Some EOQ Model for Weibull Deterioration Items with Selling Price Dependent Demand

A. Lakshmana Rao, B. Neela Rao, R. Santhi Kumar

Abstract - In this paper we develop and analyze an inventory model assumption that deterioration rate follows Weibull two parameter distributions with selling price dependent demand. Here it is assumed that demand rate is a function of selling price. With shortage and without shortage both cases have been taken care of in developing the inventory models. Shortages are fully backlogged whenever they are allowed. Through numerical examples the results are illustrated. The sensitivity analysis for the model has been performed to study the effect changes of the values of the parameters associated with the model.

Keywords: EOQ model; deteriorating items; shortage; selling price dependent demand; Weibull distribution.

I. INTRODUCTION

The influence and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over period of time. In real life, many of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items where deterioration can take place during the normal storage period of the commodity and consequently this loss must be taken into account when analyzing the system. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models.


At the beginning, demand rate were assumed to be constant which is in general likely to be time dependent and stock dependent.

Begum et al. [2010] have developed economic lot size model for price-dependent demand. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al [2003], with the motivation of C. K. Tripathy., et al. [2010] and Sushil Kumar., et al. [2013] we developed EOQ models for Weibull deteriorating items and price dependent demand.

In this paper, we have developed generalized EOQ model for deteriorating items where deterioration rate follows two-parameter Weibull and demand rate is considered to be a function of selling price. For the model where shortages are allowed they are completely backlogged. Here we have considered both the case of with shortage and without shortage in developing the model. Using differential equations, the profit rate function are obtained. By maximizing the profit rate function, the optimal production schedule and optimal production quantity are derived. Through numerical illustration the sensitivity analysis is carried. This model also includes some of the earlier models as particular cases for particular or limiting values of the parameters.

II. ASSUMPTIONS AND NOTATIONS

The following assumptions are made for developing the model:

a) The demand rate is a function of selling price which is \( f(s) = (α - bs) > 0 \)
b) Shortages, whenever allowed are completely backlogged.
c) The deterioration rate is proportional to time.
d) Replenishment is instantaneous and lead time is zero.
e) \( T \) is the length of the cycle.
f) \( Q \) : Ordering quantity in one cycle
h) \( A \) : Ordering cost
i) \( C \) : Cost per unit
j) \( h \) : Inventory holding cost per unit per unit time

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\( \pi \) : Shortages cost per unit per unit time
\( s \) : Selling price per unit and

The deterioration of units follows the two parameter Weibull distribution with probability density function \( f(t) = αβt^{\frac{\beta - 1}{\beta}}e^{-αt^\beta} \), \( 0 < α < 1 \) is scale parameter and \( β > 0 \) is shape parameter and \( t > 0 \). Therefore, the instantaneous rate of replenishment is \( αβt^{\beta - 1} \)

\( m \) : During time \( t_1 \), inventory is depleted due to deterioration and demand of the item. At time \( t_1 \) the inventory becomes zero and shortages start occurring.

III. MATHEMATICAL FORMULATION OF THE MODEL

Let \( I(t) \) be the inventory level at time \( t \) (\( 0 ≤ t ≤ T \)). The differential equations

\[ \frac{dI(t)}{dt} = -s + f(s) - r(t) \]

\[ r(t) = \begin{cases} \frac{s}{\beta}t^{\beta - 1}e^{-αt^\beta}, & 0 < α < 1 \\ 0, & α ≥ 1 \end{cases} \]

\[ s = α - bs \]

\[ h = \frac{C}{2} \]

\[ A = \frac{Q}{2}C \]

\[ C = C(A, h, s, f(s)) \]

\[ f(s) = (α - bs) > 0 \]

\[ Q = \frac{\sqrt{2Ah}}{fs} \]

\[ I(t) = \int_{0}^{t} r(t) dt \]

\[ I(t) = \frac{s}{\beta}t^{\beta - 1}e^{-αt^\beta}, \quad 0 < α < 1 \]

\[ I(t) = 0, \quad α ≥ 1 \]

\[ I(t) = \frac{Q}{2} - \frac{A}{2} \]

\[ Q = \frac{\sqrt{2Ah}}{fs} \]

\[ I(t) = \int_{0}^{t} r(t) dt \]

\[ I(t) = \frac{s}{\beta}t^{\beta - 1}e^{-αt^\beta}, \quad 0 < α < 1 \]

