A New Approach for Dealing with Uncertain Degree in Group Judgment Aggregation using Triangular Intuitionistic Fuzzy Numbers

Le Ngoc Son, Daji Ergu, Pham Xuan Kien

Abstract—The goal of this paper is to propose a new approach for aggregating group judgment using the triangular intuitionistic fuzzy number (IFN). The original group decision making (GDM) problems are converted to a triangular intuitionistic fuzzy decision making model by adding one simple conversion step which generates triangular IFNs from the mean and deviation of group judgment values to the process of GDM methods and inherits existing techniques. Using of triangular IFNs to express group judgment aggregation values keeps completely the information after aggregating and reflects evaluation more truthfully. Consequently, the application of the proposed model helps improve the efficiency and accuracy of GDM methods. In addition, an illustrative example is also presented in order to put this process in detail and comparing with conventional methods.

Index Terms— Group decision making, triangular intuitionistic fuzzy set, group judgment, group aggregation.

I. INTRODUCTION

Group decision making (GDM) becomes more and more popular recently and has been widely applied to solve decision problems in many fields such as water resources management, emergency alternative evaluation, supply chain risk assessment, supplier selection etc. [1-4]. In the real world, the decision making problems are very complex, vague and uncertain in a number of ways. It issues a challenge attracting many researchers extend the existing methods in order to improve the accuracy and support the decision makers (DM). Fuzzy sets (FS) become helpful in these situations. Therefore, since Zadeh [5] introduced FSs in 1965, many types of FSs are extended and exploited efficiently in decision making systems. Atanassov [6] proposed the intuitionistic fuzzy set (IFS), which is characterized by a membership function and a non-membership function. For the GDM purpose, different aggregation operators were proposed by many researchers [7-11].

In GDM, judgment aggregation is one of the most important procedures. Currently, there are two basic techniques for aggregating individual judgments into group judgment, i.e. aggregating individual judgments (AIJ) and aggregating individual priorities (AIP).

In AIJ approach, the aggregated group judgments are considered as judgments of a 'new individual' and the priorities of this individual are derived as a group solution. In the AIP approach, the group is considered as a collection of individuals; their priorities are separately calculated from each individual, and then aggregate them into final group priorities. The weighted arithmetic mean and geometric mean mathematical procedures are commonly used to determine group aggregation for both AIJ and AIP. However, the single crisp mean value does not present exactly and completely whole group judgment. In this case, a FS can be used to improve the uncertainty of group judgment. Even when the DMs use the linguistic terms defined by FSs to give their opinions, with existing fuzzy aggregation operators, the crisp degrees of membership assigned to any given value of x over the universe of discourse may also be subjected to uncertainty [12]. For example in Figure 1(a), three DMs provided their opinions by triangular fuzzy numbers (FN) (F: Fair, MG: Moderately good, VG: Very good), the group evaluation can be calculated with a FWA aggregation operator and presented by a triangular FN (5.3, 6.7, 7.3). It is imprecise with its mean values and membership degrees. However, the triangular intuitionistic fuzzy number (IFN) in Figure 1(b) presents this result better. The outer-boundary of aggregating evaluation can be expended to two sides of triangular FN and the membership values can be spread in range from the lower-bound μ_L to the upper-bound μ_U depending on the divergence of group judgment.

This study proposes a new approach using a triangular IFN to present the aggregation value for GDM. The triangular IFN is generated from the mean and deviation of group judgment values for one evaluation item including degree of membership, degree of hesitation and interval values (leftist and rightist deviation) of two vertices. It reflects truthfully and objectively the group judgment after aggregating. This paper integrates the aggregation operators of triangular FN and the basic operators of triangular IFN which are generally used recently. From that point, an application model is built to express the use of new aggregation technique in GDM methods. With the concentration on intuition fuzzy [13-17], methods based on IFS can be used to solve the decision making problem - converted from conventional GDM one - more efficiently and accurately.

Manuscript Received on May, 2014.

MSc. Le Ngoc Son, School of Management and Economics, University of Electronic Science and Technology of China.

MSc. Daji Ergu, School of Business Administration, Southwest University for Nationalities, Chengdu, China.

MSc. Pham Xuan Kien, School of Management and Economics, University of Electronic Science and Technology of China, Chengdu, 610054, China.

