Numerical Study on the Detailed Characterization of Ni-MH Battery Model for its Dynamic Behavior using Multi-Regression Analysis - MRA

M. Karthik, S. Vijayachitra

Abstract—A numerical study is presented in this paper to examine the dynamic behavior for the detailed characterization of commercially available Ni-MH battery. In the present study, a novel Multi-Regression Analysis (MRA) based model for the D-size HHR650D battery from Panasonic is adopted to ascertain the charge and discharge characteristics along with its SoC estimation. Oxygen gas formation at the Ni electrode during charging and overcharging that affects the pressure variations inside the battery is essential to be analyzed for its characterization. Henceforth, the effect of battery charging conditions over the pressure and temperature variations are considered in the developed MRA model and the corresponding performance profiles subjected to recurrent load cycles are reported. Model validation of the steady state behavior is performed based on the benchmark data obtained from a 6.5Ah, 1.2V Nickel-Metal Hydride battery. The result obtained shows that the regression model responses fit well with the benchmark result. Moreover, the model can also predict pressure and temperature dynamics under a sudden change in charging and discharging states. The characterization results show that the proposed regression model of Ni-MH battery could be suited effectively for any kind of model based plug-in or hybrid electric vehicle technologies.

Index Terms—Interpolation, Multi-Regression analysis, Ni-MH battery, SoC and voltage dynamics.

I. INTRODUCTION

As global fossil fuel reserves deplete whereas the release of greenhouse pollutant gases into the atmosphere and the fuel prices tremendously increases, there is a need for the researchers to provide appropriate solutions for a sustainable transportation methods instead of ICE based propulsion systems. The only way to eliminate the dependency of oil and petroleum consumption is through electrification [1] thereby, it is sensible and reliable for both environmental position and economy. An alternative technology that bridges the gap between sustainable transportation and depletion of fuel reserves [2-4] is the Hybrid Electric Vehicle (HEV). Battery plays a significant role as an energy source not only in the development of Electric vehicles and also in Hybrid Electric Vehicles. Over the past few decades, Ni-MH battery has received much of great interest and deployed in the development of electric propulsion systems because of its high specific power (1350/kg) and high energy density. In addition, they possess long durability with the reliable and safe operation [5-6].

It is evident from the literature that there a variety of Ni-MH battery modeling techniques [7] have been developed towards its design, estimation of state of charge (SoC), prediction of battery charging-discharging voltage pattern and circuit simulation. The Electrochemical battery modeling uses a set of highly nonlinear differential equations for a detailed description and design of Ni-MH batteries [8-10]. They are used to exactly describe the chemical process that takes place inside the battery as the modeling is completely based on the fundamental mechanisms involved in it and hence this type of modeling is found to be more accurate and sophisticated. However, the electrochemical models are highly complex [10] to develop as they require a detailed knowledge of the chemical process that makes the model more time consuming. Further, these models are not suitable for representing the battery dynamics [11] and particularly for the prediction of SoC and its estimation.

On the other hand, electrical circuit based modeling involves equivalent electrical circuit models that are developed using discrete electrical components such as resistors, capacitors and voltage sources. They are more intuitive, simple, easy to handle [12] and useful for exactly predicting the transient and dynamic characteristics of the batteries. There have been several electrical circuit based models are found in the literature that falls under the categories such as Thevenin [13-16] and impedance based [17-18] models. These models are easy to handle along with co-simulation of other electrical application systems and circuits with an added benefit of less computational burden. Though these types of models are easy to construct, they work only for a fixed SoC and temperature setting [17]. In addition, the electrical circuit based models are unable to predict the battery run time with respect to the battery charging-discharging response. Further, developing an impedance based model creates more complexity in its method of implementing it without a compromise of obtaining a battery run time response. As like electrochemical models, electric circuit models are also not suitable for the prediction of SoC and its estimation.

Manuscript Received on December 2014.

Karthik M. Department of Electrical and Electronics Engineering, Kongu Engineering College, Erode, Affiliated to Anna University, Chennai, Tamilnadu, India.

Vijayachitra S. Department of Electronics and Instrumentation Engineering, Kongu Engineering College, Erode, Affiliated to Anna University, Chennai, Tamilnadu, India.

Abstract—A numerical study is presented in this paper to examine the dynamic behavior for the detailed characterization of commercially available Ni-MH battery. In the present study, a novel Multi-Regression Analysis (MRA) based model for the D-size HHR650D battery from Panasonic is adopted to ascertain the charge and discharge characteristics along with its SoC estimation. Oxygen gas formation at the Ni electrode during charging and overcharging that affects the pressure variations inside the battery is essential to be analyzed for its characterization. Henceforth, the effect of battery charging conditions over the pressure and temperature variations are considered in the developed MRA model and the corresponding performance profiles subjected to recurrent load cycles are reported. Model validation of the steady state behavior is performed based on the benchmark data obtained from a 6.5Ah, 1.2V Nickel-Metal Hydride battery. The result obtained shows that the regression model responses fit well with the benchmark result. Moreover, the model can also predict pressure and temperature dynamics under a sudden change in charging and discharging states. The characterization results show that the proposed regression model of Ni-MH battery could be suited effectively for any kind of model based plug-in or hybrid electric vehicle technologies.

Index Terms—Interpolation, Multi-Regression analysis, Ni-MH battery, SoC and voltage dynamics.

I. INTRODUCTION

As global fossil fuel reserves deplete whereas the release of greenhouse pollutant gases into the atmosphere and the fuel prices tremendously increases, there is a need for the researchers to provide appropriate solutions for a sustainable transportation methods instead of ICE based propulsion systems. The only way to eliminate the dependency of oil and petroleum consumption is through electrification [1] thereby, it is sensible and reliable for both environmental position and economy. An alternative technology that bridges the gap between sustainable transportation and depletion of fuel reserves [2-4] is the Hybrid Electric Vehicle (HEV). Battery plays a significant role as an energy source not only in the development of Electric vehicles and also in Hybrid Electric Vehicles. Over the past few decades, Ni-MH battery has received much of great interest and deployed in the development of electric propulsion systems because of its high specific power (1350/kg) and high energy density. In addition, they possess long durability with the reliable and safe operation [5-6].