\[ I(t) = 0, \quad α ≥ 1 \]

\[ I(t) = \frac{Q}{2} - \frac{A}{2} \]

\[ I(t) = \int_{0}^{t} r(t) dt \]

\[ I(t) = \frac{s}{\beta}t^{\beta - 1}e^{-αt^\beta}, \quad 0 < α < 1 \]

\[ I(t) = 0, \quad α ≥ 1 \]

\[ I(t) = \frac{Q}{2} - \frac{A}{2} \]
governing the system in the cycle time [0, T] are
\[
\frac{d}{dt} I(t) + a \beta t^{\beta - 1} I(t) = -(a - bs) \quad 0 \leq t \leq t_1
\]
\[
\frac{d}{dt} I(t) = -(a - bs) \quad t_1 \leq t \leq T
\]
(1)
(2)

With I(t) = 0 at t = t_1

Solving the equations (1) and (2) and neglecting higher powers of \( a \), we get
\[
I(t) = \frac{(a - bs)}{e^{a \beta t}} \left( (t_1 - t) + \frac{1}{\beta + 1} (t_1^{\beta + 1} - t^{\beta + 1}) \right)
\]
\[
0 \leq t \leq t_1
\]
\[I(t) = (a - bs)(t - t_1) \quad t_1 \leq t \leq T
\]
(3)
(4)

Stock loss due to deterioration in the cycle of length T is
\[
L(T) = (a - bs) \int_{t_1}^{T} e^{a \beta t} dt - (a - bs) \int_{0}^{t_1} dt
\]
(5)

Ordering quantity Q in the cycle of length T is
\[
Q = L(T) + \int_{0}^{t_1} (a - bs) dt
\]
\[
= (a - bs) \frac{a t_1^{\beta + 1}}{\beta + 1} + (a - bs)T
\]
(6)

Holding cost is obtained by substituting the equations (3) and (4), we get
\[
H = h \left( \int_{0}^{t_1} I(t) dt \right)
\]
\[
= h \left[ \int_{0}^{t_1} \frac{(a - bs)}{e^{a \beta t}} (t_1 - t) + \frac{1}{\beta + 1} (t_1^{\beta + 1} - t^{\beta + 1}) \right] dt
\]
Neglecting higher powers of \( a \), we get
\[
H = h(a - bs) \left(t_1 - \frac{a t_1^{\beta + 1}}{\beta + 1} \right) \left( t_1 + \frac{a t_1^{\beta + 1}}{\beta + 1} \right)
\]
\[
- \int_{0}^{t_1} te^{-a \beta t} dt + \alpha \frac{1}{\beta + 1} \int_{t_1}^{T} t_1^{\beta + 1} e^{-a \beta t} dt
\]
(7)

Shortage cost during the cycle is
\[
S = - \int_{0}^{T} I(t) dt
\]
\[
= - \int (a - bs)(t - t_1) dt = \frac{1}{2} (a - bs)(T - t_1)^2
\]
(8)

Let \( P(T, t_1, s) \) be the profit rate function. Since the profit rate function is the total revenue per unit minus total cost per unit time, we have
\[
P(T, t_1, s) = s(a - bs) - \frac{1}{T} (A + CQ + H + \pi S)
\]
(9)

Substituting the values of equations (6), (7) and (8) in equation (9), one can get the profit rate function as
\[
P(T, t_1, s) = s(a - bs)
\]
\[- \frac{1}{T} A + C \left( \frac{a t_1^{\beta + 1}}{\beta + 1} + (a - bs)T \right)
\]
\[+ h(a - bs) \left( t_1 - \frac{a t_1^{\beta + 1}}{\beta + 1} \right) \left( t_1 + \frac{a t_1^{\beta + 1}}{\beta + 1} \right)
\]
IV. NUMERICAL EXAMPLE

A. Case – I (with shortages)

Let \( A = 500, C = 10, h = 2, \pi = 0.5, \alpha = 10, \beta = 0.5, \gamma = 0.4, a = 100, b = 2 \)

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of \( t_1^* = 1.1484, T^* = 2.871, s^* = 34.976, Q^* = 62.627, P^* (T, s) = 310.964 \)

B. Case – II (without shortages)

Based on above input data and Using the software Matcad 6.0, we calculate the optimal value of \( t_1^* = 0.8084, T^* = 2.021, s^* = 18.39, Q^* = 120.43, P^* (T, s) = 166.199 \)

V. SENSITIVITY ANALYSIS

To study the effects of changes of the parameters on the optimal profit derived by proposed method, a sensitivity analysis is performed considering the numerical example given above. Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 10% and 20% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in Table-1 and Table-2 for with shortage case and without shortage case respectively. The relationship between the parameters and the optimal values are shown in Figure 1 and 2.

