Intuitionistic Fuzzy Tree Center-Based Clustering Algorithm

G. Thamizhendhi, R. Parvathi

Abstract- In this paper, the concepts of distance, eccentricity, radius, diameter and center of an intu-itionistic fuzzy tree are defined. Some of the domination parameters like independent domina-tion, connected domination and total domination on intuitionistic fuzzy trees are investigated. The procedure for intuitionistic fuzzification for numerical data set is proposed. Further, in-tuitionistic fuzzy tree center-based clustering algorithm is designed. The e ectiveness of the algorithm is checked with a numerical dataset and compared with two existing clustering meth-ods.

Keywords: Intuitionistic fuzzy tree, distance, eccentricity, center, connected domination, connec-tivity, clustering.

I. INTRODUCTION

Graph theoretical ideas are highly utilized in data mining, image segmentation, clustering, image processing and networks. Graph theory appears to be very convenient to describe clustering prob-lems. The concept of a tree can be used to design a data structure of a model. The notion of fuzzy sets was introduced by L.A Zadeh as a method of representing uncertainty and vagueness in [19]. The theory of intuitionistic fuzzy sets (IFSs), introduced by Atanassov([1], [2]), is an extension of fuzzy set theory in which, not only membership degree is given, but also non-membership degree, which is more or less independent. Fuzziness and uncertainty in the real world existing informa-tion, the attributes of the data sets are often given with intuitionistic fuzzy sets. Intutionistic fuzzy set is a suitable tool to cope with imperfectly defined facts and data, as well as with imprecise knowledge. A.Rosenfeld introduced and examined such concepts as paths, cycle, trees and con-nectedness in fuzzy graphs [15]. In [10], various types of fuzzy cycles, fuzzy trees in fuzzy graphs defined using level sets. The concept of domination in fuzzy graphs was studied in [16]. R.Parvathi and K.Atanassov [7] defined intuitionistic fuzzy trees using index matrix interpretation. M.Akram and N.O.Alshehri [3] introduced various types of intuitionistic fuzzy trees and investigated some of their properties. Zhang and Chen [23] suggested a clustering technique of IFSs on the basis of the λ-cutting matrix of an interval-valued matrix. Xu and Yager [18] gave a clustering technique by transforming an association matrix into an equivalent association matrix, from which a k-cutting matrix is derived and used to cluster the given IFSs. Cai et al. [6] presented a clustering method based on the intuitionistic fuzzy equivalent dissimilarity matrix and (α, β)-cutting matrices.

Zahn [22] proposed clustering algorithm using the minimal spanning tree (MST). Distance between IFSs is considered to form clusters in [22]. Dong et al. [8] gave a hierarchical clustering algorithm based on fuzzy graph connectedness. H.Zhao et al.[19] developed an intuitionistic fuzzy minimum span-ning tree clustering algorithm to deal with intuitionistic fuzzy information. Hence, intuitionistic fuzzy clustering techniques are based on distance and similarity measure betweem IFSs. In this way, the authors are motivated to concentrate on *intuitionistic fuzzy trees*(IFTs) and their struc-ture and to apply these concepts to design a clustering algorithm. In this paper, distance, radius, diameter and center of intuitionistic fuzzy trees are introduced and their domination properties are analyzed. Also, intuitionistic fuzzy tree center-based clustering algorithm is proposed to cluster the numerical data set. As the existing data in real-life are crisp, S-shaped intuitionistic fuzzification function is used in the proposed method. These values give the membership and non-membership of the vertices of the IFT under consideration. A new distance measure* between two IFSs, is defined and applied it to construct the intuitionistic fuzzy distance matrix. Center of an IFT is obtained by eccentricity concept. On the basis of the (λ, δ)-cutting matrix on distance matrix is used to cluster the given dataset. This algorithm is verified with classification of the numerical data sets containing nutrients in 27 di erent kinds of meat, fish or fowl with five attributes. The developed clustering method is compared with two existing clustering methods namely Zhang et al.[23] and Z.Wang et al.[24].

II. PRELIMINARIES

In this section, some basic definitions relating to intuitionistic fuzzy graphs (IFGs) are given. Also, the definitions of partial spanning subgraph, spanning subgraph, distance, eccentricity, radius, diameter and center of IFTs are given.

Definition 2.1. [1] Let a set E be fixed. An *intuitionistic fuzzy set (IFS)* A in E is an object of the form $A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in E\}$, where the function $\mu_A : E \to [0, 1]$ and $\nu_A : E \to [0, 1]$ determine the degree of membership and the degree of non-membership of the element $x \in E$ respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$, for every $x \in E$.

Notations

1. Hereafter, $\langle \mu_i, \nu_i \rangle$ denotes the degrees of membership and non-membership of the vertex $v_i \in V$ such that $0 \le \mu_i + \nu_i \le 1$.

Revised Version Manuscript Received on February 15, 2016.

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Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

2. $\langle \mu_{ij}, \gamma_{ij} \rangle$ denotes the degrees of membership and non-membership of the edge $(\upsilon_i, \upsilon_j) \in V \times V$ such that $0 \le \mu_{ij} + \gamma_{ij} \le 1$.

Definition 2.2. [16] Let X be a universal set and let V be an IFS over X in the form $V = \{\langle v_i, \mu_i, \gamma_i \rangle | \upsilon_i \in V\}$ such that $0 \le \mu_i + \gamma_i \le 1$. *Six types of cartesian products* of n elements of V over X are defined as

$$\upsilon_1 \times_1 \upsilon_2 \times_1 \upsilon_3 ... \times_1 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \gamma_i\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}$$

$$\upsilon_1 \times_2 \upsilon_2 \times_2 \upsilon_3 ... \times_2 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \sum_{i=1}^{n} \mu_i - \sum_{i \ne j} \mu_i\mu_j + \sum_{i \ne j \ne k} \mu_i\mu_j\mu_k - \cdots + (-1)^{n-2} \sum_{i \ne j \ne k \cdots \ne n} \mu_i\mu_j\mu_k \cdots \mu_n + (-1)^{n-1} \prod_{i=1}^{n} \mu_i, \prod_{i=1}^{n} \gamma_i\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}$$

$$\upsilon_1 \times_3 \upsilon_2 \times_3 \upsilon_3 ... \times_3 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \prod_{i=1}^{n} \mu_i, \sum_{i=1}^{n} \gamma_i - \sum_{i \ne j} \gamma_i\gamma_j + \sum_{i \ne j \ne k} \gamma_i\gamma_j\gamma_k - \cdots + (-1)^{n-2} \sum_{i \ne j \ne k \cdots \ne n} \gamma_i\gamma_j\gamma_k \cdots \gamma_n + (-1)^{n-1} \prod_{i=1}^{n} \gamma_i\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}$$

$$\upsilon_1 \times_4 \upsilon_2 \times_4 \upsilon_3 ... \times_4 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \min(\mu_1, \mu_2, \cdots, \mu_n), \max(\gamma_1, \gamma_2, \cdots, \gamma_n)\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}$$

$$\upsilon_1 \times_5 \upsilon_2 \times_5 \upsilon_3 ... \times_5 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \max(\mu_1, \mu_2, \cdots, \mu_n), \min(\gamma_1, \gamma_2, \cdots, \gamma_n)\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}$$

$$\upsilon_1 \times_6 \upsilon_2 \times_6 \upsilon_3 ... \times_6 \upsilon_n = \{\langle(\upsilon_1, \upsilon_2, \cdots, \upsilon_n), \frac{\sum_{i=1}^{n} \mu_i}{n}, \frac{\sum_{i=1}^{n} \gamma_i}{n}\rangle | \upsilon_1, \upsilon_2, \cdots, \upsilon_n \in V\}.$$

It must be noted that $\upsilon_i \times_t \upsilon_j$ is an IFS, where t = 1, 2, 3, 4, 5, 6 such that the sum of their degrees of membership and non-membership lies in [0, 1].

Definition 2.3. [9] An *intuitionistic fuzzy graph* (IFG) is of the form $G = (V, E)$ where
(i) $V = \{\upsilon_1, \upsilon_2, ..., \upsilon_n\}$, such that $\mu_i : V \to [0, 1]$ and $\gamma_i : V \to [0, 1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively, and $0 \le \mu_i + \gamma_i \le 1$ for every
$\upsilon_i \in V$, $i = 1, 2, \cdots, n$
(ii) $E \subset V \times V$ where $\mu_{ij} : V \times V \to [0, 1]$ and $\gamma_{ij} : V \times V \to [0, 1]$ are such that

$$\mu_{ij} \le \mu_i \oslash \mu_j,$$

$$\gamma_{ij} \le \gamma_i \oslash \gamma_j$$

and

$$0 \le \mu_{ij} + \gamma_{ij} \le 1$$

where μ_{ij} and γ_{ij} are the degrees of membership and non-membership of the edge (υ_i, υ_j); the values $\mu_i \oslash \mu_j$ and $\gamma_i \oslash \gamma_j$ can be determined by one of the six cartesian products $\times_t$, t = 1, 2, 3, 4, 5, 6 for all i and j given in Definition 2.2.

Definition 2.4. An IFG, $H = (V', E')$ is said to be a *partial intuitionistic fuzzy subgraph* of $G = (V, E)$ if

(i) $V' \subset V$, $\mu'_i \le \mu_i$, $\gamma'_i \le \gamma_i$ for all $v_i \in V'$, $i = 1, 2, \ldots n$.

(ii) $E' \subset E$, $\mu'_{ij} \le \mu_{ij}$, $\gamma'_{ij} \le \gamma_{ij}$ for all $(\upsilon_i, \upsilon_i) \in E'$, $i, j = 1, 2, \ldots n$.

Definition 2.5. [9] An IFG, $H = (V', E')$ is said to be an *intuitionistic fuzzy subgraph* of $G = (V, E)$ if

(i) $V' \subset V$, $\mu'_i = \mu_i$, $\gamma'_i = \gamma_i$ for all $\upsilon_i \in V'$, $i = 1, 2, \ldots n$.

(ii) $E' \subset E$, $\mu'_{ij} = \mu_{ij}$, $\gamma'_{ij} = \gamma_{ij}$ for all $(\upsilon_i, \upsilon_i) \in E'$, $i, j = 1, 2, \ldots n$.

Definition 2.6. An IFG, $H = (V', E')$ is said to be a *partial intuitionistic fuzzy spanning subgraph* of $G = (V, E)$ if

(i) $V' = V$, $\mu'_i \le \mu_i$, $\gamma'_i \le \gamma_i$ for all $v_i \in V'$, $i = 1, 2, \ldots n$.