It is evident from the literature that there a variety of Ni-MH battery modeling techniques [7] have been developed towards its design, estimation of state of charge (SoC), prediction of battery charging-discharging voltage pattern and circuit simulation. The Electrochemical battery modeling uses a set of highly nonlinear differential equations for a detailed description and design of Ni-MH batteries [8-10]. They are used to exactly describe the chemical process that takes place inside the battery as the modeling is completely based on the fundamental mechanisms involved in it and hence this type of modeling is found to be more accurate and sophisticated. However, the electrochemical models are highly complex [10] to develop as they require a detailed knowledge of the chemical process that makes the model more time consuming. Further, these models are not suitable for representing the battery dynamics [11] and particularly for the prediction of SoC and its estimation.

On the other hand, electrical circuit based modeling involves equivalent electrical circuit models that are developed using discrete electrical components such as resistors, capacitors and voltage sources. They are more intuitive, simple, easy to handle [12] and useful for exactly predicting the transient and dynamic characteristics of the batteries. There have been several electrical circuit based models are found in the literature that falls under the categories such as Thevenin [13-16] and impedance based [17-18] models. These models are easy to handle along with co-simulation of other electrical application systems and circuits with an added benefit of less computational burden. Though these types of models are easy to construct, they work only for a fixed SoC and temperature setting [17]. In addition, the electrical circuit based models are unable to predict the battery run time with respect to the battery charging-discharging response. Further, developing an impedance based model creates more complexity in its method of implementing it without a compromise of obtaining a battery run time response. As like electrochemical models, electric circuit models are also not suitable for the prediction of SoC and its estimation.

Manuscript Received on December 2014.

Karthik M. Department of Electrical and Electronics Engineering, Kongu Engineering College, Erode, Affiliated to Anna University, Chennai, Tamilnadu, India.

Vijayachitra S. Department of Electronics and Instrumentation Engineering, Kongu Engineering College, Erode, Affiliated to Anna University, Chennai, Tamilnadu, India.
Besides electrochemical models and electric circuit models, mathematical models such as Peukert’s model, modified Shepherd model, etc., provides a higher level of abstraction [19] using only a few equations. This makes the mathematical models to easily predict the battery properties like efficiency, run time of the battery etc., [12]. However, these models do not have the ability to deal with the dynamic loads [19] and they are not applicable to predict the battery I-V charging-discharging response, that are necessary for the vehicular applications. Further, this model suffers from the problem of algebraic loop and model instability when it is integrated and used in the co-simulation of other electrical application systems.

All of the battery models described and reviewed here [7-19] can be a good platform in providing a detailed description about the modeling techniques that aids in analyzing the battery performance from different perspectives. Nevertheless, these models does not account the other important factors influencing the battery performance that includes temperature rise [20] and pressure variations inside the battery which are essential to be computed. This is because, as the battery charges and discharges, heat formation occurs inside the battery and even it becomes faster, when it is overcharged. Further, the pressure inside the battery is generated more during the charging reactions and it is greatly increased along with the rise in temperature. This continual increase in pressure leads to the venting of the battery as there may be a possibility of exploding which abruptly affects the battery performance. Beside temperature, pressure and SoC estimation, battery charging and discharging voltage patterns are also required to be analyzed for a predefined operation pattern.

As there is a significant complexity involved in the existing modeling techniques, it is thus important that there is a need of novel simulation based modeling with a generalized framework has to be definitely entrenched for estimating the performance of a battery. Also, to cope with the technical hitches involved in the design of battery packed Hybrid Electric vehicle systems and to quantify the battery state of charge (SoC) for its degradation, a robust simulation based modeling of the battery is essential that aids in developing an optimal design of the battery to be used for all sorts of automotive applications. Hence, a novel Multi-Regression battery model is proposed in this paper for precisely capturing the dynamic and nonlinear behaviour of the battery that incorporates the effect of temperature and pressure dynamics. The simulation results obtained from this MRA model are analyzed for its charging and discharging behaviour using the MATLAB and Simulink™ environment and the model results are validated based on the benchmark data obtained from a 6.5Ah, 1.2V Nickel-Metal Hydride battery [21]. The proposed regression model is computationally effective that can be applied to any other kind of battery types and it can be served as a benchmark tool for predicting the performance of a battery for its dynamic behaviour when integrated with other electrical application systems.

II. DYNAMIC NI-MH BATTERY MODELING

This section deals with the development of a novel Multi-Regression (MR) model for Ni-MH battery system under study. The dynamic model presented in this work is based on polynomial regression model as it relies on curve fit regression functions deduced based on the estimated relationship between various independent variables and a dependent variable that governs the complexity of the physical phenomena.

A. Modeling Framework

The modeling framework proposed in the present analysis accounts for charging and discharging voltage dynamics, State of charge variations, temperature and pressure dynamics that are crucial in the design and development of a battery assisted hybrid electric vehicle systems. The physical domain of the Ni-MH battery structure is presented at the outset, followed by the battery equations governing its operational behaviour. First, the fundamental formulation of MRA model adopted in the proposed study is presented. Second, the regression functions governing the charge acceptance efficiency of the battery are developed. Next, the regression functions governing the battery voltage for its charging and discharging process are developed. Finally, the pressure and temperature dynamics required to complete the modeling framework is presented that decides the battery life and its performance.

