Qualitative and Quantitative Approaches in Dynamics of Two Different Prey-Predator Systems

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***Abstract:** This paper describes the dynamical behavior of two different systems consisting of two preys and a predator. It also deals with the stability of tri-species community in the systems by means of both qualitative and quantitative approaches. The existence and local stability of the equilibrium points of the systems were analyzed. Harvesting activity in both prey and predator populations plays a significant role in controlling the spread of disease.*

***Keywords**: Prey-Predator system, Qualitative stability, Iteration matrix, Quantitative stability, Harvesting Activity.*

I. INTRODUCTION

Complex systems often involve both qualitative aspects and quantitative aspects that can be modelled by logical and Dynamical Systems Theory based approaches respectively [12]. This paper shows an integrative approach to investigate complex systems, combining quantitative, numerical and qualitative aspects. When species interact, the population dynamics of each species is affected. In general, there is a whole web of interacting species, sometimes called a trophic web, which makes for structurally complex communities. The harvesting of population is commonly practiced in various biological resources (wild life, Fishery, etc.) [5]. Many people have made numerical analysis of stability conditions of specific multispecies communities. These works have considered the actual magnitudes of the interactions between species in the system. This is a quantitative approach. Sometimes, we can also make deductions regarding the stability of multispecies communities, knowing only the structure of the food web. They are useful in suggesting the general tendencies of the species communities. This is a qualitative approach. This paper describes the complex behavior of the interaction between two preys (both, not infected) and a predator. Also, in this paper, we analyze a system consisting of a susceptible and an infected part of a prey population and a predator. Both prey types are included in predator's diet and thereby harvesting activity is being carried out in each species. We investigate the stability of two different prey-predator systems using both qualitative and quantitative stability conditions.

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II. QUALITATIVE APPROACH

Food webs summarize the interactions among species in natural communities. Model structure is described by an iteration matrix and for n species, this matrix has n^2 elements. This type of interaction between species sets the sign of the non-zero elements in the iteration matrix [11]. The sign structure of this $n \times n$ matrix is obtained from Odum's scheme which specifies the interaction between species using signs (+, -, or 0). He emphasize that the effect of species j upon species i as positive (+), neutral (0), or negative (-) according as whether the population of species i is increased, is unaffected or is decreased by the presence of species j. R.M. May used community matrices in which species were randomly linked with random interaction strength to show that the local stability decreases with complexity, diversity and average interaction strength among the species [14,15].

Qualitative Stability Conditions: Mathematically, the necessary and sufficient conditions for the existence of qualitative stability [13] in an $n \times n$ matrix, A are

i) $a_{ii} \leq 0$ for all i.
ii) $a_{ii} \neq 0$ for at least one i.
iii) the product $a_{ij}\, a_{ji} \leq 0$ for all $i \neq j$.
iv) For any sequence of three or more indices, i, j, k, …..,q, r (with $i \neq j \neq k \neq \ldots\ldots \neq q \neq r$), the product $a_{ij} \,.\, a_{jk} \ldots\ldots a_{qr} \,.\, a_{ri} = 0$.
v) The determinant of the matrix, $\det A \neq 0$.

If the conditions (i) – (v) are not satisfied, then it does not imply that the matrix is unstable, but rather that a complete knowledge about the magnitude of the interaction coefficients is needed.

III. QUANTITATIVE APPROACH

Lotka and Volterra described the interactions between two species in an ecosystem, a predator and a prey [6], [19]. After them, Holling suggested three kinds of functional responses for different species to model the different phenomena of predation [2]. Many people have constructed various models for studying the effect of infectious diseases in predator – prey system [9], [10], [16]-[18]. Dubey. B. *et. al.* [3], [20] have studied the effect of harvesting the prey or the predator on the coexistence of both populations. Mathematical models have been used extensively successfully to gain insight into scientific management of renewable sources [7], [8]. Existing models for the dynamics of the population are often based on quantitative, numerical methods such as Dynamical Systems Theory (DST) [1], [12]. Such approaches often use numerical variables to describe global aspects and indicate how they affect each other over time; for example, how the number of predators affects the number of prey species. An advantage of such methods is that numerical approximation methods and software environments are available for Simulation.