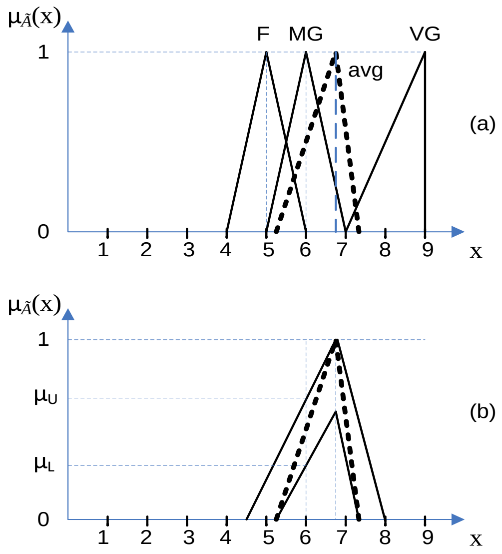


Fig 1. Presentation of a group aggregation value

The remaining parts of this paper are structured as follows: Section 2 briefly reviews the basic definitions and operators of fuzzy sets. In section 3, a new approach is proposed to present the group aggregation value and its application in group MCDM methods. Numerical examples are used to illustrate the proposed technique in section 4. Finally, section 5 concludes the paper.

II. PRELIMINARIES

In the following, some basic definitions and common operators of fuzzy sets are briefly presented. Details of these definitions are referred to [5].

Definition 2.1 (Fuzzy set)

A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0; 1]. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

Definition 2.2 (Convex fuzzy set)

A fuzzy set $\tilde{A}$ of the universe of discourse X is convex if and only if for all x_1, x_2 in X:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \qquad (1)$$

where $\lambda \in [0,1]$.

Definition 2.3 (Triangular fuzzy number)

A triangular fuzzy number $\tilde{A}$ can be defined by a triplet (l, m, u), where $l \leq m \leq u$, l and u stand for the lower and upper value of the support of $\tilde{A}$, respectively, and m is the mid-value of $\tilde{A}$. If $l=m=u$, it is a non-fuzzy number by convention (a crisp number). The graph of triangular fuzzy can be shown in Figure 2.

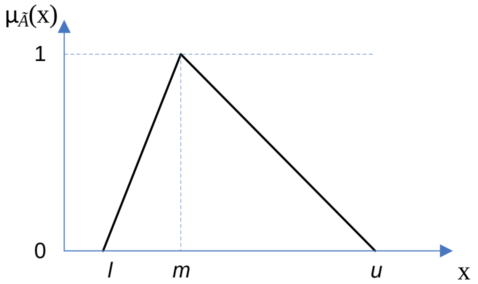


Fig.2. A triangular fuzzy number

The membership function $\mu_{\tilde{A}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < l \\ \dfrac{x-l}{m-l}, & l \leq x \leq m \\ \dfrac{x-u}{m-u}, & m \leq x \leq u \\ 0, & x > u \end{cases} \qquad (2)$$

Definition 2.4 (Triangular fuzzy number operations)

Consider two triangular fuzzy numbers $\tilde{A}_1 = (l_1, m_1, u_1)$ and $\tilde{A}_2 = (l_2, m_2, u_2)$. Their operational laws are as follows:

1. $(l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)$ (3)
2. $(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) \approx (l_1 l_2, m_1 m_2, u_1 u_2)$ (4)
3. $(\lambda, \lambda, \lambda) \otimes (l_1, m_1, u_1) \approx (\lambda l_1, \lambda m_1, \lambda u_1), \lambda > 0, \lambda \in R$ (5)
4. $(l_1, m_1, u_1)^{-1} \approx (1/u_1, 1/m_1, 1/l_1)$ (6)

The next definitions of IFS in this section are mainly borrowed from [12, 18].

Definition 2.5(Intuitionistic fuzzy set) [18]

Let a set X be fixed. An intuitionistic fuzzy set [IFS] $\tilde{A}$ in X is an object having the form:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)\rangle \mid x \in X\}, \qquad (7)$$

where the $\mu_{\tilde{A}}(x)$:X→[0,1] and $\nu_{\tilde{A}}(x)$:X→[0,1] define the degree of membership and degree of non-membership respectively, of the element $x \in X$ to the set $\tilde{A}$, which is a subset of X, for every element $x \in X$, $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. An IFS is shown in Figure 3.

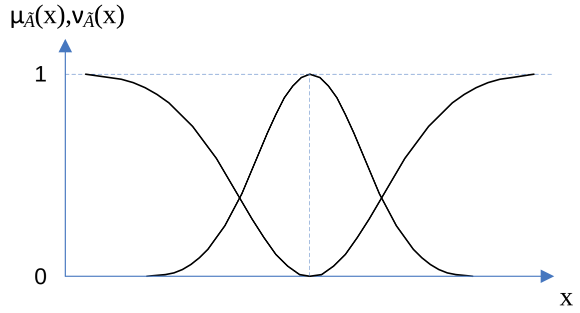


Fig.3. Membership and non-membership function of Ã

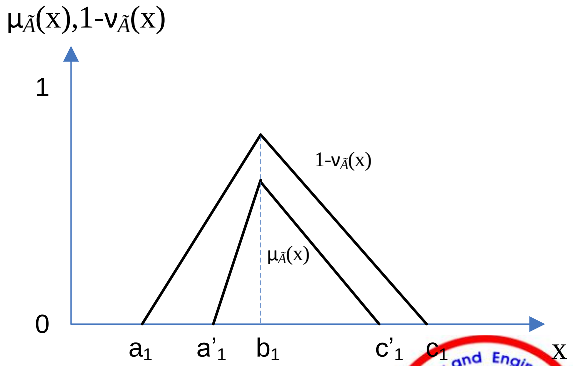
Definition 2.6 (Intuitionistic fuzzy index) [12]