B. Ni-MH battery operation - Overview

The operating principle of the Ni-MH battery lies in its capability of releasing, absorbing and transporting the hydrogen between the positive and negative electrodes. Use of rare-earth hydrogen absorbing metals at the negative electrode like Nickel is a remarkable and unique feature of the Ni-MH battery. On the other hand, positive electrode uses Nickel hydroxide, Ni(OH)_2 plate with an alkaline electrolyte sandwiched between the two electrode plates that are wound together with a separator kept in a sealed metallic container. The alkaline electrolyte offers a low resistance for the flow of protons and provides a high resistance for the conduction of electrons through it.
NiOOH in the positive electrode and forms water, \( H_2O \) in the electrolyte with a release of free electron \( e^- \).

**At Positive Electrode:**

\[
\text{Charge:} \quad \text{Ni(OH)}_2 + \text{OH}^- \rightarrow \text{NiOOH} + H_2O + e^- \tag{1}
\]

**At Negative Electrode:**

\[
\text{Discharge:} \quad M + H_2O + e^- \rightarrow \text{MH} + \text{OH}^- \tag{2}
\]

**Overall:**

\[
\text{Charge:} \quad \text{Ni(OH)}_2 + M \rightarrow \text{NiOOH} + \text{MH} \tag{3}
\]

Metal alloy \( M \) in the negative electrode combines with water, \( H_2O \) and electron, \( e^- \) to form Metal hydride, \( MH \) in the negative electrode with the formation of hydroxide, \( OH^- \) in the electrolyte. Clusters of similar battery cells are combined and connected together in series that forms the battery pack and the number of cells that forms the battery pack is based on the requirement of voltage and power for a definite application.

### C. Formulation of MRA Model

The multi-regression analysis (MRA) is a widely used statistical procedure [22] for modeling a highly nonlinear system and it is a systematic procedure for establishing and investigating a quantitative relationship between the variables of the system [23] with the aim of building a precise multi-regression model. Due to its highly nonlinear mapping ability, formulation of a multiple regression model is proposed in this study using the experimental data to predict the battery response in terms of battery charging-discharging voltage, SoC, temperature and pressure dynamics.

A general multiple linear regression function is represented in its polynomial form as follows [24]:

\[
Y = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots + b_p x^p \tag{4}
\]

where, \( Y \) is the dependent variable given by the regression model, \( x^i \) (i = 1,2, …p) is the \( i^{th} \) independent/predictor variable from total set of \( p \) variables, \( b_0 \) is the intercept/constant, \( b_i \) (i = 1,2, …p) is the \( i^{th} \) coefficient corresponding to \( x^i \), and i = 1,2 … p is the independent variables’ index that represents the degree of polynomial.

The linear regression equation seen in Eq. (4) is said to be highly linear since the form of representing the predictor variables is linear in the equation that it depends. However, the non-linear regression equation has the predictor variables with its nonlinear representation that can cover many different forms of a single curved relationship between the dependent and independent variable. As a consequence of this powerful curve fitting ability, non-linear MR model is adopted in this study.

Based on this fundamental formulation of the non-linear MR function, a novel multi-regression based model for the HHR 650D battery is developed to ascertain the charge and discharge characteristics along with its SoC estimation. Since the performance model of the battery is dependent on several key parameters viz., charge acceptance efficiency, State of Charge (SoC), charging and discharging voltage, charge input, Depth of Discharge (DoD), pressure and temperature dynamics, a comprehensive MR model that shows the dependence of all these key parameters has been developed. For this MR model, a suitable curve fits like polynomial, gaussian, rational and exponential function fits are adopted for developing the regression function. Curve fits for the regression function is selected as the best fit that fits the actual data based on least estimation of mean square error value.

### III. Model Extraction of Ni-MH Battery

A performance model of the Ni-MH battery describing the relationship between key parameters viz., charge acceptance efficiency, State of Charge (SoC), charging and discharging voltage, charge input, Depth of Discharge (DoD), pressure and temperature dynamics is discussed and presented in this section. Parameter estimations are done using the multi-regression analysis that are identified and constructed based on the benchmark characteristics obtained from a 6.5Ah, 1.2V Nickel-Metal Hydride battery [10, 21].

Charging and discharging phases are the two frequent operations that take place inside the battery in which charging occurs for replacing the energy discharged during the previous load cycles and discharging of stored energy occurs during when the load demands. Accounting these two major cycles that takes place inside the battery operation, the proposed MR model of the battery integrates all the regression fit equations relating charge efficiency, SoC, DoD, charging and discharging voltage, pressure and temperature dynamics that are described in the subsequent section.

The three selectable charging rates in the MR analysis of charging efficiency, charging voltage and battery pressure are 2.145A, 13A and 39A that correspond to a charging rate of 0.33C, 2C and 6C respectively in this 6.5Ah Ni-MH battery. Further, the selectable charging rates for the battery temperature are 0.65A and 6.5A which corresponds to a charging rate of 0.1C and 1C respectively. Moreover, the selectable discharging rates adopted for the discharging voltage are 1.3A, 13A and 65A that correspond to a discharging rate of 0.2C, 2C and 10C respectively.

