Honey Bee Colony Optimization for Multiresponse Mixed-Integer Problems

Goutam Barman

Abstract— ***In this paper, honey bee optimization (HBO) is used to solve a multiple response optimization problem with mixed-integer (MI) search space. The work reported in this paper may be classified into six parts. The first part discusses on relevant literatures. In second and third part discusses about seemingly unrelated regression and desirability function. In fourth part discusses about two metaheuristics viz., ant colony optimization (ACO) and honey bee colony optimization. The fifth part provide the methodology of this study and in sixth part, the details of this research work illustrates. Standard single response test functions are selected to compare the performance of ACO and HBO. Statistical experimentation, seemingly unrelated regression (SUR), 'maximin' desirability function and HBO is used to solve the multiresponse optimization (MRO) problem. The results confirm the suitability of honey bee colony optimization for a typical multiresponse mixed integer problem.***

Keywords *– Ant Colony Optimization (ACO), Honey Bee Optimization (HBO), Seemingly Unrelated Regression (SUR), Desirability Function, Mixed-Integer (MI) Problem, Multiple Response Optimization (MRO)*

I. INTRODUCTION

Quality of a product is generally expressed in terms of multiple important characteristics or CTQ (Critical to Quality). Determining the process setting so as to get desired optimal output (response) is a critical and difficult task for researchers and practitioners. Such types of simultaneous optimization of multiple characteristics are referred as multiple response optimization problems. Multiple response optimization is a critical and important research area in quality engineering and management field. This MRO problem may be linear or nonlinear, with or without constraints, in which some or all decision variables are restricted to have integer values. Several classical computational techniques such as branch and bound technique, cutting planes technique, outer approximation technique etc. which are reasonably efficient, have been proposed in literature for solving mixed integer programming problems [1-7]. Metaheuristic iterative search strategies have shown to provide satisfactory global solution for various multimodal, continuous multiple response optimization problems.

Harington [8] first proposed the concept of a desirability function approach to handle the multiple response optimization problems. By this approach, firstly each predicted response $\hat{Y}_i$ is transform to an individual scale free desirability function d_i where $0 \le d_i \le 1$. Their individual desirability transformation function varied according to its desired target of the response viz., nominal-the-best (NTB), larger-the-best (LTB), and smaller-the-best (STB). The value of d_i increases as the "desirability" of the corresponding response increases. $d_i = 0$ represent a completely undesirable value of the i^{th} response and $d_i = 1$ represents a completely desirable or ideal response value. Then an overall composite or overall desirability measure is obtained by aggregating individual desirability $d_i^{'}s$. Thus the approach converts a multiple response problem to a single objective optimization problem. Later on, more generalized approaches on desirability function were suggested [9, 10].

Zellner [11] first proposed seemingly unrelated regression (SUR) for multiple response RSM problems. This SUR method is basically involves the application of Aitken's [12] generalized least-squares to the whole system of equations. This technique can be very useful when response variables in a multiple response RSM problem are correlated.

II. SEEMINGLY UNRELATED REGRESSION

The SUR-model is usually estimated by using the generalized least squares (GLS) method. This estimation procedure is a two-step process. In the first step, an ordinary least squares model is fit to each individual response variable and covariance matrix (Σ) is estimated. Then in the second step, required parameters (β) is computed by generalized least squares method. The best linear unbiased estimate (BLUE) of β in regression equation $y = X\beta + \varepsilon$ by GLS is

$$\hat{\beta} = \left[X^{'} \left(\Sigma^{-1} \otimes I_n \right) X \right]^{-1} X^{'} \left(\Sigma^{-1} \otimes I_n \right) y$$

If Σ is unknown or singular, as it usually is, it is impossible to calculate $\hat{\beta}$. Zellner [11] proposed a non-singular estimate of Σ given by $\hat{\Sigma} = \left(\hat{\sigma}_{ij} \right)$ where

$$\hat{\sigma}_{ij} = \frac{y_i^{'} \left[I_n - X_i \left(X_i^{'} \right)^{-1} X_i^{'} \right] \left[I_n - X_j \left(X_j^{'} X_j \right)^{-1} X_j^{'} \right] y_j}{n}$$

Then the estimation of β is given by the following

$$\hat{\beta} = \left[X^{'} \left(\hat{\Sigma}^{-1} \otimes I_n \right) X \right]^{-1} X^{'} \left(\hat{\Sigma}^{-1} \otimes I_n \right) y$$

The assumed non-singularity of Σ does not ensure the non-singularity of $\hat{\Sigma}$. Srivasta and Giles [13] showed that if the number of responses (m) is greater than the number of observations (n), then $\hat{\Sigma}$ will be singular. They also showed that $m \le n$ is necessary but not sufficient condition for non-singularity of $\hat{\Sigma}$. The primary advantage of SUR based approach is that a different set of independent variables can be used to predict the responses.