For each IFS $\tilde{A}$ in X, if

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x), \; 0 \leq \pi_{\tilde{A}}(x) \leq 1, \qquad (8)$$

then $\pi_{\tilde{A}}(x)$ is the third parameter of IFS and is usually called the intuitionistic fuzzy index or hesitation degree. IFSs is reduced to fuzzy sets when $\nu_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$ and $\pi_{\tilde{A}}(x) = 0$.

Definition 2.7 (Triangular intuitionistic fuzzy number) [12]



Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

Fig.4. A triangular IFS Ã

A triangular intuitionistic fuzzy number [IFN] $\tilde{A}$ is represented as:

$\tilde{A}$=<[(a'_1,b'_1,c'_1); $\mu_{\tilde{A}}$], [(a_1,b_1,c_1); $\nu_{\tilde{A}}$]>. (9)

The membership functions $\mu_{\tilde{A}}$ is used to derive the lower bounds of membership μ_L for IFN $\tilde{A}$, where the upper bound of membership μ_U is derived by taking the compliment of non-membership functions $\nu_{\tilde{A}}$, respectively. A triangular IFN is shown in Figure 4.

Definition 2.8 (Triangular intuitionistic fuzzy number operator) [12]

For two triangular FNs $\tilde{A}_1$=<[(a'_1,b'_1,c'_1); $\mu_{\tilde{A}1}$], [(a_1,b_1,c_1); $\nu_{\tilde{A}1}$]> and $\tilde{A}_2$=<[(a'_2,b'_2,c'_2); $\mu_{\tilde{A}2}$], [(a_2,b_2,c_2); $\nu_{\tilde{A}2}$]>, four common arithmetic operations for IFSs (addition, subtraction, multiplication and division) are demonstrated below:

$\tilde{A}_1+\tilde{A}_2$=<[($a'_1+a'_2$, $b'_1+b'_2$, $c'_1+c'_2$); min($\mu_{\tilde{A}1},\mu_{\tilde{A}2}$)], [($a_1+a_2$, b_1+b_2, c_1+c_2); max($\nu_{\tilde{A}1},\nu_{\tilde{A}2}$)]> (10)

$\tilde{A}_1-\tilde{A}_2$=<[($a'_1-c'_2$, $b'_1-b'_2$, $c'_1-a'_2$); min($\mu_{\tilde{A}1},\mu_{\tilde{A}2}$)], [($a_1-c_2$, b_1-b_2, c_1-a_2); max($\nu_{\tilde{A}1},\nu_{\tilde{A}2}$)]> (11)

$\tilde{A}_1$x$\tilde{A}_2$=<[(a'_1xa'_2, b'_1xb'_2, c'_1xc'_2); min($\mu_{\tilde{A}1},\mu_{\tilde{A}2}$)], [($a_1xa_2$, b_1xb_2, c_1xc_2); max($\nu_{\tilde{A}1},\nu_{\tilde{A}2}$)]> (12)

$\tilde{A}_1/\tilde{A}_2$=<[(a'_1/c'_2, b'_1/b'_2, c'_1/a'_2); min($\mu_{\tilde{A}1},\mu_{\tilde{A}2}$)], [($a_1/c_2$, b_1/b_2, c_1/a_2); max($\nu_{\tilde{A}1},\nu_{\tilde{A}2}$)]> (13)

III. THE PROPOSED MODEL

This study considers the GDM problems using crisp numbers or triangular FN to give DMs' judgments. The proposed approach transfers the type of GDM problems to a triangular intuitionistic fuzzy decision making model. In the following, we will describe a way to aggregate and generate a triangular IFN from a group of triangular FNs. Figure 5 presents an application model proposed in a general GDM situation.

Let E={E_i | i=1,2,…,n} be a set of n experts under consideration and C={C_j | j=1,2,…,k} be a set of k criteria. A general hierarchical structure of group decision making is shown in Figure 6.