### A. Regression model of SoC dynamics

The State of Charge (SoC) dynamics proposed in this section includes the modeling of charge efficiency (CE), charge input (CI), available capacity (CA), charge rate (C_R) and discharge rate (D_R) as they have a significant influence on SoC. To start with, the non-linear regression fit was adopted for the charge efficiency-state of charge characteristics [10] and the best estimation fit instigated in this study for different charge rates is depicted in Figure 2. The corresponding regression equations are estimated through charge efficiency as a function of SoC at various charge rates and they are expressed by

\[
CE_{0.33} = a_0 + a_1 \text{SoC}^0 + a_2 \text{SoC}^1 + a_3 \text{SoC}^2 + a_4 \text{SoC}^3 + a_5 \text{SoC}^4 + a_6 \text{SoC}^5 + a_7 \text{SoC}^6 \tag{5}
\]

\[
CE_{2} = a_0 + a_1 \text{SoC}^0 + a_2 \text{SoC}^1 + a_3 \text{SoC}^2 + a_4 \text{SoC}^3 + a_5 \text{SoC}^4 + a_6 \text{SoC}^5 + a_7 \text{SoC}^6 + a_8 \text{SoC}^7 - a_9 \text{SoC}^8 + a_{10} \text{SoC}^9 \tag{6}
\]

\[
CE_{6} = a_0 - a_1 \text{SoC}^0 + a_2 \text{SoC}^1 + a_3 \text{SoC}^2 - a_4 \text{SoC}^3 + a_5 \text{SoC}^4 - a_6 \text{SoC}^5 + a_7 \text{SoC}^6 - a_8 \text{SoC}^7 + a_{10} \text{SoC}^8 + a_{11} \text{SoC}^9 + a_{12} \text{SoC}^{10} \tag{7}
\]
The appropriate polynomial coefficients that facilitate the optimal convergence associated with the charge efficiency equations are presented in Table 1.

### Table 1: Polynomial coefficients of charge efficiency for the charge rates of 0.33C, 2C and 6C:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
<th>$a_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE(0.33)</td>
<td>1058</td>
<td>3845</td>
<td>5715</td>
<td>4470</td>
<td>1970</td>
<td>484.6</td>
<td>61.59</td>
<td>3.38</td>
<td>0.9488</td>
</tr>
<tr>
<td>CE(2)</td>
<td>-0.9731</td>
<td>2.577</td>
<td>1.599</td>
<td>1.166</td>
<td>1.943</td>
<td>2.801</td>
<td>1.611</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CE(6)</td>
<td>72.8</td>
<td>286.5</td>
<td>444.5</td>
<td>341.4</td>
<td>135</td>
<td>27.52</td>
<td>2.1</td>
<td>0.06877</td>
<td>0.01204</td>
</tr>
</tbody>
</table>

**Fig. 2. Regression fit of Charge acceptance efficiency to SoC at charge rates of 0.33/2/6C**

As seen from the charge efficiency-state of charge characteristics [10] shown in Figure 2, the charge rate can also be related with the charge efficiency at a specified SoC. For this at 70% SoC, the effective charge efficiency can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified charge rate (0.33C, 2C and 6C) and the effective charge efficiency for which the battery it operates.

$$CE = aR^2 + b R + c$$  \(8\)

The charge rate coefficients $a$, $b$ and $c$ can be determined in terms of charge efficiency at the specified charge rates by the way of deriving and solving the 3 sets of charge efficiency equations with different charge rates of 0.33C, 2C and 6C. Thus, the effective charge efficiency is given by,

$$CE = (0.1056 CE_{0.33} - 0.1497 CE_2 + 0.0441 CE_6) C_R^2 + (-0.845 CE_{0.33} + 0.948 CE_2 - 0.1027 CE_6) C_R + 1.268 CE_{0.33} - 0.2976 CE_2 + 0.0297 CE_6$$  \(9\)

The charge rate can be obtained using the actual capacity of the battery (Q) and the charging current (I_c) and expressed as,

$$C_R = \frac{I_c}{Q}$$  \(10\)

Now, the rise in state of charge (SoC) can be computed [10] based on the charge input (CI) and the effective charge efficiency obtained in Eq. (9) as given by,

$$SoC_{R,C} = CE \times CI$$  \(11\)

Charge input (CI) is proportional to the actual state of charge expressed as a percentage input of nominal battery capacity [10] such that at the end of 100% charge input, the state of charge (SoC) is expected to be 100% when there is no oxygen formation. But practically, the oxygen formation occurs during the charging phase leads to a pressure rise inside the battery and thus it fails to maintain the exact equivalency between the charge input and state of charge. Thus the charge input is to be assessed based on the total capacity of the battery (Q) in Ah and charging current (I_c) [10] using,

$$CI = \frac{1}{Q} \int \frac{I_c}{3600} dt$$  \(12\)

Similarly, the fall in state of charge (SoC_{f,D}) can be computed based on the available capacity of the battery (CA) in Ah and the discharge current (I_d) [25] using,

$$SoC_{f,D} = \frac{CA}{6.5} \int \frac{I_d}{3600 \times 6.5} dt$$  \(13\)

A third order polynomial regression equation is estimated to evaluate the available capacity (CA) from the commercially available Ni-MH batteries in the market of different capacities (7/6.76/6.5/6.25/5.75Ah) with its discharge rate (D_k) at different coulomb rates (0.2/1/2/5/10C) and it is given by,

$$CA = -0.004173D_k^3 + 0.06953 D_k^2 -0.413D_k + 7.09$$  \(14\)

The discharge rate can be obtained using the actual capacity of the battery (Q) and the discharging current (I_d) and expressed as,

$$D_k = \frac{I_d}{Q}$$  \(15\)

Thus by knowing the initial SoC (SoC_{ini}), the actual state of charge available at any time can be obtained using,

$$SoC = SoC_{ini} + SoC_{R,C} - SoC_{f,D}$$  \(16\)

Using the dynamic equations developed in this section from Eq. (5) to (16), the influence on battery SoC can be modeled and examined for its charging and discharging conditions.

### B. Regression model of battery voltage

The battery voltage dynamics presented in this segment comprises the modeling of both the charging and discharging voltage as the battery voltage behavior differs at different charge rates and discharge rates.