(ii) $E' \subset E$, $\mu'_{ij} \le \mu_{ij}$, $\gamma'_{ij} \le \gamma_{ij}$ for all $(\upsilon_i, \upsilon_i) \in E'$, $i, j = 1, 2, \ldots n$.

Definition 2.7. An IFG, $H = (V', E')$ is said to be an *intuitionistic fuzzy spanning subgraph*(IFSSG) of $G = (V, E)$ if

(i) $V' = V$, $\mu'_i = \mu_i$, $\gamma'_i = \gamma_i$ for all $\upsilon_i \in V'$, $i = 1, 2, \ldots n$.

(ii) $E' \subset E$, $\mu'_{ij} = \mu_{ij}$, $\gamma'_{ij} = \gamma_{ij}$ for all $(\upsilon_i, \upsilon_i) \in E'$, $i, j = 1, 2, \ldots n$.

Definition 2.8. [11] Let $G = (V, E)$ be an IFG, then the *cardinality* of a subset S of V is defined as $|S| = \sum_{v_i \in S} \left(\frac{1 + \mu_i - \gamma_i}{2} \right)$ for all $v_i \in S$.

Definition 2.9. [11] The number of vertices in G is called as *order* of an IFG, $G = (V, E)$, denoted by $o(G)$, and is defined as $o(G) = \sum_{v_i \in V} \left(\frac{1 + \mu_i - \gamma_i}{2} \right)$ for all $v_i \in V$.

Definition 2.10. [11] An IFG, $G = (V, E)$ is said to be *complete IFG* if $\mu_{ij} = \min(\mu_i, \mu_j)$ and $\gamma_{ij} = \max(\gamma_i, \gamma_j)$ for every $v_i, v_j \in V$.

Definition 2.11. [10] If $v_i, v_j \in V \subseteq G$, the μ-strength of connectedness between v_i and v_j is $\mu_{ij}^{\infty} = \sup\{\mu_{ij}^k \mid k = 1, 2, \ldots, n\}$ and γ-strength of connectedness between v_i and v_j is $\gamma_{ij}^{\infty} = \inf\{\gamma_{ij}^k \mid k = 1, 2, \ldots, n\}$.
If v_i, v_j are connected by means of paths of length k then μ_{ij}^k is defined as $\sup\{\mu_{i1} \wedge \mu_{12} \wedge \mu_{23} \ldots \wedge \mu_{k-1j} \mid v_i, v_1, v_2 \ldots v_{k-1}, v_j \in V\}$ and γ_{ij}^k is defined as $\inf\{\gamma_{i1} \vee \gamma_{12} \vee \gamma_{23} \ldots \vee \gamma_{k-1j} \mid v_i, v_1, v_2 \ldots v_{k-1}, v_j \in V\}$

Definition 2.12. [10] An edge (υ_i, υ_j) is said to be a *strong edge* of an IFG $G = (V, E)$, if $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\gamma_{ij} \geq \gamma_{ij}^{\infty}$.

Definition 2.13. [16] An IFG, $G = (V, E)$ is said to be *connected* IFG if there exists a path between every pair of vertices υ_i, υ_j in V. Connected IFG is also defined using strength of connectedness as follows:

(i) $\mu_{ij}^{\infty} > 0$, and $\gamma_{ij}^{\infty} > 0$

(ii) $\mu_{ij}^{\infty} = 0$, and $\gamma_{ij}^{\infty} > 0$

(iii) $\mu_{ij}^{\infty} > 0$, and $\gamma_{ij}^{\infty} = 0$ for all υ_i, $\upsilon_j \in V$.

Definition 2.14. [17] An IFG, $G = (V, E)$ is said to be *intuitionistic fuzzy forest* (IFF), if it has an intuitionistic fuzzy spanning subgraph $H = (V', E')$, which is a forest (in crisp sense), where for all edges (υ_i, υ_j) not in H, $\mu_{ij} < \mu_{ij}'^{\infty}$ and $\gamma_{ij} > \gamma_{ij}'^{\infty}$.

Definition 2.15. [17] An connected IFF, $G = (V, E)$ is said to be *intuitionistic fuzzy tree* (IFT) if it has an intuitionistic fuzzy spanning subgraph $H = (V', E')$, which is a tree (in crisp sense), where for all edges (υ_i, υ_j) not in H, $\mu_{ij} < \mu_{ij}'^{\infty}$ and $\gamma_{ij} > \gamma_{ij}'^{\infty}$.

Definition 2.16. [17] A connected IFG, $G = (V, E)$ is said to be *intuitionistic fuzzy spanning tree* (IFST), if it has an IFSSG, $H = (V', E')$ which is a tree.

Definition 2.17. [11] A *path* in an IFG is a sequence of distinct vertices $\upsilon_1, \upsilon_2, \ldots \upsilon_n$, such that either one of the following conditions is satisfied for some $i, j = 1, 2, 3 \ldots n$:

(i) $\mu_{ij} > 0, \gamma_{ij} > 0$
(ii) $\mu_{ij} = 0, \gamma_{ij} > 0$
(iii) $\mu_{ij} > 0, \gamma_{ij} = 0$.

Definition 2.18. [17] A *strong path* in an IFG is a path $P = \upsilon_1 \upsilon_2 \ldots \upsilon_n$, in which for every edge $(\upsilon_i, \upsilon_j) \in P$, is strong edge.

Definition 2.19. [8] The *length* of a path $P = \upsilon_1 \upsilon_2 \ldots \upsilon_{n+1}$ ($n > 0$) is n.

Example 1. Consider an IFG, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$, $E = \{(\upsilon_1, \upsilon_2), (\upsilon_2, \upsilon_4), (\upsilon_1, \upsilon_3), (\upsilon_3, \upsilon_4)\}$. and its IFSSG $H = (V', E')$, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$, $E = \{(\upsilon_1, \upsilon_2), (\upsilon_2, \upsilon_4), (\upsilon_1, \upsilon_3)\}$.

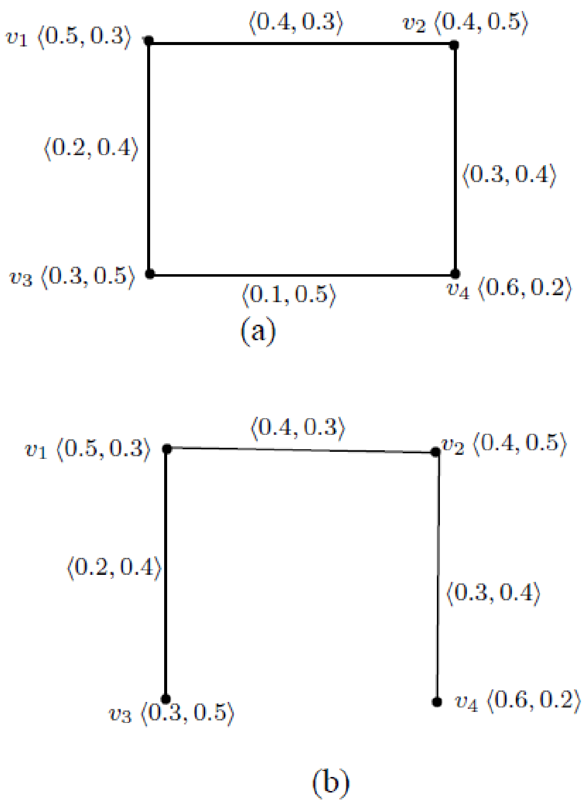

Figure 1: (a) Intuitionistic fuzzy graph *G* (b) Spanning subgraph *H*

Here G is an IFT.

Note 1. Not all intuitionistic fuzzy graphs are intuitionistic fuzzy trees.

Example 2. Consider an IFG, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$, $E = \{(\upsilon_1, \upsilon_2), (\upsilon_1, \upsilon_3), (\upsilon_3, \upsilon_4), (\upsilon_1, \upsilon_4), (\upsilon_2, \upsilon_3)\}$.

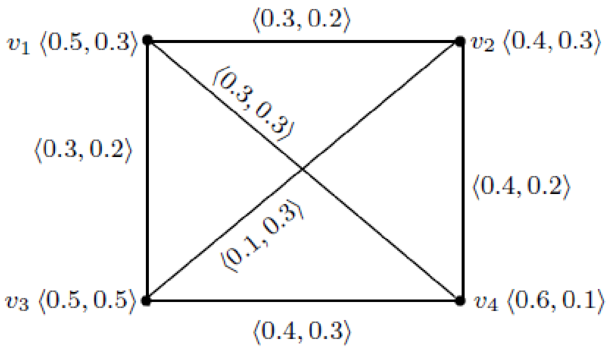

Figure 2: Intuitionistic fuzzy graph *G*

Here, G is an IFG but not an IFT.

Definition 2.20. Let $G = (V, E)$ be an IFT and let $P = \upsilon_1 \upsilon_2 \ldots \upsilon_n$ be a path. The *μ-length* of P in G, denoted by $\iota_\mu(P)$, is defined as, $\iota_\mu(P) = \sum_{(\upsilon_i, \upsilon_j) \in P} \mu_{ij}$

$i, j = 1, 2, 3 \ldots n$. The *γ-length* of path P in G, denoted by $\iota_\gamma(P)$, is defined as $\iota_\gamma(P) = \sum_{(\upsilon_i, \upsilon_j) \in P} \gamma_{ij}$,

$i, j = 1, 2, 3 \ldots n$. The *length* of P in G, denoted by $\iota(P)$, is defined as $\iota(P) = \langle \iota_\mu(P), \iota_\gamma(P) \rangle$.

Definition 2.21. Let $G = (V, E)$ be an IFT. For any two vertices υ_i and υ_j in G, let $\Omega = \{P_i : P_i$ is a $\upsilon_i - \upsilon_j$ path, $i = 1, 2, 3 \ldots n\}$. The *μ-distance* between any two vertices $\upsilon_i, \upsilon_j \in V$, denoted by $\delta_{\mu i,\mu j}$, is defined as $\delta_{\mu i,\mu j} = \min\{\iota_\mu(P_i) : P_i \in \Omega, i = 1, 2, 3 \ldots n\}$. The *γ-distance* between any two vertices $\upsilon_i, \upsilon_j \in V$, denoted by $\delta_{\gamma i,\gamma j}$, is defined as $\delta_{\gamma i,\gamma j} = \min\{\iota_\gamma(P_i) : P_i \in \Omega, i = 1, 2, 3 \ldots n\}$.

The *distance*, $\delta(\upsilon_i, \upsilon_j)$, is defined as $\delta(\upsilon_i, \upsilon_j) = \langle \delta_{\mu i,\mu j}, \delta_{\gamma i,\gamma j} \rangle$.