<table>
<thead>
<tr>
<th>Variation Parameters</th>
<th>Optimal Policies</th>
<th>Change in parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-20%</td>
</tr>
<tr>
<td>Alpha</td>
<td>( t_1 )</td>
<td>1.285</td>
</tr>
<tr>
<td></td>
<td>( T^* )</td>
<td>3.214</td>
</tr>
<tr>
<td></td>
<td>( s^* )</td>
<td>31.073</td>
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<tr>
<td></td>
<td>( Q^* )</td>
<td>35.161</td>
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<td></td>
<td>( P^* (T, s) )</td>
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<tr>
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<td>( t_1 )</td>
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</tr>
<tr>
<td></td>
<td>( T^* )</td>
<td>5.192</td>
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<tr>
<td></td>
<td>( s^* )</td>
<td>24.868</td>
</tr>
<tr>
<td></td>
<td>( Q^* )</td>
<td>99.519</td>
</tr>
<tr>
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<td>( P^* (T, s) )</td>
<td>428.809</td>
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<td>Alpha</td>
<td>( t_1 )</td>
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<td></td>
<td>( s^* )</td>
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<td></td>
<td>( Q^* )</td>
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<td></td>
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<td>350.938</td>
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<td>( s^* )</td>
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<td></td>
<td>( Q^* )</td>
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<td></td>
<td>( P^* (T, s) )</td>
<td>343.059</td>
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</table>

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We study from above Table-1 reveals the following:

i) Increase in the values of either of the parameters $a$, $b$, $s$, and $Q$, but increase $t_1^*$, $P^*(T, s)$.  

ii) Decrease in the values of either of the parameters $a$, $b$, $s$, and $Q$, but increase $t_1^*$, $P^*(T, s)$.  

iii) Increase in the values of either of the parameters $a$, $b$, $s$, and $Q$, but increase $T^*$, $Q^*$ and $P^*(T, s)$.  

iv) Decrease in the values of either of the parameters $a$, $b$, $s$, and increase $T^*$, $Q^*$ and $P^*(T, s)$.  

v) Increase in the values of either of the parameters $\alpha$, $\beta$, will result in increase of $t_1^*$, $T^*$ and $s^*$ but decrease $Q^*$ and $P^*(T, s)$.  

vi) Decrease in the values of either of the parameters $\alpha$, $\beta$, will result in decrease of $t_1^*$, $T^*$ and $s^*$ but increase $Q^*$ and $P^*(T, s)$.  

vii) Increase in the values of either of the parameters $\alpha$, $\beta$, will result in increase of $t_1^*$, $T^*$ and $s^*$ but decrease $Q^*$ and $P^*(T, s)$.  

viii) Decrease in the values of either of the parameters $\alpha$, $\beta$, will result in decrease of $t_1^*$, $T^*$ and $s^*$ but increase $Q^*$ and $P^*(T, s)$.  

Table – 2 Sensitivity analysis of the model (without shortages)