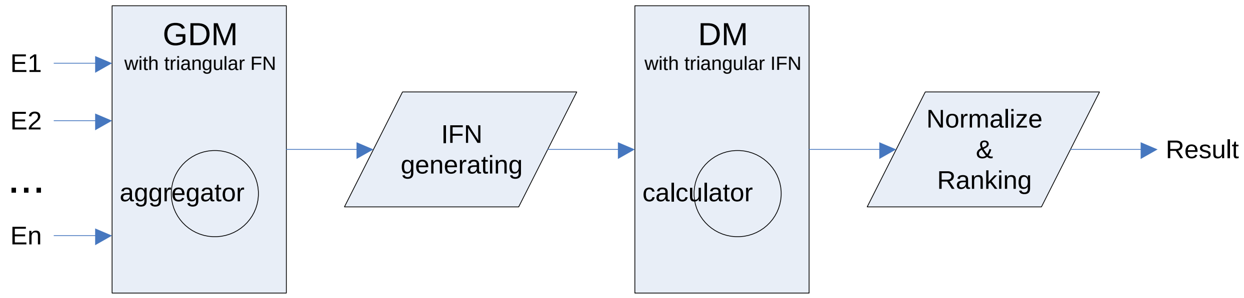


Fig.5. The proposed GDM model

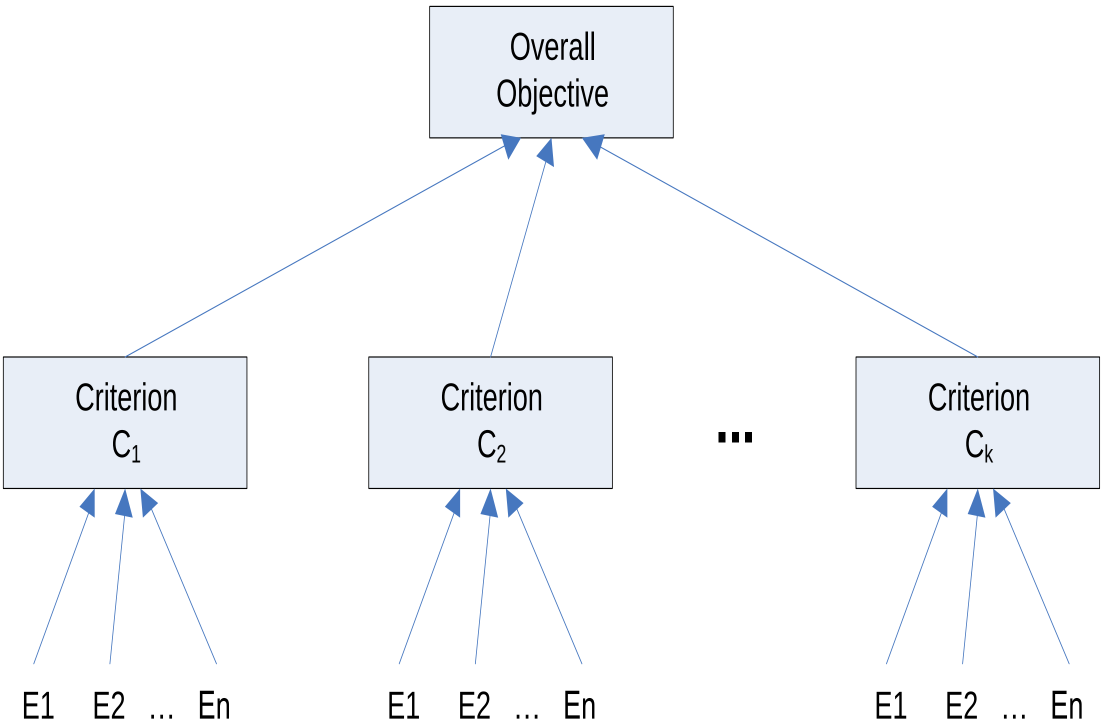


Fig.6. General hierarchical structure of group decision making

The process of proposed GDM model include the following steps:

Step 1: Identify the objectives of the decision making process and define the problem scope**.**

Step 2: Construct the fuzzy decision model and get the DMs' opinion.

For a goal object, the DMs give their judgments to each criterion using linguistic terms defined with triangular FN $\tilde{c}_j^{(i)}=(l, m, u)$, where $1 \le i \le n$ and $1 \le j \le k$.

Step 3: Aggregate the group judgment.

There are a lot of choices to aggregate group fuzzy judgments, including fuzzy weighted averaging (FWA) operator, ordered weighted averaging (OWA) operator, ordered weighted geometric averaging (OWGA) operator, weighted arithmetic averaging (WAA) operator, weighted geometric averaging (WGA) operator, generalized ordered weighted averaging (GOWA) operator, weighted ordered weighted averaging (WOWA) operator, fuzzy ordered weighted geometric averaging (FOWGA) operator, hybrid weighted averaging (HWA) operator, induced ordered weighted averaging (IOWA) operator, and induced ordered weighted geometric averaging (IOWGA) operator [19]. Depend on the chosen GDM model, group aggregation values will be represented in crisp numbers or triangular FNs.

