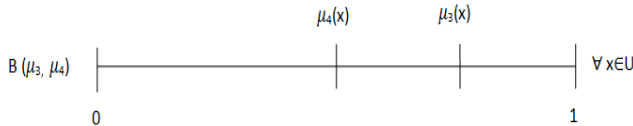


Fig 2. Representation of Fuzzy set B in number line.

Then the operations intersection and union are defined as

$$A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\} \text{ and}$$

$$A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$$

Two Fuzzy sets  $C = \{x, \mu_C(x); x \in U\}$  and  $D = \{x, \mu_D(x); x \in U\}$  in the usual definition would be expressed as

$$C(\mu_C, 0) = \{x, \mu_C(x), 0; x \in U\} \text{ and } D(\mu_D, 0) = \{x, \mu_D(x), 0; x \in U\}$$

Accordingly, we have,

$$\begin{aligned} C(\mu_C, 0) \cap D(\mu_D, 0) &= \{x, \min(\mu_C(x), \mu_D(x)), \max(0, 0); x \in U\} \\ &= \{x, \min(\mu_C(x), \mu_D(x)), 0; x \in U\} \\ &= \{x, \mu_C(x) \wedge \mu_D(x); x \in U\} \end{aligned}$$

which in the usual definition is nothing but  $C \cap D$ .

Similarly we have,

$$\begin{aligned} C(\mu_C, 0) \cup D(\mu_D, 0) &= \{x, \max(\mu_C(x), \mu_D(x)), \min(0, 0); x \in U\} \\ &= \{x, \max(\mu_C(x), \mu_D(x)), 0; x \in U\} \\ &= \{x, \mu_C(x) \vee \mu_D(x); x \in U\} \end{aligned}$$

which in the usual definition is nothing but  $C \cup D$ .

Neog and Sut [4] showed by an example that this definition sometimes gives degenerate cases and revised the above definition as follows -

Let  $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$  and  $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$  be two Fuzzy sets defined over the same universe  $U$ . The operation intersection is defined as  $A(\mu_1, \mu_2) \cap B(\mu_3, \mu_4) = \{x, \min(\mu_1(x), \mu_3(x)), \max(\mu_2(x), \mu_4(x)); x \in U\}$  with the condition that  $\min(\mu_1(x), \mu_3(x)) > \max(\mu_2(x), \mu_4(x)) \forall x \in U$ .

Now if for some  $x \in U$ ,  $\min(\mu_1(x), \mu_3(x)) < \max(\mu_2(x), \mu_4(x))$   
Then our conclusion is that  $A \cap B = \emptyset$ .

and if for some  $x \in U$ ,

$\min(\mu_1(x), \mu_3(x)) = \max(\mu_2(x), \mu_4(x))$  then also  $A \cap B = \emptyset$ .

Further the operation Union is defined as  $A(\mu_1, \mu_2) \cup B(\mu_3, \mu_4) = \{x, \max(\mu_1(x), \mu_3(x)), \min(\mu_2(x), \mu_4(x)); x \in U\}$ .

with the condition that

$$\min(\mu_1(x), \mu_3(x)) \geq \max(\mu_2(x), \mu_4(x)) \forall x \in U.$$

if for some  $x \in U$ ,

$$\min(\mu_1(x), \mu_3(x)) < \max(\mu_2(x), \mu_4(x))$$

then the union of Fuzzy sets  $A$  and  $B$  cannot be expressed as one single Fuzzy set.

The union, however can be expressed in one single Fuzzy set if for some  $x \in U$ ,  $\min(\mu_1(x), \mu_3(x)) = \max(\mu_2(x), \mu_4(x))$ .

We can clearly visualize: these extended definitions of union and intersection are valid in our Fig. 1 and Fig. 2 also.

### C. Complement of a Fuzzy Set Using Extended Definition

Baruah put forward the notion of complement of usual Fuzzy sets with Fuzzy reference function 0 in the following way –

Let  $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$  and  $B(1, \mu) = \{x, 1, \mu(x); x \in U\}$  be two Fuzzy sets defined over the same universe  $U$ .