Definition 2.22. Let $G = (V, E)$ be an IFT. For each $\upsilon_i \in V$, the μ- *eccentricity* of υ_i, denoted by $e_{\mu i}$, is defined as $e_{\mu i} = \max\{\delta_{\mu i,\mu j} : \upsilon_i \in V, \upsilon_i \neq = \upsilon_j$. For each $\upsilon_i \in V$, the γ-*eccentricity* of υ_i, denoted by $e_{\gamma i}$, is defined as $e_{\gamma i} = \max\{\delta_{\gamma i,\gamma j} : \upsilon_i \in V, \upsilon_i = \neq \upsilon_j$. For each $\upsilon_i \in V$, the *eccentricity* of υ_i, denoted by $e(\upsilon_i)$, and is defined as $e(\upsilon_i) = \langle e_{\mu i}, e_{\gamma i} \rangle$.

Definition 2.23. Let $G = (V, E)$ be an IFT. The μ-*radius* of G, denoted by $r_\mu(G)$, is defined as $r_\mu(G) = \min\{e_{\mu i} : \upsilon_i \in V\}$. The γ-*radius* of G, denoted by $r_\gamma(G)$, is defined as $r_\gamma(G) = \min\{e_{\gamma i} : \upsilon_i \in V\}$. The *radius* of G, denoted by $r(G)$ is defined as $r(G) = \langle r_\mu(G), r_\gamma(G) \rangle$.

Definition 2.24. Let $G = (V, E)$ be an IFT. The μ-*diameter* of G, denoted by $diam_\mu(G)$, is defined as $diam_\mu(G) = \max\{e_{\mu i} : \upsilon_i \in V\}$. The γ-*diameter* of G, denoted by $diam_\gamma(G)$, defined as $diam_\gamma(G) = \max\{e_{\gamma i} : \upsilon_i \in V\}$. The *diameter* of G, denoted by diam(G), is defined as diam $(G) = \langle diam_\mu(G), diam_\gamma(G) \rangle$.

Definition 2.25. A vertex $\upsilon_i \in V$ is called a *central* vertex of an IFT $G = (V, E)$, if $r_\mu(G) = e_{\mu i}$ and $r_\gamma(G) = e_{\gamma i}$. The set of all central vertices of an IFT is denoted by $C_V(G)$

Definition 2.26. An IFSG $H = (V', E')$ induced by the central vertices of G, is called *center* of G, denoted by $C(G)$.

Definition 2.27. A vertex $\upsilon_i \in V$ is called a *peripheral* vertex of an IFT $G = (V, E)$, if $diam_\mu(G) = e_{\mu i}$ and $diam_\gamma(G) = e_{\gamma i}$. The set of all peripheral vertices of an IFT is denoted by $Z(G)$.

Definition 2.28. Let $G = (V, E)$ be an IFT, then the distance function $\delta : V \times V \rightarrow [0, 1] \times [0, 1]$ is a *metric* on V, if the following conditions are satisfied:

(i) $\delta(\upsilon_i, \upsilon_j) \geq 0$ That is, $\delta_{\mu i,\mu j} \geq 0$, $\delta_{\gamma i,\gamma j} \geq 0$, for all $\upsilon_i, \upsilon_j \in V$

(ii) $\delta(\upsilon_i, \upsilon_j) = \langle 0, 1 \rangle$ if and only if $\upsilon_i = \upsilon_j$

(iii) $\delta(\upsilon_i, \upsilon_j) = \delta(\upsilon_j, \upsilon_i)$ That is, $\delta\mu i,\mu j = \delta\mu j,\mu i$, $\delta\gamma i,\gamma j = \delta\gamma j,\gamma i$

(iv) $\delta\mu i,\mu j \leq \delta\mu_j, \mu_k + \delta\mu_k, \mu_j$, $\delta\gamma_i, \gamma_j \leq \delta\gamma_j, \gamma_k + \delta\gamma_k, \gamma_j$, for all $\upsilon_i, \upsilon_j, \upsilon_k \in V$.

Definition 2.29. [5] A vertex $\upsilon_k \in V$ of an IFG $G = (V, E)$ is called *cut vertex* if $\mu_{ij}^\infty (G - \upsilon_k) < \mu_{ij}^\infty$ and $\gamma_{ij}^\infty (G - \upsilon_k) > \gamma_{ij}^\infty$ for some $\upsilon_i, \upsilon_j \in V$.

Definition 2.30. Let $G = (V, E)$ be an IFG and let $Y = \{\upsilon_1, \upsilon_2, \ldots \upsilon_n\}$ be the set of cut vertices in G. The μ-*strong weight* of Y in G, denoted by $S_\mu(Y)$, is defined as, $S_\mu(Y) = \sum_{\upsilon_j \in Y} \mu_{ij} i, j = 1, 2, 3 \ldots n$, where μ_{ij} is the minimum membership weight of strong edges incident on υ_i. The γ-*strong weight* of Y in G, denoted by $S_\gamma(Y)$, is defined as, $S_\gamma(Y) = \sum_{\upsilon_j \in Y} \gamma_{ij} i, j = 1, 2, 3 \ldots n$, where γ_{ij} is the maximum non-membership weight of strong edges incident on υ_i. The *strong weight* of Y in G, denoted by $S(Y)$, is defined as $S(Y) = \langle S_\mu(Y), S_\gamma(Y) \rangle$.

Definition 2.31. Let $G = (V, E)$ be an IFG, the μ-*vertex connectivity* of G, denoted by $k_\mu(G)$, is defined as, $k_\mu(G) = \min(S_\mu(Y))$. The γ-*vertex connectivity* of G, denoted by $k_\gamma(G)$, is defined as, $k_\gamma(G) = \min(S_\gamma(Y))$. The *vertex connectivity* of G, denoted by $k(G)$, is defined as, $\langle k_\mu(G), k_\gamma(G) \rangle$.

Definition 2.32. An edge $e_k \in E$ of an IFG $G = (V, E)$ is called *cut edge* if $\mu_{ij}^\infty (G - e_k) < \mu_{ij}^\infty$ and $\gamma_{ij}^\infty (G - e_k) > \gamma_{ij}^\infty$ for some $\upsilon_i, \upsilon_j \in V$.

Definition 2.33. Let $G = (V, E)$ be an IFG and let $E = \{e_1, e_2, \ldots e_n\}$ be set of cut edges in G.

The μ-*strong weight* of E in G, denoted by $S'_\mu(E)$, is defined as $S'_\mu(E) = \sum_{e_i \in E} \mu_{ij}, i, j = 1, 2, 3 \ldots n$.

The γ-*strong weight* of E in G, denoted by $S'_\gamma(E)$, is defined as, $S'_\gamma(E) = \sum_{e_i \in E} \gamma_{ij}, i, j = 1, 2, 3 \ldots n$,

The *strong weight* of E in G, denoted by $S'(E)$, is defined as $S'(E) = \langle S'_\mu(E), S'_\gamma(E) \rangle$.

Definition 2.34. Let $G = (V, E)$ be an IFG, the μ-*edge connectivity* of G, denoted by $k'_\mu(G)$, is defined as $k'_\mu(G) = \min(S'_\mu(E))$. The γ-*edge connectivity* of G, denoted by $k'_\gamma(G)$, is defined as

$k'_\gamma(G)$ = min($S'_\gamma(E)$). The *edge connectivity* of G, denoted by $k'(G)$, is defined as, $\langle k'_\mu(G), k'_\gamma(G)$.

Definition 2.35. A vertex $\upsilon_i \in V$ of an IFT $G = (V, E)$ is called *end vertex* if $\mu_{ij} \geq \mu^\infty_{ij}$ and $\gamma_{ij} \geq \gamma^\infty_{ij}$ for at most one $\upsilon_j \in V$.

Note 2. (1) In an IFT, the diameter not necessarily be twice of the radius.

(2) The center of intuitionistic fuzzy tree need not be k_1 or k_2.

(3) For any spanning subgraph H (which is a tree) of G contains atleast two end vertices and every vertex in G is either cut vertex or end vertex.

Example 3. A single-centered intuitionistic fuzzy tree with one central vertex is illustrated here. Consider an IFT, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$, $E = \{(\upsilon_1, \upsilon_2), (\upsilon_2, \upsilon_3), (\upsilon_1, \upsilon_5), (\upsilon_3, \upsilon_4), (\upsilon_5, \upsilon_4), (\upsilon_3, \upsilon_2)\}$.

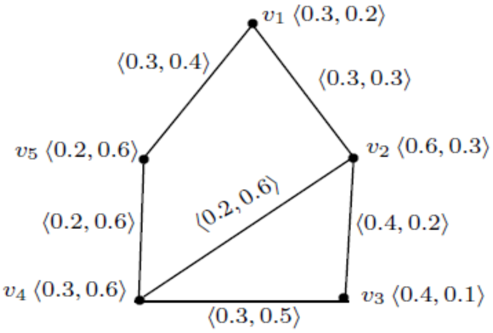

Figure 3: Intuitionistic fuzzy tree *G*

By routine computations, we have

(i) $\delta(\upsilon_1, \upsilon_2) = \langle 0.3, 0.3 \rangle$, $\delta(\upsilon_1, \upsilon_3) = \langle 0.7, 0.5 \rangle$, $\delta(\upsilon_1, \upsilon_4) = \langle 0.5, 0.9 \rangle$, $\delta(\upsilon_1, \upsilon_5) = \langle 0.3, 0.4 \rangle$, $\delta(\upsilon_2, \upsilon_3) = \langle 0.4, 0.2 \rangle$, $\delta(\upsilon_2, \upsilon_4) = \langle 0.2, 0.6 \rangle$, $\delta(\upsilon_2, \upsilon_5) = \langle 0.4, 0.6 \rangle$, $\delta(\upsilon_3, \upsilon_4) = \langle 0.3, 0.5 \rangle$, $\delta(\upsilon_3, \upsilon_5) = \langle 0.5, 0.9 \rangle$, $\delta(\upsilon_4, \upsilon_5) = \langle 0.2, 0.6 \rangle$.

(ii) Eccentricity of each vertex is $e(\upsilon_1) = \langle 0.7, 0.9 \rangle$, $e(\upsilon_2) = \langle 0.4, 0.6 \rangle$, $e(\upsilon_3) = \langle 0.7, 0.9 \rangle$, $e(\upsilon_4) = \langle 0.5, 0.9 \rangle$, $e(\upsilon_5) = \langle 0.5, 0.9 \rangle$

(iii) Radius of G is $\langle 0.4, 0.6 \rangle$, diameter of G is $\langle 0.7, 0.9 \rangle$.