Step 4: Generate the triangular IFNs from group aggregation values.

Triangular IFNs are generated based on the mean (or average value) and the deviation among group. In order to do the conversion, in the previous steps, each parameter in triangular FNs, which is the result of aggregation operator, should accompany with its deviation value ∂.

$$\tilde{c}_j = \tilde{f}(\tilde{c}_j^{(i)}) = (<a_1;\partial_1>,<a_2;\partial_2>,<a_3;\partial_3>), \qquad (14)$$

$$\tilde{c}'_j = <[(a_1,a_2,a_3);\mu], [(a_1-\Delta_L,a_2,a_3+\Delta_U);\nu]>, \qquad (15)$$

where $\tilde{c}_j$ is a triangular FN and $\tilde{c}'_j$ is a triangular IFN with $1 \le j \le k$, $\tilde{f}$ is the fuzzy aggregation operator used in previous step,

$$\Delta_L = \lambda \min(a_1\text{-}minimum\_of\_range,\ \partial_1), \qquad (16)$$

$$\Delta_U = \lambda \min(maximum\_of\_range\text{-}a_3,\ \partial_3), \qquad (17)$$

$$\mu = 1 - \nu - \pi, \qquad (18)$$

$$\nu = 0, \qquad (19)$$

$$\pi = \lambda\, \partial_2/(a_2 + \lambda\, \partial_2/2). \qquad (20)$$

$\tilde{c}'_j$ is a triangular FN of the universe of discourse X, which represent an imprecise value from *minimum_of_range* to *maximum_of_range*.

With this formula, the generated triangular IFN contain the divergence in group judgment. The intervals of the leftist Δ_L and of rightist Δ_U integrated with the hesitation degree determine the lower and upper bound of IFN. They are calculated from the deviation values to reflect the difference in group judgment. The membership value for each element varies in interval [μ_U, μ_L] as shown in Figure 1(b). At the mid-value of FN, membership value is ranged from μ to 1, where π moves to 1 if there is a convergence in group evaluation. The membership is conceded enlarge to left-side of left-value and right-side of right-value, dependence on the divergence in group evaluation. They are adjusted to keep in range of minimum and maximum scale of values. In addition, multiplier λ ($\lambda \le 1$) is a design parameter. It should be small if we want to increase the converge degree in group DMs.

Example 3.1 and 3.2 as shown in the following will make it more clearly.

Step 5: Apply a suitable intuitionistic fuzzy DM method to solve the problem.

Step 6: Normalize and rank the result.

This process can be extended from the other GDM methods. Moreover, the triangular IFN conversion step can be integrated to existing triangular intuitionistic DM methods. In case of using crisp number to give the evaluations, it is a specific case of triangular FN. Therefore, it can be reduced from triangular FN problem.

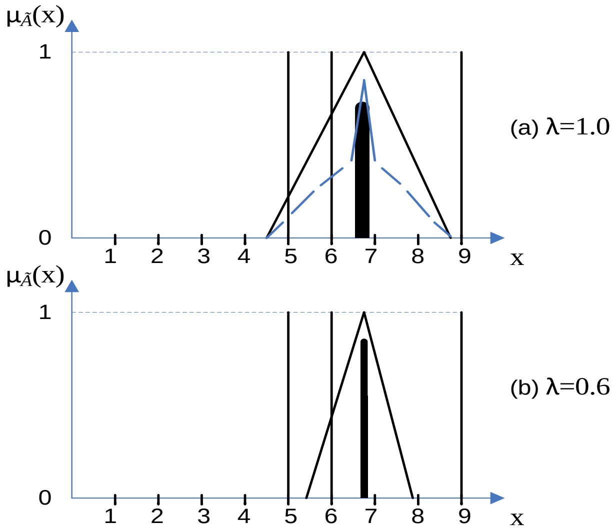


Fig.7. Representation of a reduced triangular FN

Example 3.1: Group judgment with triangular FN.

In above example (Figure 1), the aggregation result calculated from three triangular FNs (4,5,6), (5,6,7) and (7,9,9) of three DM evaluations using a simple FWA is $\tilde{c}_j$ = (<5.3;1.5>,<6.7;2.1>,<7.3;1.5>). Using equations (16) to (20), we have a triangular IFN $\tilde{c}'_j$ =<[(5.3,6.7,7.3);0.73],[(3.8,6.7,8.9);0]> with λ=1.0.

Example 3.2: Group judgment with crisp number.

Assume three DMs evaluate the performance of a criterion for one object, and the assigned values are 5, 6 and 9 respectively. The group aggregation value will be m=(5+6+9)/3=6.67 with simple arithmetic average operator and the deviation is ∂=2.08.