The charging voltage can be computed based on charge rate (C_R) and charge input (CI) obtained from Eq. (10) and (12). For this, a non-linear regression fit was realized for the charging voltage-charge input characteristics [10] and the best estimation fit formulated in the study for different charge rates is depicted in Figure 3. The resultant regression equations are assessed through charging voltage as a function of charge input at various charge rates and they are stated by,

$$V_{C(0.33)} = -b_x CI^3 + b_y CI^2 - b_y CI^2 + b_y CI^2 + b_y CI^2 + b_y CI + b_y$$  \(17\)

$$V_{C(2)} = -b_x CI^3 + b_y CI^2 + b_y CI + b_y$$  \(18\)

$$V_{C(6)} = b_y CI + b_y CI + b_y CI + b_y CI + b_y CI + b_y CI$$  \(19\)
The appropriate polynomial coefficients that facilitate the optimal convergence associated with the charging voltage equations are presented in Table 2.

Table 2: Polynomial coefficients of charging voltage at a charging rate of 0.33C, 2C and 6C:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>V_C,0.33</th>
<th>V_C,2</th>
<th>V_C,6</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>7.146</td>
<td>-</td>
<td>0.1562</td>
</tr>
<tr>
<td>b</td>
<td>41.52</td>
<td>50.4</td>
<td>0.316</td>
</tr>
<tr>
<td>c</td>
<td>0.975</td>
<td>526.8</td>
<td>4.301</td>
</tr>
</tbody>
</table>

As seen from the charging voltage-charge input characteristics [10] shown in Figure 3, the charge rate can also be related with the charging voltage at a specified charge input. Thus for a charge input at 100%, the effective charging voltage can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified charge rate (0.33C, 2C and 6C) and the effective charging voltage for which the battery it accumulates the charges for the charging current (I_0).

\[ V_{c} = a C_{k}^2 + b C_{k} + c \]  \hspace{1cm} (20)

The charge rate coefficients a, b and c can be determined in terms of charging voltage at the specified charge rates by the way of deriving and solving the 3 sets of charging voltage equations with different charge rates of 0.33C, 2C and 6C. Thus, the effective charging voltage is given by,

\[ V_{c} = \left( 0.1056 V_{c,0.33} - 0.1497 V_{c,2} + 0.0441 V_{c,6} \right) C_{k}^2 + \left( -0.845 V_{c,0.33} + 0.948 V_{c,2} - 0.1027 V_{c,6} \right) C_{k} + 1.268 V_{c,0.33} - 0.2976 V_{c,2} + 0.0297 V_{c,6} \]  \hspace{1cm} (21)

Similarly, the discharging voltage can be computed based on discharge rate (Dk) and the fall in state of charge (SoC = D_0 ... D_k) obtained from Eq. (15) and (13). Hence, a non-linear regression fit was implemented for the discharging voltage-depth of discharge characteristics [26] and a valid estimation fit implemented in the study for different discharge rates is depicted in Figure 4. The regression equations obtained from them are evaluated through discharging voltage as a function of depth of discharge at various discharge rates and they are described by,

\[ V_{D}(0.2) = -c_6 D^4 + c_5 D^3 - c_4 D^2 + c_3 D + c_0 \]  \hspace{1cm} (22)

\[ V_{D}(2) = -c_2 D^2 + c_1 D^0 \]  \hspace{1cm} (23)

\[ V_{D}(10) = -c_6 D^4 + c_5 D^3 - c_4 D^2 + c_3 D + c_0 \]  \hspace{1cm} (24)

The appropriate polynomial coefficients that facilitate the optimal convergence associated with the discharging voltage equations are presented in Table 3.

Table 3: Polynomial coefficients of discharging voltage at a discharging rate of 0.2C, 2C and 10C:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>c_0</th>
<th>c_1</th>
<th>c_2</th>
<th>c_3</th>
<th>c_4</th>
<th>c_5</th>
<th>c_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_D,0.2</td>
<td>8.315e-5</td>
<td>0.002137</td>
<td>0.02216</td>
<td>0.1183</td>
<td>0.342</td>
<td>0.5075</td>
<td>0.2941</td>
</tr>
<tr>
<td>V_D,2</td>
<td>3.128e-5</td>
<td>0.000565</td>
<td>0.00373</td>
<td>0.0045</td>
<td>0.04172</td>
<td>0.212</td>
<td>0.4105</td>
</tr>
</tbody>
</table>

From the discharging voltage-discharge capacity characteristics [26] shown in Figure 4, the discharge rate can be related with the discharging voltage at a specified discharge capacity. For this at a discharge capacity of 5Ah, the effective discharging voltage can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified discharge rate (0.2C, 2C and 10C) and the effective discharging voltage for which the battery it discharges for the current (I_0).

\[ V_{D} = a D_{k}^2 + b D_{k} + c \]  \hspace{1cm} (25)

The discharge rate coefficients a, b and c can be determined in terms of discharging voltage at the specified charge rates by the way of deriving and solving the 3 sets of discharging voltage equations with different discharge rates of 0.2C, 2C and 10C. Thus, the effective discharging voltage is given by,

\[ V_{D} = (0.0566 V_{D,0.2} - 0.0694 V_{D,2} + 0.01275 V_{D,10}) D_{k}^2 + (-0.6792 V_{D,0.2} + 0.7078 V_{D,2} - 0.028 V_{D,10}) D_{k} + 1.1324 V_{D,0.2} - 0.138 V_{D,2} + 0.005 V_{D,10} \]  \hspace{1cm} (26)
Using the dynamic equations developed in this section from Eq. (17) to (26), the influence on battery voltage can be modeled and examined for its charging and discharging conditions.

C. Regression model of Temperature dynamics

In electric vehicular applications, frequent charging of the battery leads to a temperature rise inside the battery and it is an important measure to be monitored and managed. This rise in battery temperature affects the battery performance that ultimately influences the efficiency of the battery system. Thus there is a need arises for the study of temperature behavior and its rise inside the battery system. For this, the proposed battery regression model integrates the temperature dynamics modeling in it. Hence the temperature dynamics model developed in this section includes the behavior of battery temperature only during charging while the discharging of battery over temperature is not included since it is ineffective over the temperature rise.

