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Consistency Test in ANP Method with Group Judgment under Intuitionistic Fuzzy Environment

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***Abstract*: The consistency test is one of the critical components both in AHP and ANP. It is necessary to make sure if the judgment result is accuracy and reliable. This paper stated a specific process of consistency test in ANP with group judgment under intuitionistic fuzzy environment. A two steps de-fuzzification technique with intuitionistic fuzzy number reduction and generalized mean computation is proposed to apply in this study. The group consistency is also fully tested in two stages. This proposed process exposes that it is comprehensive and feasible. Besides, the application of maximum eigenvalue threshold method, a new consistency test index to check for the consistency, is an advantage because it reduces a lot of operations.**

***Index Terms*: Consistency testing, group judgment, analytic network process, intuitionistic fuzzy.**

I. INTRODUCTION

A consistency of judgments in the pairwise comparison method can be measured using the consistency index of judgment matrices [1]. Under the uncertainty decision environment, the consistency test becomes more complex, especially with intuitionistic fuzzy set (IFS), an extension of fuzzy sets which playing an important role in decision making and have gained popularity in recent years. Many studies have focused on IFSs to handle impreciseness and vagueness in the data (such as [2]-[4]) and extended the general multiple criteria decision making (MCDM) methods under intuitionistic fuzzy environment, in which analytic network process (ANP) is a typical method (such as [5]-[8]). During the process of making decisions, there will be inconsistency issue occurring when comparing different attributes or criteria as the decision problems are complicated in nature [9]. Thus, consistency test has been getting more attention ([1], [10], [11]). In this paper, a process of consistency test in ANP under intuitionistic fuzzy environment is proposed. It bases on intuitionistic fuzzy number (IFN) reduction technique to de-fuzzy pairwise comparison matrices and consistency ratio (CR) value to classify a matrix to set of consistent or set of inconsistent one. In the other hand, the two-stages testing procedure meets the requirement of group judgment. However, it is easy to apply and can be implement simultaneously in case of whole-test mode. Furthermore, a numeric example is also present to illustrate the implementation of this process. The remaining parts of this paper are structured as follows: Section 2 briefly reviews some basic concepts related to this study. In section 3, a process of consistency testing in ANP with under intuitionistic fuzzy environment is presented. A numerical example illustrates the proposed process in section 4. Finally, section 5 concludes the paper.

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II. BASIC CONCEPT

A. Intuitionistic Fuzzy Sets (IFSs)

As a generalization of Zadeh's fuzzy set [12], Atanassov [2] proposed the IFS, which is characterized by a membership function and a non-membership function. IFS has been proven to be a very suitable tool to describe the imprecise or uncertain decision information. The next definitions of IFS in this section are mainly borrowed from [13] and [5].

Definition 1 [13]

Let a set X be fixed. An intuitionistic fuzzy set [IFS] $\tilde{A}$ in X is an object having the form:

$$\tilde{A} = \{<x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)> \mid x \epsilon X\}, \quad (1)$$

where the $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ and $\nu_{\tilde{A}}(x): X \rightarrow [0,1]$ define the degree of membership and degree of non-membership respectively, of the element $x \epsilon X$ to the set $\tilde{A}$, which is a subset of X, for every element $x \epsilon X$, $0 \leq \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1$. An IFS is shown in Figure 1.

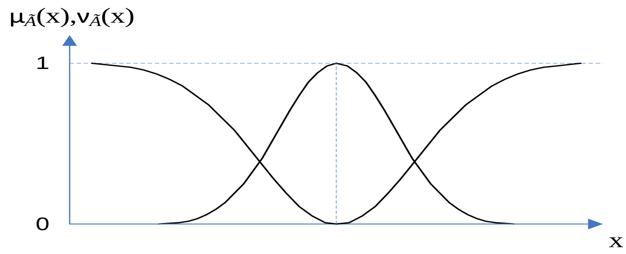

Fig.1. Membership and non-membership function of Ã

Definition 2 [5]

For each IFS $\tilde{A}$ in X, if

$$\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x), \; 0 \leq \pi_{\tilde{A}}(x) \leq 1, \quad (2)$$

then $\pi_{\tilde{A}}(x)$ is the third parameter of IFS and is usually called the intuitionistic fuzzy index or hesitation degree. IFSs is reduced to fuzzy sets when $\nu_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x)$ and $\pi_{\tilde{A}}(x) = 0$.