<table>
<thead>
<tr>
<th>Variation Parameters</th>
<th>Optimal Policies</th>
<th>-20%</th>
<th>-10%</th>
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<th>10%</th>
<th>20%</th>
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<tr>
<td>$a$</td>
<td>$t_1$</td>
<td>0.770</td>
<td>0.829</td>
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<td>1.504</td>
<td>1.542</td>
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<td>$T^*$</td>
<td>1.541</td>
<td>1.658</td>
<td>3.002</td>
<td>3.008</td>
<td>3.084</td>
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<td></td>
<td>$s^*$</td>
<td>17.675</td>
<td>17.814</td>
<td>18.107</td>
<td>18.815</td>
<td>19.411</td>
</tr>
<tr>
<td></td>
<td>$Q^*$</td>
<td>130.339</td>
<td>160.007</td>
<td>194.469</td>
<td>194.575</td>
<td>197.932</td>
</tr>
<tr>
<td></td>
<td>$P^*(T, s)$</td>
<td>171.313</td>
<td>176.242</td>
<td>221.659</td>
<td>261.284</td>
<td>305.507</td>
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<tr>
<td>$b$</td>
<td>$t_1$</td>
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<td>1.36</td>
<td>1.501</td>
<td>1.487</td>
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<tr>
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<td>$T^*$</td>
<td>2.04</td>
<td>2.72</td>
<td>3.002</td>
<td>2.975</td>
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<td></td>
<td>$s^*$</td>
<td>17.731</td>
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<td>19.139</td>
<td>19.628</td>
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<td>$Q^*$</td>
<td>130.015</td>
<td>164.106</td>
<td>194.469</td>
<td>197.072</td>
<td>198.043</td>
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<td>$P^*(T, s)$</td>
<td>182.2</td>
<td>184.828</td>
<td>221.659</td>
<td>192.267</td>
<td>182.685</td>
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<tr>
<td>$\alpha$</td>
<td>$t_1$</td>
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<td>1.530</td>
<td>1.501</td>
<td>1.411</td>
<td>1.326</td>
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<td>$T^*$</td>
<td>3.076</td>
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<td>2.653</td>
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<tr>
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<td>$s^*$</td>
<td>18.039</td>
<td>18.083</td>
<td>18.107</td>
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<td>$Q^*$</td>
<td>198.7</td>
<td>198.76</td>
<td>194.469</td>
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<td>$P^*(T, s)$</td>
<td>227.355</td>
<td>226.653</td>
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<td>208.438</td>
<td>195.789</td>
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<tr>
<td>$\beta$</td>
<td>$t_1$</td>
<td>1.256</td>
<td>1.525</td>
<td>1.501</td>
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<td>1.083</td>
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<td>$T^*$</td>
<td>2.512</td>
<td>3.043</td>
<td>3.002</td>
<td>2.597</td>
<td>2.167</td>
</tr>
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</table>
We study from above Table-2 reveals the following

i) Increase in the values of either of the parameters $a$, will result in increase of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

ii) Decrease in the values of either of the parameters $a$, will result in increase of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

iii) Increase in the values of either of the parameters $b$, will result in increase of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

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iv) Decrease in the values of either of the parameters $b$, will result in increase of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

v) Increase in the values of either of the parameters $a$, will result in decrease of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

vi) Decrease in the values of either of the parameters $a$, will result in decrease of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

vii) Increase in the values of either of the parameters $b$, will result in decrease of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

viii) Decrease in the values of either of the parameters $b$, will result in increase of $t^*$, $T^*$, $s^*$, $Q^*$ and $P^*(T,s)$.

VI. CONCLUSION

In this present paper we have developed deterministic inventory model for deteriorating items for with shortage and without shortage cases. The deterministic demand rate is assumed to be a function of selling price. Whenever shortages are allowed and they are completely backlogged. We can make a good comparative study between the results of the with-shortage case and without-shortage case. In the numerical examples, it is found that the optimum average profit in with shortage case is more than that of the without shortage case. From the above model one can calculate the optimum average profit margins for with shortage case and without shortage case for the deterministic inventory model with varying demand rate.

REFERENCES


Fig.2: Relationship between parameters and optimal values without shortages

We can make a good comparative study between the results of the with-shortage case and without-shortage case.
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AUTHOR PROFILE

Dr. A. Lakshmana Rao
Faculty Department of Basic Sciences and Humanities in Aditya Institute of Technology and Management, Tekkali, Andhra Pradesh (India). He received his Ph.D. degree from Andhra University, Visakhapatnam and he has more than Sixteen years experience in academics and research. He has published two research papers in reputed national and international journals and reviewer of International Journal of Operations Research. His area of specialization is Operations Research, Inventory Models.

B. Neela Rao
Faculty Department of Basic Sciences and Humanities in Aditya Institute of Technology and Management, Tekkali, Andhra Pradesh (India). He received his M.Phil. degree from Andhra University, Visakhapatnam and he has more than Eighteen years experience in academics and research. His area of specialization is Fluid Dynamics, Magneto Hydrodynamics.

Dr. R. Santhi Kumar
Faculty Department of Basic Sciences and Humanities in Aditya Institute of Technology and Management, Tekkali, Andhra Pradesh (India). He received his Ph.D. degree from Acharya Nagarjuna University, Guntur and he has more than Fifteen years experience in academics and research. He has published Eight research papers in reputed national and international. His area of specialization is Relativity and Cosmology.