The coefficients $a$ and $b$ can be determined in terms of battery temperature at the specified charge rates by the way of deriving and solving the set of battery temperature equations with different charge rates of 0.1C and 1C. Thus, the effective battery temperature is given by,

$$T = a + \frac{b}{\sqrt{E_s}}$$  \hspace{1cm} (29)

Using the dynamic equations developed in this section from Eq. (27) to (30), the influence on battery temperature can be modeled and examined for its charging conditions.

D. Regression model of Pressure dynamics

Ni-MH battery operation involves the generation of oxygen that takes place at the Ni electrode during charging and overcharging. This oxygen has to be transported properly to the other MH electrode where the recombination takes place. If the complete oxygen gas evolved is not utilized for recombination at the MH electrode, then the excess oxygen is liable for the rise in battery pressure. This gassing effect extremely affects the effective operational behavior of the Ni-MH battery and eventually influences over the life cycle of the battery system. This necessitates the exploration of battery pressure variations inside the battery system during charging. Hence a pressure dynamics model is also integrated in this study besides the inclusion of temperature dynamics model.

For this, as followed in the development of temperature dynamics model, the model development of pressure dynamics includes the behavior of battery pressure only during the charging period as the oxygen formation inside the battery is the only contributor to the rise in pressure that occurs at the time of charging and overcharging. Hence, the pressure dynamics model for the discharging effect is ignored in the model development as it does not have the influence over the pressure rise. The battery pressure can be computed based on charge rate (CR) and charge input (CI) obtained from Eq. (10) and (12). For this, a non-linear regression fit was adopted for the battery temperature-charge input characteristics [26] shown in Figure 5, the charge rate can be related with the battery temperature at a specified charge input. Thus for a charge input at 1.2 units, the effective battery temperature can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified charge rate (0.1C and 1C) and the effective battery temperature for which the battery it operates.

$$T = a + \frac{b}{\sqrt{E_s}}\left(\frac{1}{2.16}\right)$$  \hspace{1cm} (30)

Using the dynamic equations developed in this section from Eq. (27) to (30), the effective battery temperature can be modeled and examined for its charging conditions.

### Table 4: Polynomial coefficients of temperature at a charging rate of 0.1C and 1C:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$y_4$</th>
<th>$y_3$</th>
<th>$y_2$</th>
<th>$y_1$</th>
<th>$y_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (0.1)</td>
<td>35.13</td>
<td>11.15</td>
<td>35.69</td>
<td>11.99</td>
<td>12.33</td>
</tr>
<tr>
<td>T (1)</td>
<td>260.2</td>
<td>236</td>
<td>235.1</td>
<td>92.29</td>
<td>92.78</td>
</tr>
</tbody>
</table>

### Fig. 5. Regression fit of Battery Temperature variations to Charge input at charge rates of 0.1/1C.

The battery temperature can be computed based on charge rate (CR) and charge input (CI) obtained from Eq. (10) and (12). For this, a non-linear regression fit was adopted for the battery temperature-charge input characteristics [26] and a nominal estimation fit devised in this study for different charge rates is depicted in Figure 5. The corresponding regression equations are estimated through battery temperature as a function of charge input at various charge rates and they are expressed by,

$$T_{(0.1)} = y_4 e^{CI} - y_3 - y_2 CI - y_1 CI^2 - y_0 CI^2 + 5$$  \hspace{1cm} (27)

$$T_{(1)} = y_4 e^{CI} - y_3 - y_2 CI - y_1 CI^2 - y_0 CI^2 + 5$$  \hspace{1cm} (28)

The appropriate polynomial coefficients that facilitate the optimal convergence associated with the battery temperature equations are presented in Table 4. From the battery temperature-charge input characteristics [26] shown in Figure 5, the charge rate can be related with the battery temperature at a specified charge input. Thus for a charge input at 1.2 units, the effective battery temperature can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified charge rate (0.1C and 1C) and the effective battery temperature for which the battery it operates.

$$P_{(0.1)} = -x_5 CI^3 + x_4 CI^4 - x_3 CI^5 + x_2 CI^6 - x_1 CI + x_0$$  \hspace{1cm} (31)

$$P_{(1)} = -x_5 CI^3 + x_4 CI^4 - x_3 CI^5 + x_2 CI^6 - x_1 CI^7 + x_0$$  \hspace{1cm} (32)

$$P_{(0)} = -x_5 CI^3 + x_4 CI^4 - x_3 CI^5 + x_2 CI^6 - x_1 CI + x_0$$  \hspace{1cm} (33)
The appropriate polynomial coefficients that facilitate the optimal convergence associated with the battery pressure equations are presented in Table 5.

### Table 5: Polynomial coefficients of pressure at a charging rate of 0.33C, 2C and 6C:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Coefficient</th>
<th>Coefficient</th>
<th>Coefficient</th>
<th>Coefficient</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (0.33)</td>
<td>6.908</td>
<td>20.49</td>
<td>19.86</td>
<td>7.608</td>
<td>0.6954</td>
<td>0.05701</td>
</tr>
<tr>
<td>P (2)</td>
<td>542.4</td>
<td>3362</td>
<td>8572</td>
<td>1.1614</td>
<td>9008</td>
<td>4049</td>
</tr>
<tr>
<td></td>
<td>1015</td>
<td>128</td>
<td>6.142</td>
<td>0.09634</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P (6)</td>
<td>58.37</td>
<td>229.6</td>
<td>590.6</td>
<td>547.4</td>
<td>234.9</td>
<td>47.62</td>
</tr>
<tr>
<td></td>
<td>3.38</td>
<td>0.8806</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 6: Regression fit of Battery Pressure variations to Charge input at charge rates of 0.33/2/6C.](image)