IV. ILLUSTRATIONS

This section deals with the complex behavior and local stability with regard to two different predator-prey systems.

Case :1 In this case, we study the interaction between three species (two preys and a predator) living in a closed environment, striving for survival. **It** is assumed that there is an unlimited supply of food to the prey species; there is no threat to the preys other than the specific predator and the predator is totally dependent on the two prey species as its food supply. Also, assume that all the three species have density dependent intraspecific interactions. The Food Web for this system is as follows:

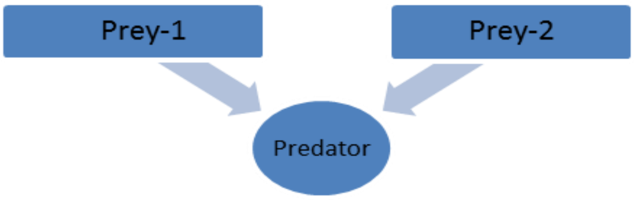

Fig.1

The iteration or community matrix for the tri-species community represented by Fig.1 is described as below.

$$A = \begin{bmatrix} - & + & + \\ - & - & 0 \\ - & 0 & - \end{bmatrix}$$

It is clear that the matrix A satisfies the qualitative stability conditions (i) – (v), and hence is qualitatively stable.

A. The Model - I

In this section, we shall study the dynamical behavior of two preys and a predator system. Here, we use the basic model known as Lotka - Volterra model. Holling type-I functional response is taken to represent the interaction between the two preys and a predator.

$$\frac{dx_1}{dt} = ax_1\left(1 - \frac{a}{k_1}\right) - p_1x_1y$$

$$\frac{dx_2}{dt} = bx_2\left(1 - \frac{b}{k_2}\right) - p_2x_2y \qquad (1)$$

$$\frac{dy}{dt} = e_1p_1x_1y + e_2p_2x_2y - dy$$

Where $x_1(t)$, $x_2(t)$ denote the prey densities, $y(t)$ represents the density of predator population, a,b are the intrinsic growth rates of two preys, k_1, k_2 are their respective carrying capacities, p_1, p_2 are the efficiencies of the predator among the preys, d is the natural death rate of the predator, e_1, e_2 are the conversion rates, the parameters $a > 0$, $b > 0$, $k_1 > 0$, $k_2 > 0$, $p_1 > 0$, $p_2 > 0$, $e_1 > 0$, $e_2 > 0$ and $d > 0$.

B. Equilibrium points of the System

In this section, we study the existence of the equilibrium points of the system. The system (1) has the following non-negative equilibrium points:

i) $E_0 = (0, 0, 0)$

ii) $E_1 = \left(\frac{d}{e_1p_1}, 0, \frac{a(k_1-a)}{p_1k_1}\right)$

iii) $E_2 = \left(0, \frac{d}{e_2p_2}, \frac{b(k_2-b)}{p_2k_2}\right)$

iv) $E_3 = (x_1^*, x_2^*, y^*)$,

Where $x_1^* = \frac{d - e_2p_2x_2^*}{e_1p_1}$, $x_2^* =$

$$\frac{adk_1k_2 - a^2dk_2 - p_1k_1k_2dy}{ae_2p_2k_1k_2 - a^2e_2p_2k_2 - e_2p_1p_2k_1k_2y + e_1p_1k_1k_2b - e_1p_1k_1b^2 - e_1p_1k_1k_2p_2},$$

and $y^* = \frac{k_1k_2(b-a) - k_1b^2 + k_2a^2}{k_1k_2(p_2 - p_1)}$.

C. Linear Stability Analysis

At this stage, we analyze the local behavior of the model (1) around each fixed point. The Jacobian matrix of the model at state variable is as follows:

$$J = \begin{bmatrix} a\left(1 - \frac{a}{k_1}\right) - p_1y & 0 & -p_1x_1 \\ 0 & b\left(1 - \frac{b}{k_2}\right) - p_2y & -p_2x_2 \\ e_1p_1y & e_2p_2y & -d \end{bmatrix}$$

One of the most useful techniques for analyzing non-linear system quantitatively is the linearized stability technique.