Now on the basis of Baruah's extended definition of Fuzzy set, we can represent these two Fuzzy sets  $A$  and  $B$  in the number line in Fig. 3 and Fig. 4 respectively.

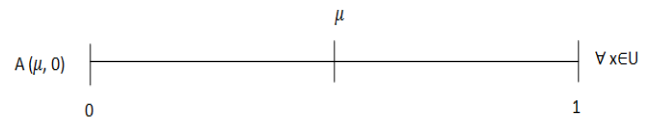


Fig 3. Representation of Fuzzy set A in number line.

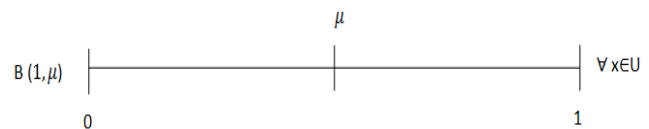


Fig 4. Representation of Fuzzy set B in number line.

Now we have

$$\begin{aligned} A(\mu, 0) \cap B(1, \mu) &= \{x, \min(\mu(x), 1), \max(0, \mu(x)); x \in U\} \\ &= \{x, \mu(x), \mu(x); x \in U\} \end{aligned}$$

which is nothing but the null/empty set  $\emptyset$  [since  $\mu(x) - \mu(x) = 0$ ] and

$$\begin{aligned} A(\mu, 0) \cup B(1, \mu) &= \{x, \max(\mu(x), 1), \min(0, \mu(x)); x \in U\} \\ &= \{x, 1, 0; x \in U\} \end{aligned}$$

which is nothing but the universal set  $U$ .

This means if we define a Fuzzy set  $(A(\mu, 0))^c = \{x, 1, \mu(x); x \in U\}$  it is nothing but the complement of  $A(\mu, 0) = \{x, \mu(x), 0; x \in U\}$ .

Neog and Sut [4] have generalized the concept of complement of a Fuzzy set when the Fuzzy reference function is not zero extending definition of complement of Fuzzy sets introduced by Baruah [2, 3] in the following manner-

Let  $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$  be a Fuzzy set defined over the universe  $U$ . The complement of the Fuzzy set  $A(\mu_1, \mu_2)$  is defined as



$$(A(\mu_1, \mu_2))^C = \{x, \mu_1(x), \mu_2(x); x \in U\}^C$$

$$= \{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}$$

Membership value of x in  $(A(\mu_1, \mu_2))^C$  is given by  $\mu_2(x) + (1 - \mu_1(x)) = 1 + \mu_2(x) - \mu_1(x)$ .

If  $\mu_2(x) = 0$ , then membership value of x is  $1 + 0 - \mu_1(x) = 1 - \mu_1(x)$ .  
Since, for  $x \in U$ ,  $\min(\mu_2(x), 1) < \max(0, \mu_1(x))$ , so the union of these two Fuzzy sets cannot be expressed as one single Fuzzy set.

The above complement properties hold good also when we take Fuzzy reference function = 0  $\forall x \in U$ .

We can clearly visualize: this extended definition of complementation is valid in our Fig. 3 and Fig. 4 also.

Thus, we have understood that for the complement of a Fuzzy set the Fuzzy membership value and the Fuzzy membership function are two different things although for a usual Fuzzy set they are not different because the value of the function is counted from 0 in the usual case.

These extended definitions of Fuzzy set has satisfied the set theoretic axioms of contradiction and exclusion in the following manner-

#### D. Law of contradiction

Let  $A(\mu_1, \mu_2)$  be a Fuzzy set defined on the set of universe U. Now with respect to our Fig. 1, we have,

$$A(\mu_1, \mu_2) \cap (A(\mu_1, \mu_2))^C = \{x, \mu_1(x), \mu_2(x); x \in U\} \cap [\{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}]$$

$$= [\{x, \mu_1(x), \mu_2(x); x \in U\} \cap \{x, \mu_2(x), 0; x \in U\}] \cup [\{x, \mu_1(x), \mu_2(x); x \in U\} \cap \{x, 1, \mu_1(x); x \in U\}]$$

