PID Control Tuning VIA Particle Swarm Optimization for Coupled Tank System


Abstract—This paper presents the use of meta-heuristic technique to obtain three parameters (K<sub>P</sub>, K<sub>I</sub> and K<sub>D</sub>) of PID controller for Coupled Tank System (CTS). Particle Swarm Optimization (PSO) is chosen and Sum Squared Error is selected as objective function. This PSO is implemented for controlling desired liquid level of CTS. Then, the performances of the system are compared to various conventional techniques which are Trial and Error, Auto-Tuning, Ziegler-Nichols (Z-N) and Cohen-Coon (C-C) method. Simulation is conducted within Matlab environment to verify the transient response specifications in terms of Rise Time (T<sub>R</sub>), Settling Time (T<sub>S</sub>), Steady State Error (SSE) and Overshoot (OS). Result obtained shows that performance of CTS can be improved via PSO as PID tuning methods.

Index Terms—Coupled Tank System (CTS), Particle Swarm Optimization (PSO), PID Controller, PID Tuning Method.

I. INTRODUCTION

Coupled Tank System (CTS) is one of the applications in industrial production as shown in Figure 1. The process control especially controlling liquid level is important and widely applied in various field such as liquid storage tank, a feeding tank, a product tank, the intermediate buffer containers and water tanks [1-2]. In CTS, the overall process need liquids to be pumped, stored in the tank and pumped again to another tank for certain desired level. The liquid is required to be maintained in a specific height or certain range [3]. Efficient and effective controls of these processes have immense economical advantage and its success depends on the type of control strategy [4].

CTS is a typical representative of the process control. It has nonlinear and complex characteristics. PID controller is implemented to the system to control the desired level of the water and this controller always been used in industrial application due to easy and simple design to implement [5-6]. Nevertheless, the conventional PID controller shows that it is difficult to reach the desired control response with the aim of high speed with short transition time and small overshoot [7]. In order to achieve the optimal specifications, Particle Swarm Optimization (PSO) algorithm is approached. The advantage of PSO is a fast convergence compares with many optimizations [5-6]. It is also easy in its concept and coding implementation.

Furthermore, PSO is potential to replace the conventional way of obtaining PID controller parameters [8].

II. PID CONTROLLER

PID controller is a control feedback mechanism controller which is widely used in industrial control system. PID controller involves three-term control which are the Proportional (P), the Integral (I) and the Derivative (D). PID controller is used to calculate an error value as the difference between a measured process variable and a desired set point. It also used to minimize the error by adjusting the process of control inputs.

\[
u(t) = K_P e(t) + K_I \int e(t) dt + K_D \frac{de(t)}{dt} \]

(1)

In the system, three parameters are needed to be tuned. One of the parameters is proportional gain, K<sub>P</sub>. This controller has the effect of reducing the Rise Time (T<sub>R</sub>) and Steady State Error (SSE) but the percentage of the Overshoot (OS) in the system is high. In the integral controller, K<sub>I</sub> as the integral gain will affect and decrease the rise time. However, it will eliminate the SSE of the system. Even though it eliminates the error but the percentage of the OS is increased and it will affect the Settling Time (T<sub>S</sub>) as well. In order to improve the performance of the system, derivative gain, K<sub>D</sub> in the derivative controller is introduced. This controller will take action to improve the transient specification and stability of the system. The effects of each of the controller on a closed-loop system are summarized in Table 1.

### Table 1: PID Controller Properties

<table>
<thead>
<tr>
<th>Effect of Performance</th>
<th>TR</th>
<th>TS</th>
<th>OS</th>
<th>SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>KP Decrease Small</td>
<td>Change</td>
<td></td>
<td>Increase</td>
<td>Decrease</td>
</tr>
<tr>
<td>KI Decrease Increase</td>
<td></td>
<td></td>
<td>Increase</td>
<td>Eliminate</td>
</tr>
<tr>
<td>KD Small Change</td>
<td>Decrease</td>
<td></td>
<td>Decrease</td>
<td>Small Change</td>
</tr>
</tbody>
</table>
In order to improve the high stability and short transient response of the system, the optimal gain value must be obtained from the PID tuning. Even though it is only three control parameters, but to adjust the parameter referred to the Table 1 are difficult. Therefore, many methods are implemented in order to obtain the best parameter of PID controller.

### III. MATHEMATICAL MODELING

It is vital to understand the mathematics modeling of the behavior of CTS. In this system, the nonlinear dynamic model is observed and the linearization process is done based on the nonlinear model.