Theorem 1:

Let $p(\lambda) = \lambda^3 + B\lambda^2 + C\lambda + D$. There are at most three roots of the equation $p(\lambda) = 0$. Then the following statements are true:

a) If every root of the equation has absolute value less than one, then the fixed point of the system is locally asymptotically stable and fixed point is called a sink.
b) If at-least one of the roots of the equation has absolute value greater than one, then the fixed point of the system is unstable and fixed point is called saddle.
c) If every root of the equation has absolute value greater than one, then the system is source.
d) The fixed point of the system is called hyperbolic if no root of the equation has absolute value equal to one. If there exists a root of the equation with absolute value equal to one, then the fixed point is called non-hyperbolic.

D. Dynamic Behavior of the Model

In this section, we discuss the local stability around each fixed point of model (1).

Preposition 1: The equilibrium point E_0 of system (1) is locally asymptotically stable if $ak_1 - a^2 < k_1$ and $bk_2 - b^2 < k_2$.

Proof: The Jacobian matrix $J(E_0)$ at the equilibrium point E_0 is

$$J(E_0) = \begin{bmatrix} a\left(1-\frac{a}{k_1}\right) & 0 & 0 \\ 0 & b\left(1-\frac{b}{k_2}\right) & 0 \\ 0 & 0 & -d \end{bmatrix}$$

The eigen values of the matrix J(E_0) are $\lambda_1 = a\left(1-\frac{a}{k_1}\right), \lambda_2 = b\left(1-\frac{b}{k_2}\right)$, and λ_3 = - d. By theorem 1, E_0 is locally asymptotically stable if ak_1 - a^2 < k_1 and bk_2 - b^2 < k_2.

Preposition 2: The equilibrium point E_1 of system (1) is locally asymptotically stable if

$p_1 k_1 (b(k_2 - b) - 1) < a p_2 k_2 (k_1 - a)$.

Proof: The Jacobian matrix J(E_1) at the equilibrium point E_1 is

$$J(E_1) = \begin{bmatrix} 0 & 0 & \frac{-d}{e_1} \\ 0 & \frac{p_1k_1b(k_2-b)-ap_2k_2(k_1-a)}{p_1k_1} & 0 \\ \frac{e_1a(k_1-a)}{k_1} & \frac{e_2p_2a(k_1-a)}{p_1k_1} & -d \end{bmatrix}$$

The eigen values of the matrix J(E_1) are $\lambda_1 = 0, \lambda_2 = \frac{p_1k_1b(k_2-b)-ap_2k_2(k_1-a)}{p_1k_1}$ and λ_3= -d.

By theorem 1, E_1 is locally asymptotically stable if $p_1 k_1 (b(k_2 - b) - 1) < a p_2 k_2 (k_1 - a)$.

Preposition 3: The equilibrium point E_2 of system (1) is locally asymptotically stable if

$p_2 k_2 (a(k_1 - a) - 1) < b p_1 k_1 (k_2 - b)$.

Proof: The Jacobian matrix J(E_2) at the equilibrium point E_2 is

$$J(E_2) = \begin{bmatrix} a\left(1-\frac{a}{k_1}\right) - \frac{p_1b(k_2-b)}{p_2k_2} & 0 & 0 \\ 0 & 0 & \frac{-d}{e_2} \\ \frac{e_1p_1b(k_2-b)}{p_2k_2} & \frac{e_2b(k_2-b)}{k_2} & 0 \end{bmatrix}$$

The eigen values of the matrix J(E_2) are

$\lambda_1 = \frac{p_2k_2a(k_1-a)-p_1k_1b(k_2-b)}{p_2k_2}, \lambda_2 = 0$, and λ_3= -d.

By theorem 1, E_2 is locally asymptotically stable if $p_2 k_2 (a(k_1 - a) - 1) < b p_1 k_1 (k_2 - b)$.

Preposition 4: The equilibrium point $E_3(x^*, y^*, z^*)$ of system (1) is locally asymptotically stable, if

$a\left(1-\frac{a}{k_1}\right) < p_1 y^*$ and $b\left(1-\frac{b}{k_2}\right) < p_2 y^*$.