$$= [\{x, \mu_2(x), \mu_2(x); x \in U\}] \cup [\{x, \mu_1(x), \mu_1(x); x \in U\}]$$

$$= (\text{Empty Set}) \cup (\text{Empty Set})$$

$$= \emptyset \cup \emptyset = \emptyset$$

#### E. Law of exclusion

Let  $A(\mu_1, \mu_2)$  be a Fuzzy set defined on the set of universe U. Now with respect to our Fig. 1, we have,

$$A(\mu_1, \mu_2) \cup (A(\mu_1, \mu_2))^C = \{x, \mu_1(x), \mu_2(x); x \in U\} \cup [\{x, \mu_2(x), 0; x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}]$$

$$= [\{x, \mu_1(x), \mu_2(x); x \in U\} \cup \{x, \mu_2(x), 0; x \in U\}] \cup [\{x, \mu_1(x), \mu_2(x); x \in U\} \cup \{x, 1, \mu_1(x); x \in U\}]$$

$$= [x, \mu_1(x), 0; x \in U] \cup [x, 1, \mu_2(x); x \in U]$$

$$= [x, 1, 0; x \in U]$$

$$= U$$

Neog and Sut [4] put forward the notion of Fuzzy subset using the extended notion of Fuzzy sets in the following manner -  
F. Fuzzy subset and properties

Let  $A(\mu_1, \mu_2) = \{x, \mu_1(x), \mu_2(x); x \in U\}$  and  $B(\mu_3, \mu_4) = \{x, \mu_3(x), \mu_4(x); x \in U\}$  be two Fuzzy sets defined over the same universe U.

With respect to our Fig. 1 and Fig. 2, the Fuzzy set  $A(\mu_1, \mu_2)$  is a subset of the Fuzzy set  $B(\mu_3, \mu_4)$  if  $\forall x \in U, \mu_1(x) \leq \mu_3(x)$  and  $\mu_2(x) \leq \mu_4(x)$ .

Two Fuzzy sets  $C = \{x, \mu_C(x); x \in U\}$  and  $D = \{x, \mu_D(x); x \in U\}$  in the usual definition would be expressed as

$$C(\mu_C, 0) = \{x, \mu_C(x), 0; x \in U\} \text{ and } D(\mu_D, 0) = \{x, \mu_D(x), 0; x \in U\}$$

Accordingly we have,

$$C(\mu_C, 0) \subseteq D(\mu_D, 0)$$

Therefore  $\forall x \in U, \mu_C(x) \leq \mu_D(x)$ , which can be obtained by putting  $\mu_2(x) = \mu_4(x) = 0$  in our extended definition of Fuzzy set.

### III. A NEW SIMILARITY MEASURE FOR FUZZY SETS

Let A and B be two elements belonging to a Fuzzy set (or sets). Now we can measure the similarity between A and B as below:

$$\text{Sim}(A, B) = \frac{I_{FS}(A, B)}{I_{FS}(A, B^C)} = \frac{a}{b} \quad (1)$$

where a is distance from  $A(\mu_m, \mu_r, \mu_v)$  to  $B(\mu_m, \mu_r, \mu_v)$  and b is a distance from  $A(\mu_m, \mu_r, \mu_v)$  to  $B^C(\mu_m, \mu_r, \mu_v)$  where  $\mu_m, \mu_r, \mu_v$  are membership function, reference function and membership value respectively.

For this similarity measure, we have,  
 $0 \leq \text{Sim}(A, B) \leq \alpha$

Similarly we can calculate the Similarity between two Fuzzy sets:

Let A and B be two Fuzzy sets defined on the same set of universe of discourse. Now we can measure the similarity between A and B by assessing similarity of the corresponding elements belonging to A and B, as defined in the eqn (1).

Now using Baruah's definition of Fuzzy set, for the Similarity measure of A and B, we can obtain the following 4 possibilities,

- A and B may be two exactly similar sets.
- or A and  $B^C$  may be two exactly similar sets.
- or A may be more similar to B than to  $B^C$ .
- or A may be more similar to  $B^C$  than to B.
- But A can never be similar to B and  $B^C$  together i.e.  $A=B=B^C$  is never possible according to the new definition of complementation of Fuzzy set [2, 3].