![Schematic diagram of CTS](image)

**Figure 2: Schematic diagram of CTS**

Based on Figure 2, $H_1$ and $H_2$ are the fluid level in Tank 1 and Tank 2. It is measured with respect to the corresponding outlet. Considering a simple mass balance, the rate of change of fluid volume in each tank equals the net flow of fluid into the tank. Therefore, the equation for Tank 1 and Tank 2 are:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - Q_{o1} - Q_3 \tag{2}$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - Q_{o2} + Q_3 \tag{3}$$

where:

- $H_1, H_2$ = height of fluid in Tank 1 and 2 respectively
- $A_1, A_2$ = cross-sectional area of Tank 1 and 2 respectively
- $Q_{i1}, Q_{i2}$ = pump flow rate into Tank 1 and 2 respectively
- $Q_{o1}, Q_{o2}$ = flow rate of fluid out of Tank 1 and 2 respectively

Each outlet drain can be modeled as a simple orifice. Bernoulli’s equation for steady, non-viscous, incompressible flow between the tanks is proportional to the square root of the head of water in the tank. Similarly, the flow rate is described by the square root of the head differential. Thus:

$$Q_{i1} = \alpha_1 \sqrt{H_1} \tag{4}$$

$$Q_{i2} = \alpha_2 \sqrt{H_2} \tag{5}$$

$$Q_{i1} = \alpha_3 \sqrt{H_1 - H_2} \tag{6}$$

where $\alpha_1, \alpha_2$ and $\alpha_3$ are proportionality constants which are depend on the coefficients of discharge, the cross sectional area of each orifice and the gravitational constant. By substitute (4), (5) and (6) into (2) and (3), the nonlinear state equations which describe the system dynamics of the CTS apparatus are:

$$A_1 \frac{dH_1}{dt} = Q_{i1} - \alpha_1 \sqrt{H_1} - \alpha_3 \sqrt{H_1 - H_2} \tag{7}$$

$$A_2 \frac{dH_2}{dt} = Q_{i2} - \alpha_2 \sqrt{H_2} + \alpha_3 \sqrt{H_1 - H_2} \tag{8}$$

In the second order configuration, $h_2$ is the process variable and $q_1$ is the manipulated variable and assume that $q_2$ is zero. The block diagram of the second-order system can be simplified as shown in Figure 3.

![Block diagram of second order system](image)

**Figure 3: Block diagram of second order system**

Thus, the nonlinear CTS can be obtained as:

$$\frac{h_2(s)}{q_1(s)} = \frac{k_1k_2}{T_1s + 1} \tag{9}$$

where:

$$T_1 = \left(\frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \tag{10}$$

$$T_2 = \left(\frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}}\right) \tag{11}$$

$$k_1 = \frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \tag{12}$$

$$k_2 = \frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \tag{13}$$

$$k_{12} = \frac{\alpha_1}{2\sqrt{H_1}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \tag{14}$$

$$k_{21} = \frac{\alpha_2}{2\sqrt{H_2}} + \frac{\alpha_3}{2\sqrt{H_1 - H_2}} \tag{15}$$

The transfer function for the plant can be obtained by substituting the parameter which was provided from the Table 2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>17</td>
<td>cm</td>
</tr>
<tr>
<td>$H_2$</td>
<td>15</td>
<td>cm</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>10.78</td>
<td>cm$^{3/2}$/sec</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>11.03</td>
<td>cm$^{3/2}$/sec</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>11.03</td>
<td>cm$^{3/2}$/sec</td>
</tr>
<tr>
<td>$A_1$</td>
<td>32</td>
<td>cm$^2$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>32</td>
<td>cm$^2$</td>
</tr>
</tbody>
</table>
Then, all the parameters in Table 1 have been inserted into (9). Thus, the actual transfer function of the plant with the completed value is:

$$G_p(s) = \frac{h_2(s)}{q_1(s)} = \frac{0.0361}{36.9406s^2 + 12.1565s + 0.4514} \quad (16)$$

IV. PARTICLE SWARM OPTIMIZATION (PSO)

PSO is introduced by Kennedy and Eberhart [10]. It was first intended for simulation social behavior, as a stylized representation of the movement of organisms in a bird flock or fish school. While searching for food, the birds are either scattered or go together before they locate the place where they can find the food. While the birds are searching for food from one place another, there is always a bird that can smell the food well, that is the bird is perceptible of the place where the food can be found, having the better food resource information. Because they are transmitting the information, especially the good information at any time while searching the food from one place to another, conducted by the good information, the birds will eventually flock to the place where food can be found [11].

The basic principle of the PSO algorithm is it uses a number of particles (agents) that constitute a swarm moving around in the search space looking for the best solution. Each of the particles is treated as a point in N-dimensional space which adjusts its flying according to its own flying experience as well as the flying experience as well as the flying experience of other particles. Each particle keeps track of its coordinates in the solution space which are associated with the best solution (fitness) that has achieved so far by that particle. This value is known as personal best, $P_{BEST}$. Another best value that is tracked by the PSO is the best value obtained so far by any particle in the neighborhood of that particle which known as global best, $G_{BEST}$.