Proof: The Jacobian matrix J(E_3) at the equilibrium point E_3 is

$$J(E_3) = \begin{bmatrix} a\left(1-\frac{a}{k_1}\right) - p_1 y^* & 0 & -p_1 x_1^* \\ 0 & b\left(1-\frac{b}{k_2}\right) - p_2 y^* & -p_2 x_2^* \\ e_1 p_1 y^* & e_2 p_2 y^* & -d \end{bmatrix}$$

The eigen values of the matrix J(E_3) are $\lambda_1 = a\left(1-\frac{a}{k_1}\right) - p_1 y^*, \lambda_2 = b\left(1-\frac{b}{k_2}\right) - p_2 y^*$, and λ_3= -d, where $y^* = \frac{k_1k_2(b-a) - k_1b^2 + k_2a^2}{k_1k_2(p_2 - p_1)}$. By theorem 1, it follows that E_3 is stable only when $a\left(1-\frac{a}{k_1}\right) < p_1 y^*$ and $b\left(1-\frac{b}{k_2}\right) < p_2 y^*$.

Case :2 In this case, we consider a system consisting of a susceptible and infected parts of a prey population and a predator. The predator is allowed to consume both types of prey. We consider that the disease spreads among both the prey and the predator species. The food web for this system is as follows.

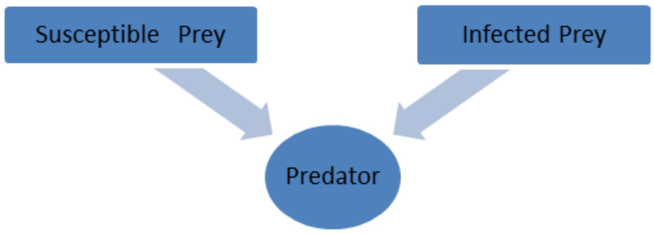

Fig.2

The iteration matrix, B for the system represented by Fig. 2 is given as follows:

$$B = \begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \end{pmatrix}$$

It follows that the matrix, B does not satisfy the condition (iv) of the qualitative stability conditions and hence is not qualitatively stable. But, it may correspond to a stable equilibrium, depending on the actual magnitudes of the various matrix elements. This part is discussed in the following section.

E. The Model - II

It is assumed that the predator population consumes both prey populations. We have considered Holling type-II functional response for the predation of susceptible prey and since infected preys are easier to catch, Holling type-I is chosen for the predation of infected prey. The proposed model is

$$\frac{dx}{dt} = ax\left(1-\frac{a}{k}\right) - Ixy - h_1x - \frac{p_1xz}{1+sx}$$

$$\frac{dy}{dt} = Ixy - p_2yz - h_2y \qquad (2)$$

$$\frac{dz}{dt} = \frac{c_1p_1}{1+sx} + c_2p_2yz - h_3z - dz$$

Where x(t), y(t) are susceptible and infected parts of a prey population; z(t) denotes the density of predator population; **a** represents the intrinsic growth rate of susceptible prey; k denotes their carrying capacity; I represents the rate of infection in prey population; p_1, p_2 are the predation rates among two types of prey; s denotes the half saturation constant; c_1, c_2 are the conversion efficiencies on the preys; h_1, h_2 and h_3 are the harvesting efforts on susceptible prey; infected prey and the predator respectively; d is the natural death rate of predator; the parameters a > 0, k > 0, I > 0, s > 0, $p_1 > 0$, $p_2 > 0$, $h_1 > 0$, $h_2 > 0$, $h_3 > 0$, $c_1 > 0$, $c_2 > 0$ and d > 0.

F. *Fixed points of the System*

In this section, we study the local stability of the equilibrium points of the system (2). we list the following non- negative fixed points:

i) $E_0 = (0, 0, 0)$

ii) $E_1 = (\frac{h_2}{I}, 0, 0)$

iii) $E_2 = (0, 0, \frac{ak-a^2-h_1kp_1}{kp_1})$

iv) $E_3 = (0, \frac{h_3+d}{c_2p_2}, \frac{a}{p_1}\left(1-\frac{a}{k}\right) - \frac{I(h_3+d)}{c_2p_1p_2} - \frac{h_1}{p_1})$