Definition 3 [5]

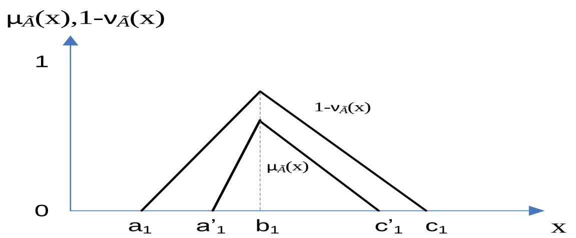

Fig.2. A triangular IFS Ã

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A triangular intuitionistic fuzzy number [IFN] $\tilde{A}$ is represented as:

$\tilde{A}$=<[(a'_1,b'_1,c'_1); $\mu_{\tilde{A}}$], [(a_1,b_1,c_1); $\nu_{\tilde{A}}$]>. (3)

The membership functions $\mu_{\tilde{A}}$ is used to derive the lower bounds of membership μ_L for IFN $\tilde{A}$, where the upper bound of membership μ_U is derived by taking the compliment of non-membership functions $\nu_{\tilde{A}}$, respectively. A triangular IFN is shown in Figure 2.

Definition 4 [5]

For two triangular FNs Ã1=<[(a'1,b'1,c'1); μÃ1], [(a1,b1,c1); νÃ1]> and Ã2=<[(a'2,b'2,c'2); μÃ2], [(a2,b2,c2); νÃ2]>, four common arithmetic operations for IFSs (addition, subtraction, multiplication and division) are demonstrated below:

Ã1+Ã2=<[(a'1+a'2, b'1+b'2, c'1+c'2); min(μÃ1,μÃ2)], [(a1+a2, b1+b2, c1+c2); max(νÃ1,νÃ2)]> (4)

Ã1-Ã2=<[(a'1-c'2, b'1-b'2, c'1-a'2); min(μÃ1,μÃ2)], [(a1-c2, b1-b2, c1-a2); max(νÃ1,νÃ2)]> (5)

Ã1xÃ2=<[(a'1xa'2, b'1xb'2, c'1xc'2); min(μÃ1,μÃ2)], [(a1xa2, b1xb2, c1xc2); max(νÃ1,νÃ2)]> (6)

Ã1/Ã2=<[(a'1/c'2, b'1/b'2, c'1/a'2); min(μÃ1,μÃ2)], [(a1/c2, b1/b2, c1/a2); max(νÃ1,νÃ2)]> (7)

B. Cardinal Inconsistency

The most widely used consistency index is the consistency ratio (*CR*) [14], that is

$$CR = \frac{CI}{RI} < 0.1 \tag{8}$$

where

$CI = \dfrac{\lambda_{\max} - n}{n-1}$ is the consistency index,

RI is the average random index based on matrix size shown in Table 1, λmax is the maximum eigenvalue of matrix A, and n is the order of matrix A. According to rule of thumb, the comparison matrix is consistent only if the value of CR is less than 0.1. The consistency test includes the following four steps [10]:

Step 1: Calculate the λmax of one comparison matrix.

Step 2: Calculate the value of CI.

Step 3: Calculate the CR.

Step 4: Compare the value of CR with the consistency threshold 0.1 to judge whether the comparison is consistent.

There is a major shortcoming when using CR as the consistency index for comparison matrices, as above steps has to be calculated repeatedly for each comparison matrix to test the consistency.

Table 1. The average random index

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.11	1.25	1.35	1.4	1.45	1.49

III. PROPOSED CONSISTENCY TESTING

This paper proposes to use the new consistency test index – maximum eigenvalue threshold method to check for the consistency [10]:

$$\lambda_{\max} < \lambda^n_{\max\_thrd}, \tag{9}$$

where $\lambda^n_{\max\_thrd}$ is the threshold of the maximum eigenvalue,

$$\lambda^n_{\max\_thrd} = 0.1RI(n-1)+n \tag{10}$$

If $\lambda^i_{\max} > \lambda^n_{\max\_thrd}$, that is $\lambda^i_{\max} - \lambda^n_{\max\_thrd} < 0$ or CR<0.1, the ith comparison matrix passes the consistency test. Therefore, the advantages of the maximum eigenvalue threshold method can be summarized into two aspects: efficient and easier to be implemented [11]. The intuitionistic fuzzy comparison matrix must be firstly converted to crisp matrix as the input of consistency test process. The intuitionistic de-fuzzification can be done by a two-step process [5] as below:

Step 1: Type reduction of IFS into a fuzzy set

Mendel [15] proposed a method to reduce IFS into a fuzzy set by taking an arithmetic mean of interval-valued memberships [μL, μU] at each xd, representing predefined discrete points over the universe of discourse.