(iv) The central vertex of G is υ_2, that is, $r(G) = e(\upsilon_2)$

(v) The center of G is displayed in Figure 4

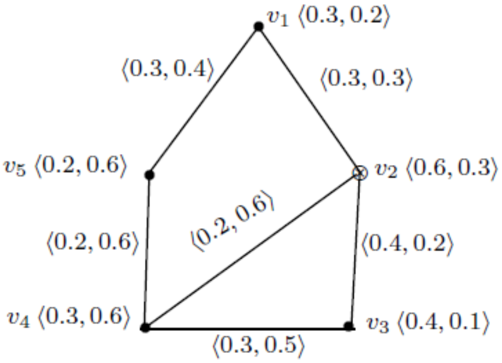

Figure 4: Single-centered IFT

(vi) The peripheral vertices of G are υ_3 and υ_1.

Example 4. A bi-centered intuitionistic fuzzy tree with two central vertices is discussed here.

Consider an IFT, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4\}$, $E = \{(\upsilon_1, \upsilon_2), (\upsilon_4, \upsilon_3), (\upsilon_2, \upsilon_4), (\upsilon_1, \upsilon_4), (\upsilon_2, \upsilon_3)\}$.

Therefore,

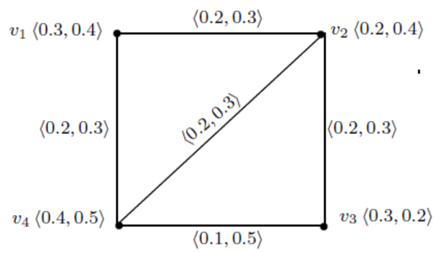

Figure 5: Intuitionistic fuzzy tree *G*

(i) $\delta(\upsilon_1, \upsilon_2) = \langle 0.2, 0.3 \rangle$, $\delta(\upsilon_1, \upsilon_3) = \langle 0.3, 0.6 \rangle$, $\delta(\upsilon_1, \upsilon_4) = \langle 0.2, 0.3 \rangle$, $\delta(\upsilon_2, \upsilon_1) = \langle 0.2, 0.3 \rangle$, $\delta(\upsilon_2, \upsilon_4) = \langle 0.2, 0.5 \rangle$, $\delta(\upsilon_2, \upsilon_3) = \langle 0.2, 0.3 \rangle$, $\delta(\upsilon_3, \upsilon_4) = \langle 0.1, 0.5 \rangle$

(ii) Eccentricity of each vertex is $e(\upsilon_1) = \langle 0.3, 0.6 \rangle$, $e(\upsilon_2) = \langle 0.2, 0.5 \rangle$, $e(\upsilon_3) = \langle 0.3, 0.6 \rangle$, $e(\upsilon_4) = \langle 0.2, 0.5 \rangle$

(iii) Radius of *G* is $\langle 0.2, 0.5 \rangle$, diameter of *G* is $\langle 0.3, 0.6 \rangle$.

(iv) The central vertices of *G* are υ_2 and υ_4. That is $r(G) = e(\upsilon_2)$, $r(G) = e(\upsilon_4)$

(v) The center *C*(*G*) is displayed in Figure 6.

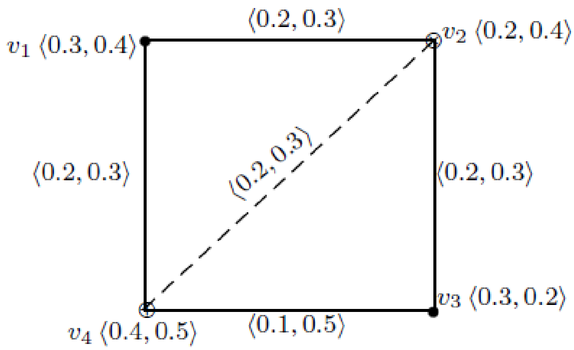

Figure 6: Bi-centered IFT

(vi) The peripheral vertices of *G* are υ_1 and υ_3.

Example 5. A tri-centered intuitionistic fuzzy tree with three central vertices is illustrated here. Consider an IFT, $G = (V, E)$, such that $V = \{\upsilon_1, \upsilon_2, \upsilon_3, \upsilon_4, \upsilon_5\}$,

$E = \{(\upsilon_1, \upsilon_2), (\upsilon_4, \upsilon_3), (\upsilon_2, \upsilon_4), (\upsilon_1, \upsilon_4), (\upsilon_2, \upsilon_3), (\upsilon_5, \upsilon_2), (\upsilon_1, \upsilon_5), (\upsilon_5, \upsilon_4), (\upsilon_1, \upsilon_5)\}$.

By routine computations, we have

(i) $\delta(\upsilon_1, \upsilon_2) = \langle 0.5, 0.2 \rangle$, $\delta(\upsilon_1, \upsilon_3) = \langle 0.5, 0.2 \rangle$, $\delta(\upsilon_1, \upsilon_4) = \langle 0.3, 0.5 \rangle$, $\delta(\upsilon_1, \upsilon_5) = \langle 0.5, 0.3 \rangle$, $\delta(\upsilon_2, \upsilon_3) = \langle 0.2, 0.5 \rangle$, $\delta(\upsilon_2, \upsilon_4) = \langle 0.3, 0.4 \rangle$, $\delta(\upsilon_2, \upsilon_5) = \langle 0.3, 0.4 \rangle$, $\delta(\upsilon_3, \upsilon_4) = \langle 0.2, 0.5 \rangle$, $\delta(\upsilon_3, \upsilon_5) = \langle 0.5, 0.7 \rangle$, $\delta(\upsilon_4, \upsilon_5) = \langle 0.5, 0.2 \rangle$

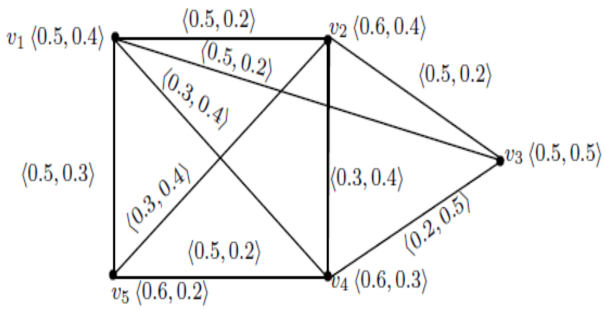

Figure 7: Intuitionistic fuzzy tree *G*

(ii) Eccentricity of each vertex is $e(\upsilon_1) = \langle 0.5, 0.5 \rangle$, $e(\upsilon_2) = \langle 0.5, 0.5 \rangle$, $e(\upsilon_3) = \langle 0.5, 0.7 \rangle$, $e(\upsilon_4) = \langle 0.5, 0.5 \rangle$, $e(\upsilon_5) = \langle 0.5, 0.7 \rangle$

(iii) Radius of G is $\langle 0.5, 0.5 \rangle$, diameter of G is $\langle 0.5, 0.7 \rangle$.

(iv) The central vertices $C(G)$ are υ_1, υ_2 and υ_4, that is, $r(G) = e(\upsilon_1)$, $r(G) = e(\upsilon_2)$, $r(G) = e(\upsilon_4)$

(v) The center $C(G)$ is displayed in Figure 8

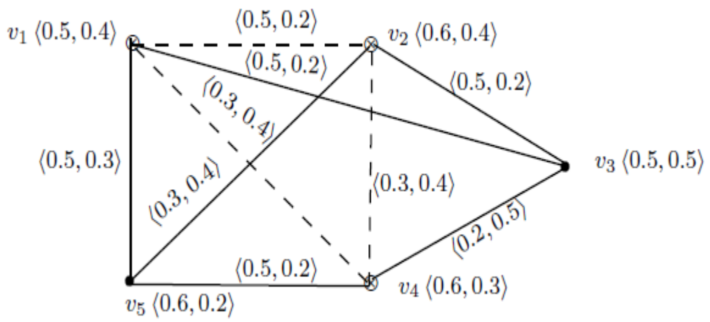

Figure 8: Tri-centered IFT

(vi) The peripheral vertices of G are υ_3 and υ_5.

III. DOMINATION IN INTUITIONISTIC FUZZY TREES

Definition 3.1. [11] Let $G = (V, E)$ be an IFG on V. Let u, $\upsilon \in V$, u is said to *dominate* υ in G if there exists a strong edge between them.
Definition 3.2. [11] A subset S of V is called a *dominating set* in G if for every $\upsilon \in V - S$, there exists $u \in S$ such that u dominates υ.
Definition 3.3. [11] A dominating set S of an IFG is said to be a *minimal dominating set* if no proper subset of S is a dominating set.
Definition 3.4. [11] Minimum cardinality among all minimal dominating set is called *lower dom-ination number* of G, and is denoted by $d(G)$.
Maximum cardinality among all minimal dominating set is called *upper domination number* of G, and is denoted by $D(G)$.
Definition 3.5. [11] Two vertices in an IFG, $G = (V, E)$ are said to be *independent* if there is no strong edge between them.
Definition 3.6. [11] A subset S of V is said to be *independent set* of G if $\mu_{ij} < \mu_{ij}^{\infty}$ and $\gamma_{ij} < \gamma_{ij}^{\infty}$ for all υ_i, $\upsilon_j \in S$. An independent set S of G in an IFG is said to be *maximal independent*, if for every vertex $\upsilon_j \in V - S$, the set $S \cup \{\upsilon_j\}$ is not independent.
Definition 3.7. [11] The minimum cardinality among all maximal independent set is called *lower independence number* of G, and it is denoted by $i(G)$.
The maximum cardinality among all maximal independent set is called *upper independence number* of G, and it is denoted by $I(G)$.
Definition 3.8. [11] Let $G = (V, E)$ be an IFG without isolated vertices. A subset D of V is a *total dominating set* if for every vertex $\upsilon_i \in V$, there exists a vertex $\upsilon_j \in D$, $\upsilon_i \neq \upsilon_j$, such that υ_j dominates υ_i.
Definition 3.9. [11] The minimum cardinality of a total dominating set is called *total domination number* of G, and it is denoted by $d_t(G)$.
Definition 3.10. [17] Let G be a connected IFG. A subset V' of V is called a *connected dominating set* of G, if

(i) For every $\upsilon_j \in V - V'$, there exists $\upsilon_i \in V'$ such that $\mu_{ij} \geq \mu_{ij}^{\infty}$ and $\gamma_{ij} \geq \gamma_{ij}^{\infty}$

(ii) The sub graph $H = (V', E')$ of $G=(V, E)$ induced by V' is connected.

Definition 3.11. [17] The minimum cardinality of a connected dominating set is called the *con-nected domination number* of G, and is denoted by $d_c(G)$.

Example 6. Consider an IFT, $G = (V, E)$, in Figure 3 in Example 3

(i) The minimal dominating set of G is $\{\upsilon_1, \upsilon_4\}$ and the domination number $d(G)$ is 0.9.

(ii) The maximal independent set of G is $\{\upsilon_1, \upsilon_4\}$ and the independent domination number $i(G)$ is 0.9.