v) $E_4 = (x^*, y^*, z^*)$

Where $x^* = \frac{p_2z^*+h_2}{I}$,

$$y^* = \frac{(h_3+d)(I+sx^*)-c_1p_1(p_2z^*+h_2)}{c_2p_2I(1+sx^*)} \text{ and}$$

$$z^* = \frac{Ic_1p_1h_2+(I+sh_2)[c_2p_2\left(a\left(1-\frac{a}{k}\right)-h_1\right]+I(h_3+d)}{s(p_2+h_2)\left[I(h_3+d)-c_2p_2(a(1-\frac{a}{k})-h_1)\right)-Ip_1p_2(c_1-c_2)}.$$

G. *Linear Stability Analysis*

In this section, we analyze the local behavior of the model (2) around each fixed point. The Jacobian matrix, J of the model (2) at state variable is as follows:

$$\begin{bmatrix} a\left(1-\frac{a}{k}\right) - Iy - h_1 - \frac{p_1z}{(1+sx)^2} & -Ix & -\frac{p_1x}{(1+sx)^2} \\ Iy & Ix - p_2z - h_2 & -p_2y \\ \frac{c_1p_1z}{(1+sx)^2} & c_2p_2z & \frac{c_1p_1x}{1+sx} + c_2p_2y - h_3 - d \end{bmatrix}$$

Theorem: 2

If the eigen values of the Jacobian matrix of the fixed point are inside the unit circle of the complex plane, then the fixed point is locally stable. Using Jury's condition, we have the necessary and sufficient condition for the eigen values $|\lambda_{1,2}|$ < 1 as follows:

i) 1 + Tr (J) + Det (J) >0

ii) 1 – Tr (J) + Det (J) > 0

iii) $|Det\,(J)|$ < 1

H. *Complex Behavior of the Model*

In this section, we discuss the local stability around each fixed point of model (2).

Preposition 5: The equilibrium point E_0 of system (2) is locally asymptotically stable only when $a(1-\frac{a}{k}) < 1+h_1$, otherwise unstable.

Proof: The Jacobian matrix $J(E_0)$ at the equilibrium point E_0 is

$$J(E_0) = \begin{bmatrix} a\left(1-\frac{a}{k}\right) - h_1 & 0 & 0 \\ 0 & -h_2 & 0 \\ 0 & 0 & -h_3-d \end{bmatrix}$$

The eigen values of the matrix $J(E_0)$ are

$\lambda_1 = a\left(1-\frac{a}{k}\right) - h_1, \lambda_2 = -h_2$ and $\lambda_3 = -h_3 - d$.

By theorem (1), E_0 is locally asymptotically stable if $a(1-\frac{a}{k}) < 1+h_1$.

Preposition 6: The equilibrium point E_1 of system (2) is locally asymptotically stable, if

$-1 < [\left(1-\frac{a}{k}\right) + \frac{c_1p_1h_2}{I+sh_2} - h_3 - d] < 1.$

Proof: The Jacobian matrix $J(E_1)$ at the equilibrium point E_1 is

$$J(E_1) = \begin{bmatrix} a\left(1-\frac{a}{k}\right) - \frac{h_1h_2}{I} & -h_2 & \frac{-p_1h_2I}{I+sh_2} \\ 0 & 0 & 0 \\ 0 & 0 & \frac{c_1p_1h_2}{I+sh_2} - h_3 - d \end{bmatrix}$$

By Jury's condition in theorem (2), it follows that E_1 is locally stable if and only if

$-1 < [\left(1-\frac{a}{k}\right) + \frac{c_1p_1h_2}{I+sh_2} - h_3 - d] < 1.$

Preposition 7: The equilibrium point E_2 of system (2) is locally asymptotically stable, if $h_1(p_1-1) < 1$, otherwise unstable.