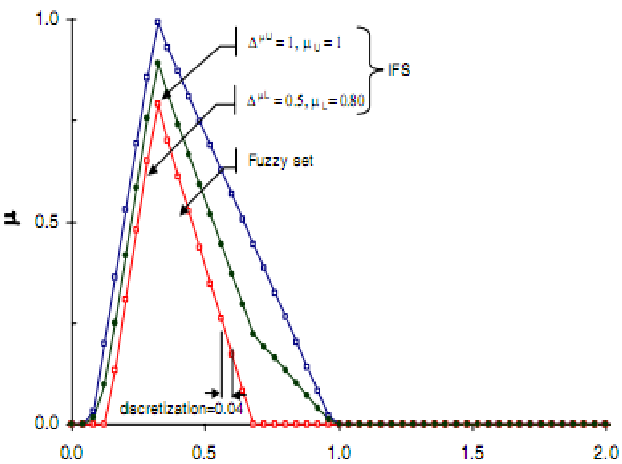

Fig.3. Reduction of IFS into a fuzzy set

In Fig. 3, an intuitionistic fuzzy number is derived by selecting discrete points xd at an interval of 0.04. Lee and Li [16] and Chen and Hwang [17] have proposed the use of generalized mean and standard deviation of an intuitionistic fuzzy number:

$$\bar{x}(\tilde{F}) = \frac{\int_a^b x\mu_{\tilde{F}}(x)dx}{\int_a^b \mu_{\tilde{F}}(x)dx}, \tag{11}$$

$$\sigma(\tilde{F}) = \left[\frac{\int_a^b x^2\mu_{\tilde{F}}(x)dx}{\int_a^b \mu_{\tilde{F}}(x)dx} - \left[\bar{x}(\tilde{F})\right]^2\right]^{1/2}, \tag{12}$$

where a and b are the lower and upper bounds when the membership is not equal to zero.

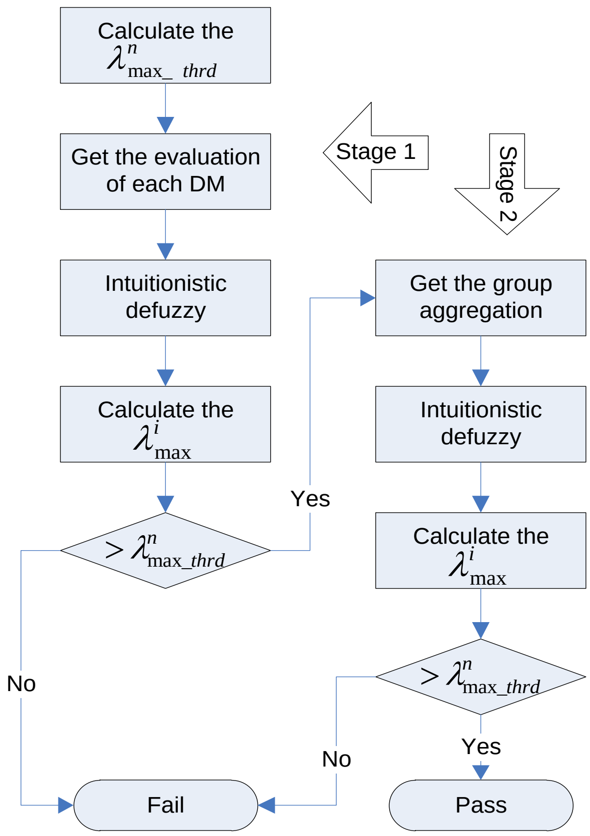

Fig.4. Flow chart of an IFS comparison matrix consistency test

In this study, the consistency of each IFS comparison matrix in ANP which is represented with mean values of the reduced IFSs needs to be tested using the maximum eigenvalue threshold method. If a comparison matrix fails the consistency test, then this comparison matrix has to be revised by DMs. If each individual comparison matrix passes the test in the first stage, then the group aggregation matrices are tested in the second stage as shown in Fig. 4. Number comparison matrices may be large depend on number levels of ANP hierarchy structure, including the comparison matrix between criteria with respect to the goal, the dimensions pairwise comparisons matrices with respect to each related dimension, the inner-dependence fuzzy pairwise comparison matrices in each dimension, the outer-dependence fuzzy pairwise comparison matrices in each dimension influenced by the other sub-criterion, etc. These comparison matrices can be tested independently. It means that test the consistencies of the comparison matrices with the same order in the same level one by one (level-by-level test) and test the whole consistencies of the comparison matrices simultaneously (whole-level test) are possible. Under the intuitionistic fuzzy environment, the intuitionistic de-fuzzification step must be done firstly. In case of group judgment, there are two basic techniques for aggregating individual judgments into group judgment currently, those are aggregating individual judgments (AIJ) and aggregating individual priorities (AIP). For each comparison matrix, the individual comparison matrices should be tested in the first stage. If they pass, in AIJ approach, the second stage continues with the group aggregation comparison matrix, which also contains intuitionistic fuzzy number.