A crisp number 5 for example can be represented like a triangular FN as (5,5,5). Thus, the intermediate triangular FN can be written as (<6.67;2.08>,<6.67;2.08>,<6.67;2.08>) and showed in Figure 7(a). And the triangular FN is <[(6.67, 6.67, 6.67);0.73], [(4,59,6.67,8.75);0]> with λ=1.0. In this approach, the aggregation value is not exact at 6.67 but it line in range from 4.59 to 8.75. The degree membership is 0.73 and vague interval in [0.73;1.0]. The degree of accuracy is higher at the mean value, lesser and lesser in the direction of its two sides. To know it more visually, we reduce from this IFN into a FN by taking arithmetic mean of interval-valued memberships [μ_L, μ_U] at each x_d=0.5 [20],

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

it is showed as a convex FN with dotted line in Figure 7(a). In addition, if we decrease the value of parameter λ to 0.6, lessen the uncertainty. It reflects that group of DMs have a higher unanimity, as showed in Figure 7(b). However, it is not easy to determine a suitable value for λ.

IV. ILLUSTRATIVE EXAMPLE

In this section, a numerical example is introduced to illustrate the application of the proposed GDM model on supplier selection in fashion market. For simplicity, the selection criteria under consideration are borrowed and modified from a study conducted by Teng and Jaramillo and after that by Chan and Chan [21]. They are: quality (Q), cost (C), delivery (D), assurance (A) and flexibility (F). The levels of the judgment value are range from 1 to 9 and are defined with the linguistic terms (as shown in Table 1).

Table 1. Definitions of linguistic terms for the ratings

	Linguistic terms	IFS values
VP	Very poor	(1,1,2)
P	Poor	(1,2,3)
MP	Medium poor	(2,3,4)
F	Fair	(3,5,7)
MG	Medium good	(6,7,8)
G	Good	(7,8,9)
VG	Very good	(8,9,9)

Table 2. Group judgments

	Q	C	D	A	F
Decision maker - DM1					
A1	VG	MP	G	MG	G
A2	G	F	MG	MG	G
A3	G	VG	F	G	VG
A4	MG	MG	P	F	G
Decision maker - DM2					
A1	MG	MG	G	G	G
A2	MG	F	G	MG	G
A3	G	MG	G	G	G
A4	G	G	VG	G	MG
Decision maker - DM3					
A1	G	MG	MP	MG	F
A2	MP	G	F	F	MG
A3	VG	VG	MG	MG	G
A4	G	VG	F	G	MG

A group of three DMs (DM1, DM2 and DM3) gave their opinions using defined linguistic terms, as shown in Table 2. Suppose the same weight is assigned to all experts and the weights of criteria are 0.35, 01.5, 0.2, 0.1 and 0.2, respectively. The group evaluations are aggregated using FWA operator. The main step in this process is to generate a triangular TFN matrix of aggregation group judgment using equations (15) to (20). Table 3 presents the evaluating results provided by DMs for each alternative with respect to each criterion.

Table 3. Group aggregation result

Q	**A1**	<[(7.00,8.00,8.67);0.92],[(6.00,8.00,9.00);0]>
	A2	<[(5.00,6.00,7.00);0.71],[(2.35,6.00,9.00);0]>
	A3	<[(7.33,8.33,9.00);0.95],[(6.76,8.33,9.00);0]>
	A4	<[(6.67,7.67,8.67);0.95],[(6.09,7.67,9.00);0]>
C	**A1**	<[(4.67,5.67,6.67);0.73],[(2.36,5.67,8.98);0]>
	A2	<[(4.33,6.00,7.67);0.81],[(2.02,6.00,8.82);0]>
	A3	<[(7.33,8.33,8.67);0.91],[(6.18,8.33,9.00);0]>
	A4	<[(7.00,8.00,8.67);0.92],[(6.00,8.00,9.00);0]>
D	**A1**	<[(5.33,6.33,7.33);0.70],[(2.45,6.33,9.00);0]>
	A2	<[(5.33,6.67,8.00);0.85],[(3.25,6.67,9.00);0]>
	A3	<[(5.33,6.67,8.00);0.85],[(3.25,6.67,9.00);0]>
	A4	<[(4.00,5.33,6.33);0.56],[(1.00,5.33,9.00);0]>
A	**A1**	<[(6.33,7.33,8.33);0.95],[(5.76,7.33,8.91);0]>
	A2	<[(5.00,6.33,7.67);0.88],[(3.27,6.33,8.24);0]>
	A3	<[(6.67,7.67,8.67);0.95],[(6.09,7.67,9.00);0]>
	A4	<[(5.67,7.00,8.33);0.84],[(3.36,7.00,9.00);0]>
F	**A1**	<[(5.67,7.00,8.33);0.84],[(3.36,7.00,9.00);0]>
	A2	<[(6.67,7.67,8.67);0.95],[(6.09,7.67,9.00);0]>
	A3	<[(7.33,8.33,9.00);0.95],[(6.76,8.33,9.00);0]>
	A4	<[(6.33,7.33,8.33);0.95],[(5.76,7.33,8.91);0]>