Proof: The Jacobian matrix $J(E_2)$ at the equilibrium point E_2 is

$$J(E_2) = \begin{bmatrix} a\left(1-\frac{a}{k}\right) - h_1 - \frac{ak-a^2-hkp_1}{k} & 0 & 0 \\ 0 & \frac{-p_2(ak-a^2-hkp_1)}{kp_1} - h_2 & 0 \\ c_1p_1(\frac{ak-a^2-hkp_1}{kp_1}) & c_2p_2(\frac{ak-a^2-hkp_1}{kp_1}) & -h_3-d \end{bmatrix}$$

The eigen values of the matrix $J(E_2)$ are

$\lambda_1 = h_1(p_1-1),\ \lambda_2 = \frac{-p_2(ak-a^2-hkp_1)}{kp_1} - h_2$ and

$-h_3 - d$. By theorem 1, E_2 is locally asymptotically stable if $h_1(p_1-1) < 1$, otherwise unstable.

Dynamic behavior of the system around the Fixed point E_3: In this section, we investigate the stability and bifurcation of E_3. The Jacobian matrix of E_3 is as follows:

$$J(E_3) = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

Where $b_{11} = \left(1-\frac{a}{k}\right) - \frac{I(h_3+d)}{c_2p_2} - h_1 - p_1z^*$, $b_{12} = 0$, $b_{13} = 0$, $b_{21} = \frac{I(h_3+d)}{c_2p_2}$, $b_{22} = -p_2z^* - h_2$, $b_{23} = -\frac{(h_3+d)}{c_2}$, $b_{31} = c_1p_1z^*$, $b_{32} = c_2p_2z^*$, $b_{33} = 0$. The eigen values are $\lambda_1 = b_{11}$ and $\lambda_{2,3} = \frac{R_1 \pm \sqrt{R_1^2-4R_2}}{2}$, where $R_1 = b_{11} + b_{33}$ and $R_2 = b_{11}b_{33} - b_{13}b_{31}$. By theorem (1),

E_3 is locally asymptotically stable if and only if $h_2 > Ix^* - p_2z^*$,

$R_1 + \sqrt{R_1^2 - 4R_2} < 2$ and

$R_1 - \sqrt{R_1^2 - 4R_2} < 2$.

Local Stability of the system around the interior fixed point E_4: The Jacobian matrix of system (2) at the equilibrium point $E_4(x^*, y^*, z^*)$ is given below.

$$J(E_4) = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix}$$

Where $H_{11} = a\left(1 - \frac{a}{k}\right) - Iy^* - h_1 - \frac{p_1 z^*}{(1+sx^*)^2}$, $H_{12} = -Ix^*$, $H_{13} = \frac{-p_1 x^*}{(1+sx^*)^2}$, $H_{21} = Iy^*$, $H_{22} = Ix^* - p_2 z^* - h_2$, $H_{23} = -p_2 y^*$, $H_{31} = \frac{c_1 p_1 z^*}{(1+sx^*)^2}$, $H_{32} = c_2 p_2 z^*$ and $H_{33} = \frac{c_1 p_1 x^*}{1+sx^*} + c_2 p_2 y^* - h_3 - d$.

The characteristic equation of $J(E_4)$ is $\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0$, where $A_1 = -(H_{11} + H_{22} + H_{33})$,
$A_2 = H_{11}H_{22} - H_{12}H_{21} - H_{23}H_{32} + H_{11}H_{33}$ and
$A_3 = H_{11}H_{23}H_{32} - H_{11}H_{22}H_{33} + H_{12}H_{21}$.

According to the Routh-Hurwitz criterion, $E_4(x^*, y^*, z^*)$ is locally asymptotically stable if only if $A_1 > 0$, $A_3 > 0$ and $A_1 A_2 > A_3$.

V. CONCLUSION

In the first case, we have studied the dynamical behavior of two preys and a predator system. We have proved that system (1) described by Fig.1 is qualitatively stable and the system is examined using the techniques of local stability analysis of the equilibrium points. It is clear that a population in the absence of harvesting is free of extinction. In the second case, we have considered a prey-predator system with disease in both populations and harvesting in each species. In order to maintain a healthy population, the infected species were harvested. We have proved that system (2) is not qualitatively stable. In brief, for the trophic web described by Fig.2, we cannot say from the structure of web alone, whether the equilibrium system (if it exists) is stable or not . Various conditions for the stability of the equilibrium points for the three species populations were obtained. Under certain parametric conditions, the interior point of the system enters a bifurcation phenomenon. We may conclude that harvesting of diseased species may be used as a biological control for the spread of diseases.

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