IV. NUMERICAL EXAMPLE

This part presents a numerical example on supplier selection in fashion market with group judgment using triangular IFNs. For simplicity, the selection criteria are: quality (Q), cost (C), delivery (D), assurance (A) and flexibility (F) with their sub-criteria. The levels of the judgment value are range from 1 to 9 and are defined with the linguistic IFS terms (as in Table 2). The vagueness is express with $\Delta\mu L = 0.5$, $\Delta\mu U = 1.0$ and a membership interval of [0.8, 0.9] to account for non-specificity. In this example, only the comparison matrix between criteria with respect to the goal is presented.

Table 2. Definitions of linguistic terms for the importance

	Linguistic terms	IFS values
JE	Just Equal	<[(1 ,1, 1),μ], [(1 ,1, 1),ν]>
EI	Equally important	<[(1 ,1, 1 1/2),μ], [(1 ,1, 2),ν]>
WMI	Weakly more important	<[(2 1/2 ,3, 3 1/2),μ], [(2 ,3, 4),ν]>
SMI	Strongly more important	<[(4 1/2 ,5, 5 1/2),μ], [(4 ,5, 6),ν]>
VSMI	Very strongly more important	<[(6 1/2 ,7, 7 1/2),μ], [(6 ,7, 8),ν]>
AMI	Absolutely more important	<[(8 1/2 ,9, 9 1/2),μ], [(8 ,9, 10),ν]>

The evaluation of three decision makers (DM1, DM2, DM3) are shown in comparison matrix (Table 3). With discrete points xd =0.10, the result of de-fuzzification is calculated and presented in Table 4. It is easy to have $\lambda^{n}_{max\_thrd} = 0.1RI(n-1)+n$ = 5.444, with n=5 and RI=1.11 referred to Table 1. By using Matlab, the λmax values are 5.479 (>5.444, inconsistency), 5.094 (<5.444, consistency) and 5.065 (<5.444, consistency). DM1 should revive his judgment. The important degree between Assurance and Flexibility is change to WMI instead of 1/SMI. This change decreases the λmax to 5.029 (<5.444, consistency). With the similar way, the group judgment in Table 5 also satisfies the consistency condition with λmax =5.0126.

Table 3. Criteria weight pairwise comparison matrix

Criteria		Q	C	D	A	F
Quality	DM1	JE	WMI	1/WMI	EI	WMI
	DM2	JE	WMI	1/WMI	1/WMI	EI
	DM3	JE	1/WMI	1/SMI	1/AMI	EI
Cost	DM1		JE	1/VSMI	1/SMI	EI
	DM2			1/SMI	1/SMI	1/WMI
	DM3			EI	1/WMI	WMI
Delivery	DM1			JE	WMI	SMI
	DM2			JE	EI	VSMI
	DM3			JE	EI	WMI
Assurance	DM1				JE	1/SMI
	DM2				JE	SMI

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	DM3				JE	SMI
Flexibility	DM1					JE
	DM2					JE
	DM3					JE

Table 4. De-fuzzification pairwise comparison matrix

		Q	C	D	A	F
DM1	Q	1.00	2.00	0.50	1.00	2.00
	C	0.50	1.00	0.25	0.33	1.00
	D	2.00	4.00	1.00	2.00	3.00
	A	1.00	3.00	0.50	1.00	0.33
	F	0.50	1.00	0.33	3.00	1.00
DM2	Q	1.00	2.00	0.50	0.50	1.00
	C	0.50	1.00	0.33	0.33	0.50
	D	2.00	3.00	1.00	1.00	4.00
	A	2.00	3.00	1.00	1.00	3.00
	F	1.00	2.00	0.25	0.33	1.00
DM3	Q	1.00	0.50	0.33	0.20	1.00
	C	2.00	1.00	1.00	0.50	2.00
	D	3.00	1.00	1.00	1.00	2.00
	A	5.00	2.00	1.00	1.00	3.00
	F	1.00	0.50	0.50	0.33	1.00

Table 5. Group aggregation matrix

	Quality	Cost	Delivery	Assurance	Flexibility
Quality	1.000	1.260	0.437	0.464	1.260
Cost	0.794	1.000	0.437	0.382	1.000
Delivery	2.289	2.289	1.000	1.260	2.884
Assurance	2.154	2.621	0.794	1.000	2.621
Flexibility	0.794	1.000	0.347	0.382	1.000

V. CONCLUSIONS

This paper stated a specific process of consistency testing in ANP with group judgment under intuitionistic fuzzy environment. It is necessary to make sure if the judgment result is accuracy and reliable. A two steps de-fuzzification technique with IFN reduction and generalized mean computation is proposed to apply in this study. The group consistency is also fully tested in two stages. This proposed process exposes that it is comprehensive and feasible. Besides, the application of maximum eigenvalue threshold method, a new consistency test index to check for the consistency, is an advantage because it reduces a lot of operations.

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