From the battery pressure-charge input characteristics [26] shown in Figure 6, the charge rate can be related with the battery pressure at a specified charge input. Thus for a charge input at 1.4 units, the effective battery pressure can be computed and approximated based on the following quadratic polynomial equation acquired by the interpolation method that relates the specified charge rate (0.33C, 2C and 6C) and the effective battery pressure for which the battery it operates.

$$ P = a C_r^2 + b C_r + c $$  (34)

The charge rate coefficients a, b and c can be determined in terms of battery pressure at the specified charge rates by the way of deriving and solving the 3 sets of battery pressure equations with different charge rates of 0.33C, 2C and 6C. Thus, the battery pressure is given by,

$$ P = \left(0.1056 P_{0.33} - 0.1497 P + 0.0441 P_r\right) C_r^2 + \left(-0.845 P_{0.33} + 0.948 P - 0.1027 P_k\right) C_r + \left(1.268 P_{0.33} - 0.2976 P + 0.0297 P_k\right) $$  (35)

Using the dynamic equations developed in this section from Eq. (31) to (35), the influence on battery pressure can be modeled and examined for its charging conditions.

### IV. Ni-MH Battery System and Benchmark Study

For the experimental assessment, the significant parameters obtained from Panasonic battery system are adopted to rationalize the Multi-Regression model developed. A brief system description about the Ni-MH battery pack accounting its specifications is discussed in the subsequent section.

### A. Battery System Description:

The Panasonic Ni-MH 1.2V, 6.5Ah Panasonic HHR650D battery combines the power and suitability of an alkaline battery by means of the environmental concern and cost saving benefits of a re-chargeable battery. Moreover, the Ni-MH battery technology guarantees for its long operating life and optimal performance. Since the heat formation takes place inside the battery during charging phase, it inherently initiates the pressure variations inside the system that leads to venting of the battery. Thus the proposed regression model accounts not only the charging and discharging voltage behaviour and also includes the temperature and pressure dynamics modelling. The specification and characteristics of a Ni-MH battery is acquired from [10] and [21] for developing the proposed multi-regression model. The completely charged voltage of the battery is maintained at 1.39V with an internal cell resistance of 0.002 ohms. The maximum capacity of the battery is 7Ah attained for a charging current of 1.3A in a time period of 5.38hour. However, the safe operating limit of the battery capacity lies between 6.25Ah and 1.3Ah for a cell voltage of 1.18V and 1.28V respectively.

### B. Benchmark study

Several simulation and its experimental studies were available in the literature for extracting the static and dynamic behaviour of a Panasonic Ni-MH battery system. Benchmark data obtained from [21] was adopted to illustrate the validity of the developed Multi-Regression model of the 6.5Ah Panasonic battery. The battery voltage and discharging time obtained from them were the significant variables used for the comparative evaluation with the simulated results obtained from the regression model.

### V. Dynamic Battery Model Implementation

A multi-regression dynamic model for a 6.5Ah, 1.2V Nickel-Metal Hydride battery is established in MATLAB/Simulink platform, based on regression equations and other battery fundamental equations related with SoC dynamics, charging and discharging voltage dynamics, temperature and pressure dynamics of the battery as discussed earlier. The significant objective of this proposed MRA model is to serve as a benchmark resource in providing battery characteristics viz., SoC, voltage, temperature and pressure dynamics for any loading effect. For any battery request current in a specified time scale, the charge rate and charge input are determined that decides the SoC and charging/discharging voltage of the battery. The Ni-MH battery regression model is established completely based on the regression equations presented in Eq. (5) to (35) that are used to evaluate the charge efficiency, SoC, charging/discharging voltage, Temperature and pressure dynamics. To estimate these dynamics, the charge rate is computed using Eq. (10), and the charge input is obtained using Eq. (12) based on the battery current and its capacity. To determine the available capacity Eq. (14) of the battery, the discharge rate obtained using Eq. (15) is used. From this, the SoC rise and its fall can be determined using Eq. (11) and (13). Ultimately, the actual State of Charge can be found from the initial SoC using Eq. (16) used to describe the battery.
capacity in terms of percentage of its maximum capacity. Thus a structured way of representing all the equations described above as a battery model in the MATLAB/Simulink environment provides a detailed description in demonstrating and analyzing the battery characteristics.

VI. MODEL VALIDATION AND RESULT ANALYSIS
In this section, the developed battery model based on multiple linear regression analysis is verified and validated with the experimental data of Ni-MH battery [21] in order to verify the accuracy of the proposed regression model. For simulation purpose, a series of test data from the manufacturer data sheet [21] corresponding to a Panasonic battery of a single 1.2V, 6.5Ah cell is used. Firstly, the simulation results of the developed regression model is obtained for the standard operating conditions and the acquired profile of cell voltage is validated with the experimental results available in the open literature [21]. Secondly, the detailed characterization of Ni-MH battery for its dynamic behavior is examined numerically based on the models previously explained. The performance of the regression based battery model is studied in terms of its voltage response, temperature and pressure variations.

A. Steady-state behavior of the battery:
The proposed regression based battery model is developed using MATLAB/Simulink environment for a 6.5Ah, 1.2V Nickel-Metal Hydride battery cell. To validate the developed model, the experimental data of Panasonic battery with a specification of 6.5Ah was used [21] in this initial model assessment. The performance of the single cell battery model in terms of its discharge voltage is shown in Figure 7.