Table 4. IF-VIKOR - The best and the worst values

	The best A^+
Q	<[(7.33,8.33,9.00);0.71],[(6.76,8.33,9.00);0]>
C	<[(7.33,8.33,8.67);0.73],[(6.18,8.33,9.00);0]>
D	<[(5.33,6.67,8.00);0.56],[(3.25,6.67,9.00);0]>
A	<[(6.67,7.67,8.67);0.84],[(6.09,7.67,9.00);0]>
F	<[(7.33,8.33,9.00);0.84],[(6.76,8.33,9.00);0]>
	The worst A^-
Q	<[(5.00,6.00,7.00);0.71],[(2.35,6.00,9.00);0]>
C	<[(4.33,6.00,7.67);0.81],[(2.02,6.00,8.82);0]>
D	<[(4.00,5.33,6.33);0.56],[(1.00,5.33,9.00);0]>
A	<[(5.00,6.33,7.67);0.84],[(3.27,6.33,8.24);0]>
F	<[(6.33,7.33,8.33);0.95],[(5.76,7.33,8.91);0]>

Table 5. Comparison the results

	Simple IF-weight		F-VIKOR [22]		IF-VIKOR [23]	
	W_i	Rank	Q_i	Rank	Q_i	Rank
A1	0.245	3	0.639	2	0.437	3
A2	0.230	4	1.000	4	0.709	4
A3	0.274	1	0.000	1	-0.031	1
A4	0.252	2	0.656	3	0.356	2

We apply IF-VIKOR method [23] to select the best suppliers. The best A^+ and the worst values A^- is shown in Table 4. After that,

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

the values of R, S, Q are calculated for all suppliers using above triangular IFN operators - equations (10)-(13).

The results calculated by Simple IF-weight, F-VIKOR method [22] and IF-VIKOR [23] are compared, in which each supplier is weighted by using IFWA operator over all criteria's scores. Table 5 shows that supplier A3 is the best one under three methods. However, the rankings of A1 and A4 are different under three methods, i.e., suppliers A1 is preferred to A4 under Simple IF-weight and IF-VIKOR, while A4 is preferred to A1 under F-VIKOR, indicating supplier A4 has better indicators when using triangular IFNs generated from the first step instead of supplier A1.

V. CONCLUSIONS

The proposed model converts a GDM problem into intuition fuzzy environment. This model adds one simple step to the process of GDM methods and inherits existing techniques. With this new approach, group judgment is simply aggregated and represented with a triangular IFN. Using of triangular IFNs to express group judgment aggregation values keeps completely the information after aggregating and reflects evaluation more truthfully.

In addition, an illustrative example is also presented to illustrate the application steps of the proposed model. Comparing with some conventional methods, the result shows that the intuitionistic fuzzy approach extends the capability of representing vague values. Consequently, the application of the proposed model helps improve the efficiency and accuracy of GDM methods.

ACKNOWLEDGMENT

Thanks to Prof. Kou Gang, who gave us useful information for this paper.