Each particle can be shown by its current velocity and position as shown in (17) and (18). Figure 4 shows the overall process of PSO.

$$v^{i+1} = \omega v^i + c_1 r_1(P_{BEST} - x^i) + c_2 r_2(G_{BEST} - x^i) \quad (17)$$

$$x^{i+1} = x^i + v^{i+1} \quad (18)$$

where:

- $v^{i+1}$ = velocity of particle at iteration $k$
- $\omega$ = inertia weight factor
- $c_1$, $c_2$ = acceleration coefficients
- $r_1$, $r_2$ = random numbers between 0 and 1
- $x^{i+1}$ = position of particle at iteration $k$

Start

$\text{Initialize particles with random position and velocity vectors}$

For each particles position (p) evaluate fitness

If fitness (p) better than fitness ($P_{BEST}$) then $P_{BEST} = p$

Set best of $P_{BEST}$ as $G_{BEST}$

Update particles velocity (17) and position (18)

Stop: giving $G_{BEST}$, optimal solution

Loop until all particles exhaust

Loop until maximum iteration

V. RESULTS AND DISCUSSION

The plant of the CTS is obtained from the mathematical modeling in previous chapter. The input voltage injected in the system is 1 Volt and the level converter (gain) will convert the input voltage to the water level. The desired level is 1 cm. The control structure with PID Controller of the CTS is shown in Figure 5 [7].

![Flow chart depicting of general PSO](image)

Figure 4: Flow chart depicting of general PSO

![Control structure with PID Controller](image)

Figure 5: Control structure with PID Controller

Simulation exercise are conducted with AMD Turion 64 X2 Processor, 4GB RAM, Microsoft Window 7 and MATLAB as a simulation platform. In this study, 20 particles are considered with 100 iterations. As default values, $c_1$ and $c_2$ are set as 2. The initial value of $\omega$ is 0.9 and linearly decreased to 0.4 at some stage of iteration. Table 3 shows the optimal PID parameter ($K_P$, $K_I$ and $K_D$) obtained using PSO.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_P$</td>
<td>250.9928</td>
</tr>
<tr>
<td>$K_I$</td>
<td>4.3478</td>
</tr>
<tr>
<td>$K_D$</td>
<td>171.6427</td>
</tr>
</tbody>
</table>
The control structure in Figure 5 is then simulated with the PSO-tuned controller parameter. The response of the system is shown in Figure 6. It shows that, PSO achieved the desired water level and improved the settling time of the system. Table 4 summaries system specifications obtained with PID controller. Figure 7 shows the performance response of the system.

Table 4: Performance of CTS based on PSO

<table>
<thead>
<tr>
<th>Method</th>
<th>$T_s$ (sec)</th>
<th>$T_r$ (sec)</th>
<th>OS (%)</th>
<th>SSE (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>17.7519</td>
<td>3.2691</td>
<td>16.1877</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Figure 8 and Table 5 shows summarize system performances obtained with PID controller. It shows that PSO-tuned method had better performance for CTS compared to the conventional tuning methods which were trial and error, auto-tuning, Z-N and C-C.

Table 5: Performances of CTS based on conventional tuning method and PSO

<table>
<thead>
<tr>
<th>Method</th>
<th>$T_s$ (sec)</th>
<th>$T_r$ (sec)</th>
<th>OS (%)</th>
<th>SSE (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial and error</td>
<td>84.4029</td>
<td>24.0334</td>
<td>6.8592</td>
<td>0.0000</td>
</tr>
<tr>
<td>Auto-tuning</td>
<td>53.3368</td>
<td>9.1404</td>
<td>1.8112</td>
<td>0.0000</td>
</tr>
<tr>
<td>Z-N</td>
<td>32.0856</td>
<td>3.2943</td>
<td>38.5350</td>
<td>0.0000</td>
</tr>
<tr>
<td>C-C</td>
<td>23.5904</td>
<td>2.8105</td>
<td>33.6989</td>
<td>0.0000</td>
</tr>
<tr>
<td>PSO</td>
<td>17.7519</td>
<td>3.2691</td>
<td>16.1877</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

As conclusion, this paper presents the study of an optimization method which was PSO and other various traditional tuning methods in order to obtain the optimal PID controller parameters. From the analysis, PID-tuned by PSO shown a better performance and successfully reduce the values of $T_s$, $T_r$, OS and SSE than conventional methods. However, this optimization might not be the best tuning method in order to obtain the best parameter for PID controller for CTS. Further research with other optimization is required to compare the performance of the system.

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REFERENCES

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