Fig. 7. Model validation of the Battery Voltage Response for the proposed model of a single cell at 0.2 C (1.3A)
A completely charged single cell battery at around 1.4V is discharged with a constant discharge current of 1300mA at a 0.2 Coulomb discharge rate. Results obtained in this analysis are for a discharging condition of 6.5Ah Ni-MH battery at an ambient temperature of 25 degree Celsius. The Model validation of the battery voltage response for the proposed single cell regression model that has been carried out for a discharge rate of 0.2 C (1.3A) shows its compatibility with the experimental findings provided by the manufacturer. For analyzing the detailed characterization of a Ni-MH battery model developed using a set of nonlinear equations, the measurement of a single parameter is sufficient to state the validity of the proposed regression model and hence the discharge voltage. The discharge voltage is captured by a constant current of 1300mA over a time period of 3.5hrs that corresponds to 12600 seconds. The predicted discharge voltage of the regression model has good agreement with the manufacturer data as shown in Figure 7. The MLRA model developed can predict the performance of the Ni-MH battery cell over a fairly large range of discharging voltages corresponding to the discharge time as high as up to 3.5hrs.

B. Battery characterization
To study the characterization of the developed battery model, a simplified battery pack consisting of two battery cells connected in series with each cell having a specification of 1.2V, 6.5Ah forming a 2.4V, 6.5Ah pack is considered with an initial SOC of 0.8. Characterization of the battery pack includes the measurement of SoC change, charging/discharging voltage and current. Then, the obtained results are used to verify the validity of the multiple linear regression analysis proposed for the developed model in terms of battery temperature and pressure variations.

Fig. 8. SoC and Battery Voltage Response to Dynamic Profile of charging and discharging current between 10A and -10A
A time varying load profile was developed that switches between 10A and -10A representing the discharging and charging current of 1.53C respectively. The dynamic simulation results for the load profile applied to the developed battery pack are shown in Figure 8. It shows the battery pack SOC variations, current profile with its battery pack voltage over a simulation run of 3500seconds for few on and off cycles where each current pulse has 500 seconds on time and 500 seconds off time. From the results shown in Figure 8, it is clear that the SOC varies at a constant gradient between maximum and minimum value that corresponds to charging and discharging of battery. It is obvious that for a falling current (1.53C discharge), the batteries are being charged and for a rising current (1.53C charge), the batteries are being discharged. Also, it is evident that the battery voltage rises for a falling current and it falls for a rising current to maintain the constant output power delivered to the load. Thus the proposed regression model accurately captures the SOC variations and battery voltage for a continuous charging and discharging current.
C. Dynamic model of the Ni-MH battery:
The temperature changes incurred inside the battery pack for the applied load profile of charging and discharging current of 10A is shown in Figure 9. It is clearly seen that the rise in temperature is from 24°C to 28°C over a period of 3000 seconds and this increase in temperature happens only at the time of charging while it remains constant during the discharging period. The constant temperature at the time of discharging is due to the reason that the Ni-MH battery regression model is developed only for charging condition and omitted for the discharging condition. This is due to the fact that the discharging of battery has inefficient effect over the rise in temperature inside the battery pack and hence the battery temperature is held constant during this period.

![Fig. 9. Battery Temperature variations for the Dynamic Profile of charging and discharging current between 10A and -10A](image)

As like temperature dynamics, the pressure dynamics are considered only for the period of charging and not during discharging period as the pressure variations are predominant only during charging period than the discharging period. This is due to the fact that the formation of oxygen gas contributes more for the pressure variations that occurs only during charging and overcharging period. This can be seen from the Figure 10 that the pressure profile responds only during the charging period and it remains constant during discharging. The pressure dynamics presented in Figure 10 is in terms of percentage variation since the magnitude of the actual pressure variations (atm) are considered to be very less as it will vary only when the battery is charged beyond a charge input of 100% and it is clearly seen in Figure 6. In addition, the dynamic behavior of battery pressure is quite negligible in the present analysis as the battery is subjected to a charging and discharging rate of 1.53C (10A). Further, it can be seen that the battery pressure tends to increase only after a definite period during the charging period where the formation rate of the oxygen gas at the Ni electrode starts to exceed the recombination rate of the oxygen that takes place at the MH electrode [10]. In addition to the model validation, simulation studies were done and analyzed in this section to examine the effects of charging and discharging current over the battery voltage, state of charge, temperature and pressure variations that takes place in the battery pack. Nevertheless, the factors considered like temperature and pressure have not been validated with any of the experimental data as these disputes are ignored by most of the researchers in their work. Further these issues are comparatively a new area of challenge and investigation in the characterization of Ni-MH battery. From this point of view, the developed regression model of Ni-MH battery is well suitable for any kind of model-based analysis and also the simulation results shows that the multiple linear regression analysis could be effectively adopted for any kind of battery based technologies.

VII. CONCLUSION
In the present study, a novel Multi-Regression Analysis (MLRA) model is proposed for the detailed characterization of a 6.5Ah, 1.2V Ni-MH battery to investigate its dynamic behavior. Multi-Regression model developed here is for fitting complicated profiles of Ni-MH battery such as the experimental charging voltage curve, discharging voltage curve, temperature and pressure curves. The fitting profile with the least root mean square error is selected as the best fit that fits the actual data.Higher order polynomial and rational fits are preferred for the most cases to obtain the regression equation model that provides the curve suitably matched with the experimental data. Systematic procedure is carefully followed and developed for implementing the battery model that governs the state of charge, charge input. Depth of discharge and charge efficiency, as well as their temperature and pressure dependence. Finally, an extensive validation of cell voltage is done against the benchmark data to illustrate the ability of the regression model for its accurate prediction. Besides the steady state performance, characterization of the battery model also includes the dynamic load behavior in terms of charging and discharging voltage with its temperature and pressure dynamics. In summary, a regression model using a higher order polynomial curve fit equation is recommended to predict the dynamic behavior of the Ni-MH battery because such a curve fit provides the sufficient accuracy needed within a valid operating range.

REFERENCES
Numerical Study on the Detailed Characterization of Ni-MH Battery Model for its Dynamic Behavior using Multi-Regression Analysis - MRA