Therefore from the above analysis, for the Similarity measure of A and B, we can conclude four possible cases as follows:

- Case 1:  $\text{Sim}(A, B) = 0$  when  $A=B$  i.e.  $AB=0$ .
- Case 2:  $\text{Sim}(A, B) = \infty$  when  $A=B^C$  i.e.  $AB^C=0$ .
- Case 3:  $\text{Sim}(A, B) > 1$  when  $AB > AB^C$ .
- Case 4:  $\text{Sim}(A, B) < 1$  when  $AB < ABC$ .

Hence to measure the similarity between the two Fuzzy sets A and B, one should be interested in the values  $0 \leq \text{Sim}(A, B) < 1$ .

Let us explain the above idea for a new Similarity measure into details:

Let A and B be two Fuzzy sets defined on the same set of universe of discourse  $U = \{e_1, e_2, e_3, e_4, e_5\}$ . Now we can calculate the similarity measure for A and B assessing the similarity measure for the every corresponding elements of A and B i.e. for the every element  $e_1, e_2, e_3, e_4, e_5$  of the set of universe of discourse U, considered for A and B. This means similarity measure for A and B has to be calculated with

respect to every  $e_1, e_2, e_3, e_4, e_5 \in U$ .

Now, based on the new definition of Fuzzy set, the similarity measure for the Fuzzy set  $A(e_k, k=1,2,3,4,5)$  and the Fuzzy set  $B(e_k, k=1,2,3,4,5)$  can be obtained under the 3 possible cases in the following manner:

We can visualize the Fuzzy set  $A(e_k)$  and the Fuzzy set  $B(e_k)$  in the number line in Fig. 5 and Fig. 6 respectively.

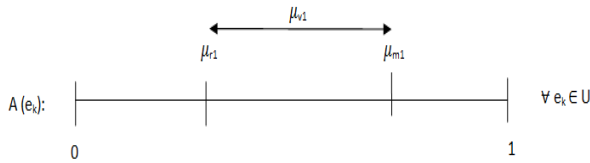


Fig 5. Representation of Fuzzy set A(ek) in number line.

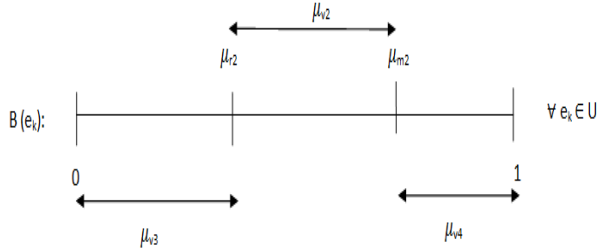


Fig 6. Representation of Fuzzy set B(ek) in number line.

Where  $\mu_{r1}, \mu_{m1}, \mu_{v1}$ ;  $\mu_{r2}, \mu_{m2}, \mu_{v2}$ ;  $0, \mu_{r2}, \mu_{v3}$ ;  $\mu_{m2}, 1, \mu_{v4}$  are reference function, membership function and membership value of the Fuzzy set  $A$ , the Fuzzy set  $B$  and the two complement sets of  $B$  respectively for every  $e_k \in U$ .

Now the 3 possible cases are:

Case 1: when  $\mu_{r2} \neq 0, \mu_{m2} \neq 1$ .

Case 1 can be visualized in Fig. 5 and Fig. 6 and Similarity Measure can be defined as,

$$\frac{AB}{AB^c} =$$

$$Sim(A, B) = \sum_{k=1}^5 \left( \frac{|A(S_k(\mu_{r1})) - B(S_k(\mu_{r2}))| + |A(S_k(\mu_{m1})) - B(S_k(\mu_{m2}))| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v2}))|}{|A(S_k(\mu_{r1})) - 0| + |A(S_k(\mu_{r1})) - B(S_k(\mu_{m2}))| + |A(S_k(\mu_{m1})) - B(S_k(\mu_{r2}))| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v3}))| + |A(S_k(\mu_{v2})) - B(S_k(\mu_{v4}))|} \right)$$

Case 2: when  $\mu_{r2} = 0, \mu_{m2} \neq 1$ .