(iii) The total dominating set of G is $\{\upsilon_1, \upsilon_4\}$ and the total domination number $d_t(G)$ is 0.9.

(iv) The connected dominating set of G is $\{\upsilon_1, \upsilon_2, \upsilon_3\}$ and the connected domination number $d_c(G)$ is 0.85

Theorem 3.1. Let $G = (V, E)$ is an IFT, then the distance between any two vertices in V is a metric.
Proof:
Let $G = (V, E)$ be a connected IFT. Then, there exists a unique strong path between any two vertices in V.

That is, $\delta_{\mu_i,\mu_j} \geq 0$, $\delta_{\gamma_i,\gamma_j} \geq 0$, which implies $\delta(\upsilon_i, \upsilon_j) \geq 0$ for every $\upsilon_i, \upsilon_j \in V$.

$\delta_{\mu_i,\mu_j} = 0$, $\delta_{\gamma_i,\gamma_j} = 0$, this implies that $\delta(\upsilon_i, \upsilon_j) = 0$.

The reversal of a path from υ_i, υ_j is a path from υ_j υ_i and vice versa. That is, $\delta_{\mu_i,\mu_j} = 0$, $\delta_{\gamma_i,\gamma_j} = 0$,

implies that $\delta(v_i, v_j) = \delta(v_j, v_i)$. Let P_1 is a $v_i v_k$ path in G such that $\delta_{\mu_i,\mu_k} = \sum_{(v_i,v_k)\in P_1} \mu_{ik}$, $\delta_{\gamma_i,\gamma_k} = \sum_{(v_i,v_k)\in P_1} \gamma_{ik}$, and P_2 be a $v_k v_j$ path such that $\delta_{\mu_k,\mu_j} = \sum_{(v_k,v_j)\in P_2} \gamma_{kj}$, $\delta_{\gamma_k,\gamma_j} = \sum_{(v_k,v_j)\in P_2} \gamma_{kj}$.

The path P_1 followed by P_2 is a υ_i, υ_j walk and since every walk contains one path, there exists a υ_i, υ_j path in G whose length is at most $\delta_{\mu_i,\mu_k} + \delta_{\mu_k,\mu_j}$, $\delta_{\gamma_i,\gamma_k} + \delta_{\gamma_k,\gamma_j}$. Therefore, $\delta_{\mu_i,\mu_j} \leq \delta_{\mu_j,\mu_k} + \delta_{\mu_k,\mu_j}$, $\delta_{\gamma_i,\gamma_j} \leq \delta_{\gamma_j,\gamma_k} + \delta_{\gamma_k,\gamma_j}$. This implies that $\delta(\upsilon_i, \upsilon_j) \leq \delta(\upsilon_i, \upsilon_k) + \delta(\upsilon_k, \upsilon_j)$. Hence, the distance δ is a metric on V.

Theorem 3.2. For any IFT $G = (V, E)$, the radius and diameter satisfy $r_\mu(G) \leq diam_\mu(G) \leq 2r_\mu(G)$ and $r_\gamma(G) \leq diam_\gamma(G) \leq 2r_\gamma(G)$.

Proof:

By the definition of radius and diameter, $r_\mu(G) \leq diam_\mu(G)$ and $r_\gamma(G) \leq diam_\gamma(G)$. Let υ_k be a central vertex and υ_i, υ_j be two peripheral vertices of G. Then $r_\mu(G) = e_\mu(\upsilon_k)$, $r_\gamma(G) = e_\gamma(\upsilon_k$ and $diam_\mu(G) = e_\mu(\upsilon_i)$, $diam_\gamma(G) = e_\gamma(\upsilon_i)$ $diam_\mu(G) = e_\mu(\upsilon_j)$, $diam_\gamma(G) = e_\gamma(\upsilon_j)$.

By triangle inequality,

$\delta_{\mu_i,\mu_j} \leq \delta_{\mu_j,\mu_k} + \delta_{\mu_k,\mu_j} = r_\mu(G) + r_\mu(G) = 2r_\mu(G)$

$\delta_{\gamma_i,\gamma_j} \leq \delta_{\gamma_j,\gamma_k} + \delta_{\gamma_k,\gamma_j}{}' = r\gamma(G) + r\gamma(G) = 2r\gamma(G)$

Therefore, $r_\mu(G) \leq diam_\mu(G) \leq 2r_\mu(G)$ and $r_\gamma(G) \leq diam_\gamma(G) \leq 2r_\gamma(G)$.

Theorem 3.3. For any two vertices υ_i, υ_j in an IFT $G = (V, E)$, $|e_\mu(\upsilon_i) - e_\mu(\upsilon_j)| \leq \delta_{\mu_i,\mu_j}$ and $|e_\gamma(\upsilon_i) - e_\gamma(\upsilon_j)| \leq \delta_{\gamma_i,\gamma_j}$.

Proof:

By the Definition 2.22, $e_{\mu_i} = \max\{\delta_{\mu_i,\mu_j} : \upsilon_i \in V, \upsilon_i \neq \upsilon_j\}$ and $e_{\gamma_i} = \max\{\delta_{\gamma_i,\gamma_j} : \upsilon_i \in V, \upsilon_i \in \upsilon_j$

Let υ_k be a vertex farthest from υ_i such that $e_{\mu_i} = \delta_{\mu_i,\mu_k}$ and $e_{\gamma_i} = \delta_{\gamma_i,\gamma_k}$. Then, by triangle inequality $e_{\mu_i} = \delta_{\mu_i,\mu_k} \leq \delta_{\mu_i,\mu_j} + \delta_{\mu_j,\mu_k}$ for any υ_k of G. Therefore,

$e_{\mu_i} \leq \delta_{\mu_i,\mu_k} + \delta_{\mu_k,\mu_j}$ for any υ_k of G. That is, $e_{\mu_i} \leq \delta_{\mu_i,\mu_j} + e_{\mu_j}$, since $\delta_{\mu_k,\mu_j} \leq e_{\mu_j}$ which implies, $e_{\mu_i} - e_{\mu_j} \leq \delta_{\mu_i,\mu_j}$.

Interchanging of vi and υ_j, we get $e_{\mu_j} - e_{\mu_i} \leq \delta_{\mu_j,\mu_i}$, υ_i, That is $-\delta_{\mu_j,\mu_i} \leq e_{\mu_i} - e_{\mu_j}$.

Combining these results give $-\delta_{\mu_j,\mu_i} \leq e_{\mu_i} - e_{\mu_j} \leq \delta_{\mu_j,\mu_i}$. Similarly, $-\delta_{\gamma_j,\gamma_i} \leq e_{\gamma_i} - e_{\gamma_j} \leq \delta_{\gamma_j,\gamma_i}$.

Hence, $|e_\mu(\upsilon_i) - e_\mu(\upsilon_j)| \leq \delta_{\mu_i,\mu_j}$ and $|e_\gamma(\upsilon_i) - e_\gamma(\upsilon_j)| \leq \delta_{\gamma_i,\gamma_j}$.

Theorem 3.4. Let υ_i and υ_j be any two vertices in an IFT $G = (V, E)$. Then $\delta_{\mu_i,\mu_k} - \delta_{\mu_k,\mu_j} \leq \delta_{\mu_i,\mu_j}$, $\left|\delta_{\gamma_i,\gamma_k} - \delta_{\gamma_k,\gamma_j}\right| \leq \delta_{\gamma_i,\gamma_j}$, for all v_k in V.

Proof:

Let v_i, v_j be any two vertices in V. Since $\delta(v_i, v_j)$ is a metric,
$\delta_{\mu_i,\mu_j} \leq \delta_{\mu_i,\mu_k} + \delta_{\mu_k,\mu_j}$ and $\delta_{\gamma_i,\gamma_j} \leq \delta_{\gamma_i,\gamma_k} + \delta_{\gamma_k,\gamma_j}$
for all v_k in V. Also $\delta_{\mu_k,\mu_j} \leq \delta_{\mu_k,\mu_i} + \delta_{\mu_i,\mu_j}$ and $\delta_{\gamma_k,\gamma_j} \leq \delta_{\gamma_k,\gamma_i} + \delta_{\gamma_i,\gamma_j}$.
That is , $\delta_{\mu_i,\mu_j} \geq \delta_{\mu_k,\mu_i} - \delta_{\mu_k,\mu_j}$, $\delta_{\gamma_i,\gamma_j} \geq \delta_{\gamma_k,\gamma_i} - \delta_{\gamma_k,\gamma_j}$.
Combining the above results, $\delta_{\mu_k,\mu_i} - \delta_{\mu_k,\mu_j} \leq \delta_{\mu_i,\mu_j} \leq \delta_{\mu_k,\mu_i} + \delta_{\mu_k,\mu_j}$ $\delta_{\gamma_k,\gamma_i} - \delta_{\gamma_k,\gamma_j} \leq \delta_{\gamma_i,\gamma_j} \leq \delta_{\gamma_k,\gamma_i} + \delta_{\gamma_k,\gamma_j}$.
That is, $-(\delta_{\mu_k,\mu_j} - \delta_{\mu_k,\mu_k}) \leq \delta_{\mu_i,\mu_j} \leq \delta_{\mu_k,\mu_i} + \delta_{\mu_k,\mu_j}$ and $-(\delta_{\gamma_k,\gamma_j} - \delta_{\gamma_k,\gamma_k}) \leq \delta_{\gamma_i,\gamma_j} \leq \delta_{\gamma_k,\gamma_i} + \delta_{\gamma_k,\gamma_j}$.
Therefore, δ_{μ_i,μ_j}, $\left|\delta_{\gamma_i,\gamma_k} - \delta_{\gamma_k,\mu_j}\right| \leq \delta_{\gamma_i,\gamma_j}$, for all v_k in V.

Theorem 3.5. Let $G = (V, E)$ be an IFT on with minimum 3 vertices. Let $M(H)$ be the maximum cardinality of end vertices in any spanning forest $H = (V', E')$ in G, then $d_c = o(G) - M(H)$.

Proof:

Let H be spanning forest of G and let $X = \{\upsilon_i \in V', \upsilon_i$ is an end vertex of $H\}$. Clearly, $V - X$ is a connected dominating set of G and the cardinality of $V - X$ is $o(G) - M(H)$. Hence $d_c(G) \leq o(G) - M(H)$.

Now, let S be a connected dominating set of G. Let H_S be any spanning forest of the induced subgraph $G[S]$. Since S is a connected dominating set G, for each υ_i in $V - S$, there exists a vertex υ_j in S such that $\mu_{ij} \geq \mu^{\infty}_{ij}$ and $\gamma_{ij} \geq \gamma^{\infty}_{ij}$. Let H be the subgraph adding the vertices of $V - S$ and the edges $\upsilon_i\ \upsilon_j$ for each υ_i in $V - S$. Clearly H is a spanning forest of G and $M(H) \geq o(G) - d_C(G)$.