REFERENCES

1. Morais, Danielle C., and Adiel Teixeira de Almeida. "Group decision making on water resources based on analysis of individual rankings." *Omega* 40.1 (2012): 42-52.
2. Ju, Yanbing, and Aihua Wang. "Emergency alternative evaluation under group decision makers: A method of incorporating DS/AHP with extended TOPSIS." *Expert Systems with Applications* 39.1 (2012): 1315-1323.
3. Chaudhuri, Atanu, Bhaba Krishna Mohanty, and Kashi Naresh Singh. "Supply chain risk assessment during new product development: a group decision making approach using numeric and linguistic data." *International Journal of Production Research* 51.10 (2013): 2790-2804.
4. Sanayei, Amir, S. Farid Mousavi, and A. Yazdankhah. "Group decision making process for supplier selection with VIKOR under fuzzy environment." *Expert Systems with Applications* 37.1 (2010): 24-30.
5. Zadeh, Lotfi A. "Fuzzy sets." *Information and control* 8.3 (1965): 338-353.
6. Atanassov, Krassimir T. "Intuitionistic fuzzy sets." *Fuzzy sets and Systems* 20.1 (1986): 87-96.
7. Xu, Zeshui, and Ronald R. Yager. "Some geometric aggregation operators based on intuitionistic fuzzy sets." *International journal of general systems* 35.4 (2006): 417-433.
8. Xu, Zeshui. "Intuitionistic fuzzy aggregation operators." *Fuzzy Systems, IEEE Transactions on* 15.6 (2007): 1179-1187.
9. Wei, Guiwu. "Some induced geometric aggregation operators with intuitionistic fuzzy information and their application to group decision making." *Applied Soft Computing* 10.2 (2010): 423-431.
10. Zhao, Hua, et al. "Generalized aggregation operators for intuitionistic fuzzy sets." *International Journal of Intelligent Systems* 25.1 (2010): 1-30.
11. Yu, Xiaohan, and Zeshui Xu. "Prioritized intuitionistic fuzzy aggregation operators." *Information Fusion* 14.1 (2013): 108-116.
12. Sadiq, Rehan, and Solomon Tesfamariam. "Environmental decision-making under uncertainty using intuitionistic fuzzy analytic hierarchy process (IF-AHP)." *Stochastic Environmental Research and Risk Assessment* 23.1 (2009): 75-91.
13. Zhao, Hua, et al. "Generalized aggregation operators for intuitionistic fuzzy sets." *International Journal of Intelligent Systems* 25.1 (2010): 1-30.
14. Li, Deng-Feng. "TOPSIS-based nonlinear-programming methodology for multiattribute decision making with interval-valued intuitionistic fuzzy sets." *Fuzzy Systems, IEEE Transactions on* 18.2 (2010): 299-311.
15. Li, Jinquan, et al. "The relationship between similarity measure and entropy of intuitionistic fuzzy sets." *Information Sciences* 188 (2012): 314-321.
16. Boran, F. E., K. Boran, and T. Menlik. "The evaluation of renewable energy technologies for electricity generation in Turkey using intuitionistic fuzzy TOPSIS." *Energy Sources, Part B: Economics, Planning, and Policy* 7.1 (2012): 81-90.
17. Wan, Shu-Ping, Qiang-Ying Wang, and Jiu-Ying Dong. "The extended VIKOR method for multi-attribute group decision making with triangular intuitionistic fuzzy numbers." *Knowledge-Based Systems* 52 (2013): 65-77.
18. Atanassov, Krassimir T. *Intuitionistic fuzzy sets*. Physica-Verlag HD, 1999.
19. Xu, Zeshui. "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment." *Information Sciences* 168.1 (2004): 171-184.
20. Mendel, Jerry M. "Fuzzy sets for words: why type-2 fuzzy sets should be used and how they can be used." *presented as two-hour tutorial at IEEE FUZZ, Budapest, Hongrie* (2004).
21. Chan, Felix TS, and Hing Kai Chan. "An AHP model for selection of suppliers in the fast changing fashion market." *The International Journal of Advanced Manufacturing Technology* 51.9-12 (2010): 1195-1207.
22. Sanayei, Amir, S. Farid Mousavi, and A. Yazdankhah. "Group decision making process for supplier selection with VIKOR under fuzzy environment." *Expert Systems with Applications* 37.1 (2010): 24-30.
23. Devi, Kavita. "Extension of VIKOR method in intuitionistic fuzzy environment for robot selection." *Expert Systems with Applications* 38.11 (2011): 14163-14168.

AUTHORS PROFILE

MSc. Le Ngoc Son is with School of Management and Economics, University of Electronic Science and Technology of China (UESTC), Chengdu, China. Le's from Ho Chi Minh city, Vietnam. He got Masters' degree of computer science at Ho Chi Minh City University of Technology, Vietnam. Le is a lecturer in IT department of Industrial University of HCM city (IUH). Recently, he is researching in information management and data mining. His current research interests are MCDM and CBR. Now he is a PhD. candidate in UESTC. Son has published some papers in new combination of AHP and CBR, and proposed a model for firm's technological capability assessment under fuzzy environment.

MSc. Daji Ergu is an associate professor of College of Electrical and Information Engineering, Southwest University for Nationalities. He received his M.S. degrees in Southwest University for Nationalities and Coventry University in 2003 and 2004 respectively. He has worked at Southwest University for Nationalities since 2003. He has participated in several research projects of National Natural Science Foundation of China. His research interests are in Multiple Criteria Decision Making. He has published more than ten papers in international journals and conferences.

MSc. Pham Xuan Kien is with School of Management and Economics, University of Electronic Science and Technology of China (UESTC), Chengdu, China. He's from Ho Chi Minh city, Vietnam. Pham got MIS Masters' degree at HCM City University of Technology, Vietnam. His major is management information system, data mining and his current research interests are MCDM, data mining. Pham is doing his PhD. research in School of Management and Economics of UESTC, China. He has published some papers in MCDM in uncertainty and information communication technology measurement.