Case 2 can be visualized in Fig. 7 and Fig. 8.

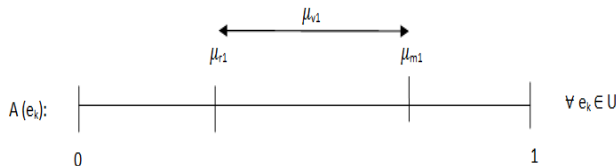


Fig 7. Representation of Fuzzy set A(ek) in number line

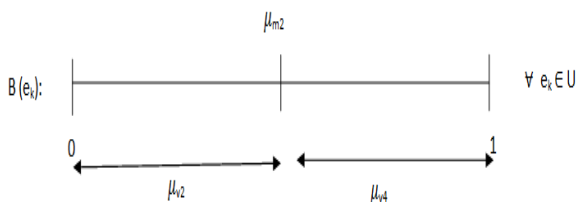


Fig 8. Representation of Fuzzy set B(ek) in number line and Similarity Measure can be defined as,

$$\frac{AB}{AB^c} =$$

$$Sim(A, B) = \frac{1}{5} \sum_{k=1}^5 \left( \frac{|A(S_k(\mu_{r1})) - 0| + |A(S_k(\mu_{m1})) - B(S_k(\mu_{m2}))| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v2}))|}{|A(S_k(\mu_{r1})) - B(S_k(\mu_{m2}))| + |A(S_k(\mu_{m1})) - 1| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v4}))|} \right)$$

Case 3: when  $\mu_{r2} \neq 0, \mu_{m2} = 1$ .

Case 3 can be visualized in Fig. 9 and Fig. 10.

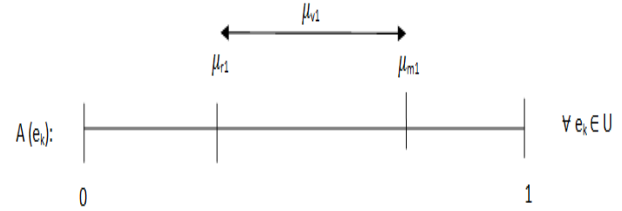


Fig 9. Representation of Fuzzy set A(ek) in number line.

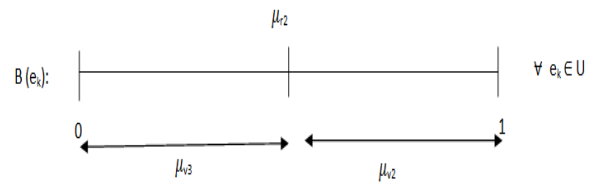


Fig 10. Representation of Fuzzy set B(ek) in number line and Similarity Measure can be defined as,

$$\frac{AB}{AB^c} =$$

$$Sim(A, B) = \frac{1}{5} \sum_{k=1}^5 \left( \frac{|A(S_k(\mu_{r1})) - B(S_k(\mu_{r2}))| + |A(S_k(\mu_{m1})) - 1| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v2}))|}{|A(S_k(\mu_{r1})) - 0| + |A(S_k(\mu_{m1})) - B(S_k(\mu_{r2}))| + |A(S_k(\mu_{v1})) - B(S_k(\mu_{v3}))|} \right)$$

#### IV. APPLICATION OF THE PROPOSED SIMILARITY MEASURE

We apply our proposed measure on some collected data[5].

Taking larger value (membership or non-membership function value) [5] as membership function value,  $\mu_v$  and the smaller value as reference function value,  $\mu_r$  (since, always  $0 \leq \mu_r(x) \leq \mu_v(x) \leq 1$ ), we can represent the collected data[5] as dataset1 = {A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, A<sub>4</sub>, A<sub>5</sub>} and dataset2 = {B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub>, B<sub>4</sub>} defined on the same set of universe of discourse  $U = \{a, b, c, d, e\}$  as given in Table I and Table II.

Table I: dataset 1 = {A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, A<sub>4</sub>, A<sub>5</sub>}.