Hence, $d_C(G) = o(G) - M(H)$.

Theorem 3.6. Let $G = (V, E)$ be an IFT with minimum three vertices. Suppose that $d(G - e_{i_j}) = d(G)$, where e_i is a strong edge in G. Then for each strong edge e_{i_j} , there exists a dominating set D satisfying either one of the following conditions:

(i) $\upsilon_i, \upsilon_j \in D$

(ii) $\upsilon_i, \upsilon_j \in V - D$

(iii) If $\upsilon_i \in D$ and $\upsilon_j \in V - D$, then there exists $\upsilon_k \in D - \{\upsilon_j\}$ such that $\mu_{j_k} \geq \mu^{\infty}_{jk}$ and $\gamma_{j_k} \geq \gamma^{\infty}_{jk}$.

Proof:

Suppose there is no dominating set D in G satisfying any of the statements (*i*),(*ii*), (*iii*). Then, any dominating set D of G is not a dominating set of $G - e_{i_j}$. Further, any dominating set of $G - e_{i_j}$ is a dominating set of G also. Hence, it follows that, $d(G - e_{i_j}) \neq d(G)$.

Theorem 3.7. In an IFT $G = (V, E)$, set of all cut vertices is a dominating set of G.

Proof:

Let D be the set of all cut vertices of G. Since, every vertex in a spanning subgraph is either a cut vertex or an end vertex. Then, $V - D$ is the set of all end vertices of G. Then, for each $\upsilon_i \in V - D$, there exists a strong neighbor $\upsilon_i \in D$. Hence, each $\upsilon_i \in V - D$, is dominated by some $\upsilon_j \in D$. So D is a dominating set of G.

Theorem 3.8. If $G = (V, E)$ is an IFT, then G is not complete.

Proof:

Suppose G be a complete IFG. Let H be spanning subgraph of G. Then $\mu^{\infty}_{ij} = \mu_{ij}$ and $\gamma^{\infty}_{ij} = \gamma_{ij}$ for all υ_i, υ_j in V. Now G being an IFT, $\mu_{ij} < \mu^{\infty}_{ij}$ and $\gamma_{ij} < \gamma^{\infty}_{ij}$ for all υ_i, υ_j not in H, where H is a spanning subgraph of G. Thus, $\mu^{\infty}_{ij} < \mu_{ij}$ and $\gamma^{\infty}_{ij} < \gamma_{ij}$, contradicting the definition of complete IFG.

IV. INTUITIONISTIC FUZZY TREE CENTER-BASED CLUSTERING ALGORITHM

The objective of clustering is to classify the observations into groups such that the degree of associ-ation is high among the members of a group and is less among the members of other groups. Graph theoretical clustering is nothing but partitioning the graph based on qualitative aspects of the data. Most of the clustering methods group the data based on distance and similarity. Rosenfeld [15] introduced distance based clustering on fuzzy graphs. Xu and Wu [20] developed an intuitionistic fuzzy c-means algorithm to cluster IFSs, which is based on the well-known fuzzy c-means clustering method and the basic distance measures between IFSs such as the Hamming distance, normalized Hamming distance, Euclidean distance and normalized Euclidean distance. Zhang and Chen [23] defined the concept of intuitionistic fuzzy similarity matrix and presented a clustering method based on λ-cutting matrix. Zhong Wang et al. [24] presented a netting method to make cluster analysis of intuitionistic fuzzy sets. Zhao et al. [21] developed an intuitionistic fuzzy minimum spanning tree clustering algorithm to deal with intuitionistic fuzzy information. In this section, a new clustering method namely, intuitionistic fuzzy tree center-based clustering method is proposed to classify the given crisp data set. The intuitionistic fuzzification of the data set is obtained by S-shaped intuitionistic fuzzification function. The classical similarity and distance measures are characterized by real numbers. The proposed distance measure is an intuitionistic fuzzy value. Cluster center is obtained by using eccentricity concept in IFTs instead of random selection. The computation procedure of this method is comparatively easier. The proposed clus-tering algorithm is verified with a numerical data set.

4.1 Notations and assumptions

Let $V = \{\upsilon_1, \upsilon_2, \upsilon_3 \ldots \upsilon_n\}$ be the data set of n objects to be clustered. Let $A = \{A_1, A_2, A_3 \cdots, A_m\}$ is the set of m attributes for each object v_i. The data set is represented as a matrix $G = \left[\upsilon^p_i\right]$, $i = 1, 2, 3 \ldots, n$, $p = 1, 2, 3 \ldots, m$. The columns (i) of the matrix G indicate the set of n objects and rows (p) represent the number of numerical attributes of each data. The object υ^p_i in the data matrix represents i^{th} object with p^{th} attribute.

The entries of the data matrix G are of the form $I_G = \left[\langle \mu^p_i, \gamma^p_i \rangle\right]_{n\times m}$, $i = 1, 2, 3, \ldots, n$ $p = 1, 2, 3 \ldots, m$ where $\langle \mu^p_i, \gamma^p_i \rangle$ represents the degree of membership and non-membership of i^{th} object with p^{th} attribute.

Definition 4.1. Let $D = (d_{ij})_{n\times n}$ be an intuitionistic fuzzy distance matrix, where $d_{ij} = \langle \mu_{ij}, \gamma_{ij} \rangle$, $i, j =$

$1, 2, 3, \ldots n$. Then $\langle \lambda, \delta \rangle_D = \langle \lambda, \delta \rangle_{d_{ij}} = \langle \lambda_{\mu_{ij}}, \delta_{\gamma_{ij}} \rangle$ is called $\langle \lambda, \delta \rangle$-*cut matrix* of D where $\langle \lambda, \delta \rangle$ $0 \le \lambda, \delta \le 1, 0 \le \lambda + \delta \le 1$, and

$$\langle \lambda, \delta \rangle_{d_{ij}} = \begin{cases} \langle 1, 0 \rangle & \text{if } \mu_{ij} \ge \lambda,\ \gamma_{ij} < \delta \\ \langle 0, 1 \rangle & \text{if } \mu_{ij} < \lambda \text{ and } \gamma_{ij} \ge \delta \end{cases} \quad (1)$$

Definition 4.2. [23] Let $\alpha, \beta \in X_{1\times n}$, where $X_{1\times n}$ denotes the set of intuitionistic fuzzy vectors. Then $(\alpha, \beta) = (\max\{\min\{\mu_{\alpha i}, \mu_{\beta i}\}\}, \min\{\max\{\gamma_{\alpha i}, \gamma_{\beta i}\}\})$ is called the *inner product* of α and β.

4.2 Algorithm

The step by step procedure of proposed intuitionistic fuzzy tree center based algorithm is described here.

Step 1: Consider the set of n objects $V = \{\upsilon_1, \upsilon_2, \upsilon_3 \cdots, \upsilon_n\}$, and a set of m attributes $A = \{A_1, A_2, A_3 \cdots, A_m\}$ in a data set. Form the data matrix G.

Step 2: The intuitioistic fuzzification for the data set of n objects is done as follows:

The degree of membership υ_i^p, is calculated using

$$\mu_i^p = \begin{cases} 0 & \text{if} \quad v_i^p \le a \\ 2\left(\frac{v_i^p - a}{b-a}\right)^2 - \epsilon & \text{if} \quad a < v_i^p \le \frac{a+b}{2} \\ 1 - 2\left(\frac{v_i^p - b}{b-a}\right)^2 - \epsilon & \text{if} \quad \frac{a+b}{2} < v_i^p < b \\ 1 - \epsilon & \text{if} \quad v_i^p \ge b \end{cases} \quad (2)$$

The degree of non-membership γ_i^p is calculated by

$$\gamma_i^p = \begin{cases} 1 - \epsilon & \text{if} \quad v_i^p \le a \\ 1 - 2\left(\frac{v_i^p - a}{b-a}\right)^2 & \text{if} \quad a < v_i^p \le \frac{a+b}{2} \\ 2\left(\frac{v_i^p - b}{b-a}\right)^2 & \text{if} \quad \frac{a+b}{2} < v_i^p < b \\ 0 & \text{if} \quad v_i^p \ge b \end{cases} \quad (3)$$

where a, b, c are arbitrary constants.

Step 3: Calculate the distance between two objects using the formula

$$d(v_i, v_j) = \begin{cases} \langle 0, 1 \rangle, & i = j \\ \left\langle 1 - \frac{1}{m}\sum_{p=1}^{m} \left|\mu_i^p - \mu_j^p\right|, \frac{1}{m}\sum_{p=1}^{m} \left|\gamma_i^p - \gamma_j^p\right| \right\rangle, & i \ne j, i, j = 1, 2, \cdots, n. \end{cases} \quad (4)$$

Form the IF distance matrix $D = d(\upsilon_i, \upsilon_j)_{n\times n}$.

Step 4: Draw the IFT $G = (V, E)$ with n vertices associated with the objects υ_i in the data set V to be clustered. The distance $d(\upsilon_i, \upsilon_j)$ is treated as the membership and non-membership values of the edges.

Step 5: Compute the eccentricity of each data object $\upsilon_i \in V$ by using the formula

$e(\upsilon_i) = \langle e_{\mu_i}, e_{\gamma_i} \rangle = \langle \max(d_\mu(\upsilon_i, \upsilon_j)), \max(d_\gamma(\upsilon_i, \upsilon_j)) \rangle$.

Step 6: Calculate the radius as $r(G) = \langle r_\mu(G), r_\gamma(G) \rangle$, where, $r_\mu(G) = \min\langle e_{\mu_i} : \upsilon_i \in V \rangle$ and $r_\gamma(G) = \min\{e_{\gamma_i} : \upsilon_i \in V\}$.

Step 7: The threshold for the center of the cluster is

[i] $e(\upsilon_i)$, if $r_\mu(G) = e_{\mu_i}$ and $r_\gamma(G) = e_{\gamma_i}$ or the corresponding e_{μ_j} of $e_{\gamma_j} = r_\gamma(G)$ is less than e_{μ_j}.

[ii] $e(\upsilon_i)$, if $r_\mu(G) = e_{\mu_i}$ and $r_\gamma(G) = e_{\gamma_j}$, the corresponding e_{μ_j} of $e_{\gamma_j} = r_\gamma(G)$ is greater than or equal to e_{μ_i}.

Step 8: Treat the center $e(\upsilon_i)$ obtained in Step 7 as $\langle \lambda, \delta \rangle$ −cut.

Step 9: Calculate the $\langle \lambda, \delta \rangle$ -cutting matrix on $\langle \lambda, \delta \rangle_{d(\upsilon_i, \upsilon_j)}$ using Definition 4.1.