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Goutam Barman, Assistant Professor, Department of Statistics, Krishnagar Government College, West Bengal, India.

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III. DESIRABILTY FUNCTION APPROACH

Depending on whether a particular response Y_i is to be maximized, minimized or assigned a target value, different desirability function can be used. Let d_i be the i^{th} individual desirability function, L_i, U_i and T_i be the lower, upper and target values respectively that are desired for that response with $L_i \le T_i \le U_i$.

If a response is LTB (larger the better) type that is., the response is to be maximized then the individual desirability is defined as,

$$d_i = \begin{cases} 0, & \text{if} \quad \hat{Y}_i \le L_i \\ \left(\dfrac{\hat{Y}_i - L_i}{U_i - L_i}\right)^r, & \text{if} \quad L_i < \hat{Y}_i < U_i \\ 1.0, & \text{if} \quad \hat{Y}_i \ge U_i \end{cases}$$

If the response is STB (smaller the better) type that is., the response is to be minimized then

$$d_i = \begin{cases} 1.0, & \text{if} \quad \hat{Y}_i \le L_i \\ \left(\dfrac{\hat{Y}_i - U_i}{L_i - U_i}\right)^s, & \text{if} \quad L_i < \hat{Y}_i < U_i \\ 0, & \text{if} \quad \hat{Y}_i \ge U_i \end{cases}$$

Again if the response is NTB (nominal the best) type that is., for this response target is the best, then

$$d_i = \begin{cases} 0, & \text{if} \quad \hat{Y}_i < L_i \text{ or } \hat{Y}_i > U_i \\ \left(\dfrac{\hat{Y}_i - L_i}{T_i - L_i}\right)^t, & \text{if} \quad L_i \le \hat{Y}_i \le T_i \\ \left(\dfrac{\hat{Y}_i - U_i}{T_i - U_i}\right)^q, & \text{if} \quad T_i < \hat{Y}_i \le U_i \end{cases}$$

Here r, s, t and q are the user-specified exponential parameters that determine the shape of desirability function and in this work we take all values of these as 1.

Harrington [8] first suggested a geometric mean approach for composite desirability. Later on, Derringer and Suich [9] suggested a weighted composite desirability function. Kim and Lin [10] extended the concept and employed a “minimum operator” to aggregate the individual desirability function and proposed a composite desirability. In this study “minimum operator” desirability approach were selected.

IV. OPTIMIZATION STRATEGY

Two different metaheuristic strategies, viz $\mathrm{ACO_R}$ [14] and Bee algorithm [15] are used in this study to find out optimal solutions. The multi-response optimization problem is formulated based on regression analysis, and desirability functions.

Dorigo [16] first proposed the concept of an ant colony optimization (ACO) method to resolve discrete optimization problem. A pheromone deposition concept is used in ACO to probabilistically direct the search in feasible solution space. Socha and Dorigo [14] proposed a variant of ACO, so-called $\mathrm{ACO_R}$ which is suitable for continuous problem. $\mathrm{ACO_R}$ performs an incremental construction of solutions using mixture of Gaussian kernel probability density function. The pseudo code for ant colony optimization is given below:

1. Randomly generate R solutions in the search space
2. According to the fitness value, sort these R solutions
3. For all solutions, set the pheromone value and age equal to 1 and 0 resp.
4. Repeat until the termination condition is met
   i. Send G global ants and replace G weakest solutions by global search
   ii. Send L local ants to selected solutions
   iii. If the fitness value improved, move the local ants to the new solutions
   iv. Update pheromone value and global maximum
   v. Evaporate trail for all solutions

Pham et al. [15] proposed the honey bee optimization (HBO) algorithm, which is inspired by the food foraging behaviour of honey bees. In bee algorithm, local search is so-called neighborhood search. It has intensification scheme on each superior solution using recruited bees. Global search is a diversification scheme on inferior solutions. Pham et al. [17] introduced two new procedure, neighbourhood shrinking and site abandonment to increase the search accuracy. The pseudo code for honey bee colony optimization is given below:

1. Generate n random solutions in the search space to make the population
2. Evaluate the fitness of the population
3. Repeat until the termination criterion met
   i. For neighbourhood search, select m sites
   ii. Send Recruit bees for selected sites and evaluate fitness
   iii. Select the fittest bee from each site
   iv. Apply neighbourhood shrinking and site abandonment procedure
   vi. Randomly assign $(n-m)$ bees in the search space

V. METHODOLOGY

To test the performance of HBO in multiresponse mixed integer space, two metaheuristic viz. ACO and HBO are selected. Firstly, comparing the performance of these two algorithms in standard six benchmark test functions in ten dimensional mixed integer search space and then the best algorithm was select to solve multiresponse mixed integer problems. Secondly, from multiresponse mixed integer dataset develop the prediction equations by using seemingly unrelated regression modeling approach. The predicted multiple responses are then converted to a single objective problem by using desirability function approach. Subsequently, the single objective function optimized by the selected best optimization algorithm.

VI. RESULTS AND DISCUSSION

In this section, the experimental setup for evaluating the performance of the algorithms, ACO and HBO and also their performance are shown. To comparing the performance of these two algorithms, 30 times independent simulation run (replicates) for six benchmark unconstrained test functions, viz. hyper sphere, hyper ellipsoid, Rosenbrock, Griewank, Rastrigin, Ackley [18] were used. In Table 1 and Table 2, are separately shown the performance of ACO and HBO for all test functions.

All the program and simulations were performed in Matlab 7.0 environment. The laptop configuration used to run the program code is 2.10 GHz Intel dual core processor with 4

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GB RAM. Here for all test functions, success in a run by a metaheuristic is achieved in any particular run, if the following condition holds:

$$| f_{\text{obtained}} - f_{\text{known}} | < \varepsilon_1 + \varepsilon_2 * | f_{\text{known}} |$$

where f_{obtained} is the optimum value found by a metaheuristic, f_{known} is the known global optimum of the test function, ε_1 and ε_2 are two accuracy parameters which is set to be $\varepsilon_1 = \varepsilon_2 = 10^{-4}$.

Table 1. Performance of ACO in mixed integer state space

Test Function	Successes	Average fitness value	Std. dev. of fitness value	Average Number of Fitness Evaluation	RunTime (sec.)
Hyper sphere	30	0	0	1860400	173.6718
Hyper ellipsoid	30	0	0	2267300	214.1717
Rosenbrock	11	0	0	2374253	248.6341
Griewank	19	10	0	2199805	211.5382
Rastrigin	6	0.0001	0	3183299	310.3125
Ackley	30	0	0	2050271	188.3962

Table 2. Performance of HBO in mixed integer state space

Test Function	Successes	Average fitness value	Std. dev. of fitness value	Average Number of Fitness Evaluation	RunTime (sec.)
Hyper sphere	30	0	0	450	0.0629
Hyper ellipsoid	30	0	0	469.1667	0.0650
Rosenbrock	30	0	0	372.5000	0.0546
Griewank	30	9.9998	0	45175	2.1133
Rastrigin	30	0	0	925.8333	0.1050
Ackley	30	0	0	47153	2.4702

Comparing the results of Table 1 and Table 2, it is evident that HBO better than ACO in respect of success rate, solution quality and run time for the reported test functions. Instigated by the above results, HBO algorithm is selected for investigating the performance in multiple responses mixed integer state space.

In our dataset, there are five response (three NTB type and two STB type) and ten independent variables. The object of this experiments is to determine the best solutions of control variables that minimize all the response variables. Seemingly unrelated regression model [11, 13] is used to modeling the dataset. SAS statistical software is used to estimate the regression coefficients. Then all response variable is converted to a single response variable by using desirability approach [10]. The overall desirability value achieved is 0.8295

Table 3. Best solution by using HBO

Optimization Technique	Optimum Desirability Value
HBO	0.8295

VII. CONCLUSION

This paper illustrates the suitability of HBO strategy for multiresponse optimization problems with mixed integer search space. The search strategy was verified by using single and multiple response optimization problems. However scope exists to test and improve HBO in complex real life industrial mixed integer multiresponse optimization problems.

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Goutam Barman is an assistant professor of statistics in West Bengal Education Service (presently posted at Krishnagar Govt. College). He did his masters in Applied Statistics and Informatics (ASI) from IIT Bombay, India. He teaches mathematics and statistics for under graduate. His research focuses on multiple response optimization, applied statistics and operation research.

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