	A <sub>1</sub>	A <sub>2</sub>	A <sub>3</sub>	A <sub>4</sub>	A <sub>5</sub>
a	(0.7,0.1)	(0.8,0.1)	(0.9,0.1)	(0.7,0.2)	(0.8,0.1)
b	(0.4,0.3)	(0.7,0.0)	(0.6,0.2)	(0.7,0.2)	(0.8,0.2)
c	(0.7,0.1)	(0.9,0.0)	(0.7,0.2)	(0.8,0.0)	(0.8,0.2)
d	(0.5,0.3)	(0.6,0.2)	(0.6,0.1)	(0.4,0.2)	(0.8,0.0)
e	(0.4,0.0)	(0.7,0.0)	(0.3,0.3)	(0.7,0.1)	(0.8,0.1)

Table II: dataset 2 = {B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub>, B<sub>4</sub>}.

	B <sub>1</sub>	B <sub>2</sub>	B <sub>3</sub>	B <sub>4</sub>
a	(0.6,0.1)	(0.8,0.1)	(0.5,0.0)	(0.4,0.3)
b	(0.6,0.1)	(0.7,0.1)	(0.7,0.2)	(0.7,0.2)
c	(0.8,0.2)	(0.6,0.1)	(0.6,0.0)	(0.4,0.3)



<b>d</b>	(0.6,0.1)	(0.4,0.4)	(0.8,0.1)	(0.5,0.4)
<b>e</b>	(0.8,0.1)	(0.8,0.0)	(0.8,0.1)	(0.6,0.1)

Each element (a, b, c, d or e) ∈ U in Table I and Table II is described by: a reference function and a membership function value.

Now to calculate a similar set from the dataset 1 for a particular set in dataset 2, we proceed in the following way:

Step 1: At first we calculate the similarity measure  $\frac{B_j A_i}{B_j A_i^c}$  for each set  $B_j \in$  dataset 2, (where  $j=1,2,3,4$ ) with every set  $A_i \in$  dataset 1, where ( $i=1,2,3,4,5$ ) separately, assessing the similarity measure for the every corresponding elements of the two sets i.e. a,b,c,d,e ∈ U, the set of universe of discourse considered for the two datasets.

Step 2: Then we find out the smallest value from the obtained similarity measures between a set  $B_j$  and every set  $A_i$ , we considered in Step 1. From that value we can decide which  $A_i \in$  dataset 1 is similar to a particular set  $B_j \in$  dataset 2.

Now we calculate the similarity measure values between the dataset 1 and the dataset 2 and represent the calculated values in table III.

**Table III: Similarity measure values between dataset 1 and dataset 2.**

	<b>A<sub>1</sub></b>	<b>A<sub>2</sub></b>	<b>A<sub>3</sub></b>	<b>A<sub>4</sub></b>	<b>A<sub>5</sub></b>
<b>B<sub>1</sub></b>	0.49	0.23	0.28	0.19	0.15
<b>B<sub>2</sub></b>	0.41	0.24	0.37	0.20	0.33
<b>B<sub>3</sub></b>	0.54	0.28	0.35	0.21	0.12
<b>B<sub>4</sub></b>	0.45	0.61	0.49	0.38	0.57

Hence from table III we can conclude that, Set  $B_1$  is similar to set  $A_5$ , set  $B_2$  is similar to set  $A_4$ , set  $B_3$  is similar to set  $A_5$  and set  $B_4$  is similar to set  $A_4$ .

## V. CONCLUSION

In this article we intended to draw attention on some existing similarity measures for Fuzzy sets in the Fuzzy set theory and found that they are based on traditional Zadehian definition of complement of a Fuzzy set which is already proved to be wrong. Also we have discussed the new definition of complementation and come to the conclusion that the Fuzzy membership value and the Fuzzy membership function for the complement of a Fuzzy set are two different things. It is observed that the complementation defined with the help of reference function seems more logical than the Zadehian definition. It is due to this reason we have proposed a new Similarity measure for Fuzzy sets on the basis of complementation based on reference function. Finally the application of this proposed measure has been demonstrated to show the evaluation of some example dataset to calculate their similarity measures.

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