Step 10: Calculate the inner products of the column vectors of the $(\lambda, \delta)_{d(\upsilon_i, \upsilon_j)}$-cut matrix. Then, the objects are clustered based on the inner product values (1, 1) or (1, 0) using Definition 4.2. Step 11: Go to Step 6, repeat the process until the desired number of clusters are obtained.

V. EXPERIMENTAL ANALYSIS

The algorithm has been implemented and tested with datasets available in the University of Cologne, Germany [25]. The data sets contains the nutrients in 27 di erent kinds of meat, fish or fowl with five attributes as food energy, protein, fat, calcium and iron. The data set is divided into five disjoint subsets. The data set with 5 attributes is given in Table 1. The intuitionistic fuzzification of the data set is presented in Table 2.

5.1 Step wise algorithm

The steps involved to cluster the numerical data set with 27 nutrients and 5 attributes, are given as follows:

Step 1. Consider the data set given in Table 1 to produce cluster.

Step 2. Compute the degrees of membership and non-membership for the given data set using S-shape intuitionistic fuzzification function using Equation 2, 3 These values are displayed in Table 2.

Step 3. Obtain IFT, by treating 27 nutrients as vertices $\upsilon_1, \cdots, \upsilon_{27}$. The distance between υ_i and υ_j is treated as the weight of e_{i_j}. Calculate distance between the objects υ_i, υ_j using the Equation 4.

Step 4. The eccentricities are given by

$e(\upsilon_1) = \langle 0.9960, 0.6559 \rangle$, $e(\upsilon_2) = \langle 0.9358, 0.4994 \rangle$, $e(\upsilon_3) = \langle 0.8847, 0.6653 \rangle$,
$e(\upsilon_4) = \langle 0.9771, 0.6739 \rangle$, $e(\upsilon_5) = \langle 0.9614, 0.4516 \rangle$, $e(\upsilon_6) = \langle 0.8838, 0.5522 \rangle$,
$e(\upsilon_7) = \langle 0.9949, 0.4859 \rangle$, $e(\upsilon_8) = \langle 0.8329, 0.6653 \rangle$, $e(\upsilon_9) = \langle 0.9358, 0.5393 \rangle$,
$e(\upsilon_{10}) = \langle 0.9266, 0.5928 \rangle$, $e(\upsilon_{11}) = \langle 0.9360, 0.6599 \rangle$, $e(\upsilon_{12}) = \langle 0.9847, 0.6513 \rangle$,
$e(\upsilon_{13}) = \langle 0.9847, 0.6666 \rangle$, $e(\upsilon_{14}) = \langle 0.9353, 0.4003 \rangle$, $e(\upsilon_{15}) = \langle 0.9439, 0.4710 \rangle$,
$e(\upsilon_{16}) = \langle 0.9486, 0.5089 \rangle$, $e(\upsilon_{17}) = \langle 0.9669, 0.6739 \rangle$, $e(\upsilon_{18}) = \langle 0.9669, 0.6717 \rangle$,
$e(\upsilon_{19}) = \langle 0.9393, 0.5115 \rangle$, $e(\upsilon_{20}) = \langle 0.9393, 0.4524 \rangle$, $e(\upsilon_{21}) = \langle 0.9353, 0.4421 \rangle$,
$e(\upsilon_{22}) = \langle 0.9298, 0.4815 \rangle$, $e(\upsilon_{23}) = \langle 0.9295, 0.3819 \rangle$, $e(\upsilon_{24}) = \langle 0.9298, 0.5129 \rangle$,
$e(\upsilon_{25}) = \langle 0.7802, 0.6589 \rangle$, $e(\upsilon_{26}) = \langle 0.9949, 0.49907 \rangle$, $e(\upsilon_{27}) = \langle 0.9106, 0.5545 \rangle$.

Step 5. The radius is calculated as $\langle 0.7802, 0.3819 \rangle$

Step 6. The center is e_{23}. Treat e_{23} as $\langle \lambda, \delta \rangle$ -cut in the distance matrix $d(\upsilon_i, \upsilon_j)$, and obtain the clusters.

If $\langle \lambda, \delta \rangle = \langle 0.9295, 0.3819 \rangle$, then the objects v_i, 1 = 1, 2 . . . , 27 fall into the following eighteen cate-gories:

$\{\upsilon_1, \upsilon_4, \upsilon_{10}, \upsilon_{11}, \upsilon_{12}, \upsilon_3\}, \{\upsilon_5, \upsilon_7, \upsilon_{26}\}, \{\upsilon_{17}, \upsilon_{18}\}, \{\upsilon_{22}, \upsilon_{24}\},$
$\{\upsilon_2\}, \{\upsilon_3\}, \{\upsilon_6\}, \{\upsilon_8\}, \{\upsilon_9\}, \{\upsilon_{14}\}, \{\upsilon_{15}\}, \{\upsilon_{16}\}, \{\upsilon_{19}\}, \{\upsilon_{20}\}, \{\upsilon_{21}\}, \{\upsilon_{23}\}, \{\upsilon_{25}\}, \{\upsilon_{27}\}$

If $\langle \lambda, \delta \rangle = \langle 0.9266, 0.5928 \rangle$, then the objects υ_i, 1 = 1, 2 . . . , 27 fall into the following seventeen categories:

$\{\upsilon_1, \upsilon_4, \upsilon_{10}, \upsilon_{11}, \upsilon_{12}, \upsilon_3\}, \{\upsilon_5, \upsilon_7, \upsilon_{16}, \upsilon_{26}\}, \{\upsilon_{17}, \upsilon_{18}\}, \{\upsilon_{22}, \upsilon_{24}\},$
$\{\upsilon_2\}, \{\upsilon_3\}, \{\upsilon_6\}, \{\upsilon_8\}, \{\upsilon_9\}, \{\upsilon_{14}\}, \{\upsilon_{15}\}, \{\upsilon_{19}\}, \{\upsilon_{20}\}, \{\upsilon_{21}\}, \{\upsilon_{23}\}, \{\upsilon_{25}\}, \{\upsilon_{27}\}$

If $\langle \lambda, \delta \rangle = \langle 0.9106, 0.5545 \rangle$, then the objects υ_i, 1 = 1, 2 . . . , 27 fall into the following sixteen categories:

$\{\upsilon_1, \upsilon_4, \upsilon_{10}, \upsilon_{11}, \upsilon_{12}, \upsilon_3\}, \{\upsilon_5, \upsilon_7, \upsilon_{16}, \upsilon_{26}\}, \{\upsilon_{17}, \upsilon_{18}\}, \{\upsilon_{22}, \upsilon_{24}\},$
$\{\upsilon_2, \upsilon_9\}, \{\upsilon_3\}, \{\upsilon_6\}, \{\upsilon_8\}, \{\upsilon_{14}\}, \{\upsilon_{15}\}, \{\upsilon_{19}\}, \{\upsilon_{20}\}, \{\upsilon_{21}\}, \{\upsilon_{23}\}, \{\upsilon_{25}\}, \{\upsilon_{27}\}.$

5.2 Comparison

There are many clustering algorithm existing in the literature. The results of the proposed algorithm is compared with two algorithms namely Zhang et al. [23] and netting method by Z.Wang et al.[24]. The derived results are compared and presented in Table 3. Though, all the three methods produce same clusters, the proposed IFT center-based algorithm reduces the complexity of calculations in forming equivalence matrix, which ultimately reduces the running time of the algorithm.

Table 1: Data set with 5 attributes and 27 objects

Objects	Name of the item	Food Energy (Calories) A_1	Protein (Grams) A_2	Fat (Grams) A_3	Calcium (Mgs) A_4	Iron (Mgs) A_5
v_1	Beef braised	340	20	28	9	2.6
v_2	Hamburger	245	21	17	9	2.7
v_3	Beef roast	420	15	39	7	2.0
v_4	Beef steak	375	19	32	9	2.6
v_5	Beef canned	180	22	10	17	1.4
v_6	Chicken broiled	115	20	3	8	3.7
v_7	Chicken canned	170	25	7	12	1.5
v_8	Beef Heart	160	26	5	14	6.9
v_9	Lamp leg roast	265	20	20	9	2.6
v_{10}	Lamb shoulder roast	300	18	25	9	2.3
v_{11}	Smoked ham	340	20	28	9	2.5
v_{12}	Pork roast	340	19	29	9	2.5
v_{13}	Pork simmered	355	19	30	9	2.4
v_{14}	Beef tongue	205	18	14	7	2.5
v_{15}	Veal cutlet	185	23	9	9	2.7
v_{16}	Bluefish baked	135	22	4	25	0.6
v_{17}	Clams raw	70	11	1	82	6.0
v_{18}	Clams canned	45	7	1	74	5.4
v_{19}	Crab meat canned	90	14	2	38	0.8
v_{20}	Haddock fried	135	16	5	15	0.5
v_{21}	Mackerel broiled	200	19	13	5	1.0
v_{22}	Mackerel canned	155	16	9	157	1.8
v_{23}	Perch fried	195	16	11	14	1.3
v_{24}	Salmon canned	120	17	5	159	0.7
v_{25}	Sardines canned	180	22	9	367	2.5
v_{26}	Tuna canned	170	25	7	7	1.2
v_{27}	Shrimp canned	110	23	1	98	2.6

Table 2: Intuitionistic fuzzification of Data set

Objects	Name the item	Food Energy (Calories) A_1	Protein (Grams) A_2	Fat (Grams) A_3	Calcium (Milli Grams) A_4	Iron (Milli Grams) A_5
v_1	Beef braised	$\langle 0.9089, 0.0910\rangle$	$\langle 0.7806, 0.1994\rangle$	$\langle 0.8321, 0.1676\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.2150, 0.7847\rangle$
v_2	Hamburger	$\langle 0.5643, 0.4356\rangle$	$\langle 0.8415, 0.1386\rangle$	$\langle 0.3542, 0.6454\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.2360, 0.7637\rangle$
v_3	Beef roast	$\langle 0.999, 0\rangle$	$\langle 0.3346, 0.6454\rangle$	$\langle 0.9997, 0\rangle$	$\langle 0.0005, 0.9999\rangle$	$\langle 0.1096, 0.8901\rangle$
v_4	Beef steak	$\langle 0.9711, 0.0288\rangle$	$\langle 0.7085, 0.2715\rangle$	$\langle 0.9318, 0.0679\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.2150, 0.7847\rangle$
v_5	Beef canned	$\langle 0.2591, 0.7408\rangle$	$\langle 0.8914, 0.0886\rangle$	$\langle 0.1119, 0.8878\rangle$	$\langle 0.0021, 0.9978\rangle$	$\langle 0.0393, 0.9604\rangle$
v_6	Chicken broiled	$\langle 0.0696, 0.9303\rangle$	$\langle 0.7806, 0.1994\rangle$	$\langle 0.0052, 0.9944\rangle$	$\langle 0.0001, 0.9999\rangle$	$\langle 0.4997, 0.500\rangle$
v_7	Chicken canned	$\langle 0.2221, 0.7777\rangle$	$\langle 0.9744, 0.005\rangle$	$\langle 0.0496, 0.9501\rangle$	$\langle 0.0007, 0.9993\rangle$	$\langle 0.0485, 0.9512\rangle$
v_8	Beef Heart	$\langle 0.1880, 0.8119\rangle$	$\langle 0.9800, 0\rangle$	$\langle 0.0219, 0.9778\rangle$	$\langle 0.0012, 0.9988\rangle$	$\langle 0.9997, 0\rangle$
v_9	Lamp leg roast	$\langle 0.6582, 0.3417\rangle$	$\langle 0.7806, 0.1994\rangle$	$\langle 0.4997, 0.500\rangle$	$\langle 0.0002, 0.9997\rangle$	$\langle 0.2150, 0.7847\rangle$
v_{10}	Lamb shoulder roast	$\langle 0.7951, 0.2048\rangle$	$\langle 0.6254, 0.3546\rangle$	$\langle 0.7282, 0.2715\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.1579, 0.8418\rangle$
v_{11}	Smoked ham	$\langle 0.9089, 0.0910\rangle$	$\langle 0.7806, 0.1994\rangle$	$\langle 0.8321, 0.1676\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.1950, 0.8047\rangle$
v_{12}	Pork roast	$\langle 0.9089, 0.0910\rangle$	$\langle 0.7085, 0.2714\rangle$	$\langle 0.8612, 0.1385\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.1950, 0.8047\rangle$
v_{13}	Pork simmered	$\langle 0.9398, 0.0601\rangle$	$\langle 0.7085, 0.2715\rangle$	$\langle 0.8875, 0.1122\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.1760, 0.8237\rangle$
v_{14}	Beef tongue	$\langle 0.3640, 0.359\rangle$	$\langle 0.6254, 0.3546\rangle$	$\langle 0.2338, 0.7659\rangle$	$\langle 0, 0.9999\rangle$	$\langle 0.1950, 0.8047\rangle$
v_{15}	Veal cutlet	$\langle 0.2787, 0.7212\rangle$	$\langle 0.9301, 0.0499\rangle$	$\langle 0.0883, 0.9114\rangle$	$\langle 0.0002, 0.9998\rangle$	$\langle 0.2360, 0.7637\rangle$
v_{16}	Bluefish baked	$\langle 0.1151, 0.8848\rangle$	$\langle 0.8914, 0.0886\rangle$	$\langle 0.0122, 0.9875\rangle$	$\langle 0.0060, 0.9939\rangle$	$\langle 0.0001, 0.9995\rangle$
v_{17}	Clams raw	$\langle 0.0088, 0.9911\rangle$	$\langle 0.0686, 0.9114\rangle$	$\langle 0, 0.9997\rangle$	$\langle 0.0945, 0.9095\rangle$	$\langle 0.9601, 0.0395\rangle$
v_{18}	Clams canned	$\langle 0, 0.9999\rangle$	$\langle 0, 0.9800\rangle$	$\langle 0, 0.9997\rangle$	$\langle 0.0727, 0.9273\rangle$	$\langle 0.8898, 0.1099\rangle$
v_{19}	Crab meat canned	$\langle 0.0287, 0.9712\rangle$	$\langle 0.2515, 0.7285\rangle$	$\langle 0.0010, 0.9986\rangle$	$\langle 0.0166, 0.9833\rangle$	$\langle 0.0041, 0.0056\rangle$
v_{20}	Haddock fried	$\langle 0.1151, 0.8848\rangle$	$\langle 0.4287, 0.5512\rangle$	$\langle 0.0218, 0.9778\rangle$	$\langle 0.0015, 0.9984\rangle$	$\langle 0, 0.9997\rangle$
v_{21}	Mackerel broiled	$\langle 0.3415, 0.6583\rangle$	$\langle 0.7085, 0.2714\rangle$	$\langle 0.1991, 0.8005\rangle$	$\langle 0, 0.9999\rangle$	$\langle 0.0119, 0.9877\rangle$
v_{22}	Mackerel canned	$\langle 0.1720, 0.8279\rangle$	$\langle 0.4287, 0.5512\rangle$	$\langle 0.0883, 0.9113\rangle$	$\langle 0.3526, 0.6473\rangle$	$\langle 0.0822, 0.9174\rangle$
v_{23}	Perch fried	$\langle 0.3199, 0.6800\rangle$	$\langle 0.4287, 0.5512\rangle$	$\langle 0.1382, 0.8614\rangle$	$\langle 0.0012, 0.9987\rangle$	$\langle 0.0310, 0.9688\rangle$
v_{24}	Salmon canned	$\langle 0.0799, 0.92\rangle$	$\langle 0.5312, 0.4487\rangle$	$\langle 0.0218, 0.9778\rangle$	$\langle 0.3619, 0.6380\rangle$	$\langle 0.0016, 0.9980\rangle$
v_{25}	Sardines canned	$\langle 0.2591, 0.7408\rangle$	$\langle 0.8913, 0.0886\rangle$	$\langle 0.0883, 0.9113\rangle$	$\langle 0.9999, 0\rangle$	$\langle 0.1950, 0.8046\rangle$
v_{26}	Tuna canned	$\langle 0.2221, 0.7777\rangle$	$\langle 0.9744, 0.0055\rangle$	$\langle 0.0495, 0.9501\rangle$	$\langle 0, 0.9999\rangle$	$\langle 0.0236, 0.9760\rangle$
v_{27}	Shrimp canned	$\langle 0.0599, 0.9399\rangle$	$\langle 0.9301, 0.0498\rangle$	$\langle 0, 0.9997\rangle$	$\langle 0.1319, 0.8679\rangle$	$\langle 0.2150, 0.7846\rangle$

Table 3: Comparisons of the derived results

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Clusters	The results derived by center of IFT method	Zhang et.al method [23]	Zhong Wang et.al method[24]
1	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}v_{24}, v_{25}, v_{26}, v_{27}\},$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}v_{24}, v_{25}, v_{26}, v_{27}\},$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}v_{24}, v_{25}, v_{26}, v_{27}\},$
2	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\}$
3	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\}$	-	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\}$
4	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{22}, v_{23}, v_{24}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}$
5	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}\{v_{22}, v_{24}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}\{v_{22}, v_{24}\}$	$\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\}\{v_{22}, v_{24}\}$
6	$\{v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}$	$\{v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\}, \{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}$	$\{v_1, v_2, v_4, v_5, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}$
7	$\{v_1, v_2, v_4, v_5, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}, \{v_6\}$	$\{v_1, v_2, v_4, v_5, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}, \{v_6\}$	$\{v_1, v_2, v_4, v_5, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}, \{v_6\}$
8	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_2, v_5, v_7, v_9, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}, \{v_6\}$	-	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_2, v_5, v_7, v_9, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\}, \{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\}, \{v_6\}$
9	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{19}, v_{20}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}$
10	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\}, \{v_{19}, v_{20}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\}, \{v_{19}, v_{20}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{23}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\}, \{v_{19}, v_{20}\}$
11	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\}, \{v_{25}\}, \{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\}, \{v_{25}\}, \{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{14}, v_{15}, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\},\{v_8\}, \{v_{25}\}, \{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}$
12	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}\{v_5, v_7, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}, v_{15}\}$	-	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}, v_{15}\}$
13	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{21}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\}$
14	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}\{v_5, v_7, v_{16}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}\{v_5, v_7, v_{16}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}\{v_5, v_7, v_{16}, v_{26}, v_{27}\},\{v_{17}, v_{18}\}, \{v_8\},\{v_{25}\},\{v_{22}, v_{24}\},\{v_3\},\{v_6\},\{v_2, v_9\}, \{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\}$
15	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{16}, v_{26}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\}, \{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\},\{v_{27}\}$	-	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\}, \{v_5, v_7, v_{16}, v_{26}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\}, \{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}, v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\},\{v_{27}\}$
16	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{26}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\}, \{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\},\{v_{27}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{26}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\}, \{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\},\{v_{27}\}$	$\{v_1, v_4, v_{10}, v_{11}, v_{12}, v_{13}\},\{v_5, v_7, v_{16}, v_{26}\},\{v_{17}, v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}, v_{24}\}, \{v_3\},\{v_6\}, \{v_2, v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\}, \{v_{14}\}, \{v_{15}\},\{v_{21}\},\{v_{27}\}$

Clusters	The results derived by center of IFT method	Zhang et.al method [23]	Zhong Wang et.al method[24]
17	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{16},v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{16},v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{16},v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\}$
18	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
19	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{10},v_{11},v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
20	$\{v_1,v_4,v_{10},v_{11}\},\{v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	-	$\{v_1,v_4,v_{10},v_{11}\},\{v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
21	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5,v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
22	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17},v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
23	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$	-	$\{v_1,v_4,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\}$
24	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12},v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$
25	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12}\},\{v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$	-	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12}\},\{v_{13}\},\{v_5\},\{v_7,v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$
26	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12}\},\{v_{13}\},\{v_5\},\{v_7\},\{v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12}\},\{v_{13}\},\{v_5\},\{v_7\},\{v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$	$\{v_1,v_{11}\},\{v_{10}\},\{v_{12}\},\{v_{13}\},\{v_5\},\{v_7\},\{v_{26}\},\{v_{17}\},\{v_{18}\},\{v_8\},\{v_{25}\},\{v_{22}\},\{v_{24}\},\{v_3\},\{v_6\},\{v_2\},\{v_9\},\{v_{19}\},\{v_{20}\},\{v_{23}\},\{v_{14}\},\{v_{15}\},\{v_{21}\},\{v_{27}\},\{v_{16}\},\{v_4\}$

VI. CONCLUSION

In this paper, the concept of distance, center, eccentricity of an intuitionistic fuzzy tree is intro-duced. The procedure for intuitionistic fuzzification for numerical data set is proposed. This paper also provides intuitionistic fuzzy tree center based clustering techniques for numerical data set with multiple attributes to produce clusters. The algorithm is tested on a data set containing informa-tion of 27 nutrients with five features .

ACKNOWLEDGMENT

The authors would like to thank the University Grants Commission, Hyderabad, India for its financial support to Minor Research Project No.F MRP-4956(SERO/UGC) dated 17th March 2014 and to Major Research Project No.F MRP- 43-418/2014(SR) dated on August 2014.

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