International Journal of Soft Computing and Engineering (IJSCE)
ISSN: 2231-2307, Volume-6 Issue-4, September 2016

Comparative Investigation of Trial load and Finite Element Methods in Analysis of Arch Dams

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Abstract*: Because of important role of dams and dam construction in human life, in the present paper the method of analysis of an important kind of dam (arch dam) has been presented in two different scientific ways and their results have been compared. In the method presented herein, the dam has been divided into horizontal elements of arcs and vertical elements of cantilevers, and using compatibility of displacements and trial and error (trial load method) the share of cantilevers and arcs from applied loads on dam have been determined. Then another analysis has been performed using Finite Element Method (FEM) by indicating stiffness matrix using iso-parametric hexahedral elements with eight nodes. Using the available equations, the displacements of nodes have been calculated. Because of high volume of calculations, computer has been used and software has been prepared. The results of these two methods have been compared to each-other. The results show that the trial load method is a reliable method in spite of the fact that simplifying assumptions have been used in its theory. As a result, an arch dam can be easily analyzed by trial load method. Also, to get more accurate results, more complete methods are necessary to solve FEM equations.

***Index Terms*: Arch dam, Trial load method, Finite Element Method, Arc analysis, Cantilever analysis.**

I. INTRODUCTION

Dam and dam construction have been of great importance in human life for optimum use of surface water reservoirs. Engineers have been trying to construct this hydraulic structure by the best available method. Arch dam is of much more importance amongst all other types of dams. An arch dam is constructed on a narrow valley with a strong foundation. This type of dams is more economical, because of less depth and volume in comparison with other types of concrete dams, but its analysis is of special difficulty and has strict need for knowledge of mathematics and strength of materials. For this purpose researchers have presented various methods of analysis. Because of complexity of the structure all methods are approximate methods so the accurate output is not presented by these methods. Amongst the different methods, cylinder theory, arc analysis method, method of trial load [1] and Finite Element Method [2], can be mentioned. In the present paper it will be tried to explain the last two methods and compare the results and through that the method of arc analysis will be presented.

Revised Version Manuscript Received on July 09, 2016.

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II. THEORY OF ANALYSIS OF ARCH DAMS BY TRIAL LOAD METHOD

Two assumptions can often be considered to analyze arch dams. At first the arch dam is divided into horizontal arcs with known lengths, each arc supported on rocky foundations of the narrow valley at two ends. Then the forces are acted upon any arc and the arcs are analyzed and the efficiency of its cross section is investigated. In this case only the arcs transfer the loads towards lateral walls of the dam. The second method of analysis, which is more accurate and is not far from reality, is that the dam is divided into vertical elements in the form of cantilever beams as well as horizontal arcs. These cantilevers are fixed at the base of the dam. In this case a fraction of external forces is transferred in the rocky walls through horizontal arcs and the remaining part is transferred through cantilevers in the foundation of the dam. Both methods can be used. The only difference is that, in the first method one of the characteristics of the structure has been ignored so that the factor of safety is higher than the second method and as a result the designed dam will not be an economical.

In second method the important point is how to divide the load between the arcs and cantilevers. As mentioned before it is assumed that arcs support a part of total load and the other part by cantilevers, but how much is it? To indicate the share of arcs and cantilevers from the loads, method of trial and error is used. At first a part of the load is given to cantilevers and the remaining part is given to arcs and then at particular points which are common in cantilever and arc, the tangential, radial, and rotational deflections are calculated for both systems and their differences are compared to each other. The attributed magnitudes of loads for arcs and cantilevers are varied until the deflections become equal at common points. Usually for the simplicity of calculations, only the compatibility of radial deflections is taken into account.

The analysis of cantilevers is similar to the analysis of a fixed end beam with variable cross section under linearly distributed load. For analysis of arcs the compatibility equations, which are usually applied in statically indeterminate structures, are used.

A. Analysis of the arcs

Any arc from a dam is an indeterminate structure with three degrees of indeterminacy. For analysis, it is divided into two halves from the crown and the internal forces are considered in both sides of the structure and principles of compatibility equations are used. It means that:

$$\delta_H^L = \delta_H^R \ , \ \delta_V^L = \delta_V^R \ , \ \theta_M^L = \theta_M^R \tag{1}$$

in which L and R indicate the left and right sides of the arc and H,V and θ denote to horizontal ,vertical and rotation displacements respectively.

Solving the system of equations, the internal forces at the crown of the arc are determined.

The arc is divided into small elements and the resultant of external forces acting on each element will be exerted as a concentric load. The rotation resulting from the moment (M) can be obtained from equation (2) in element No 3 (as an example), of Fig. 1 as follows:

$$\partial\alpha = \frac{M\Delta S}{EI} \tag{2}$$

in which

ΔS = length of the element

E = modulus of elasticity

I = moment of inertia of the section

If the depth of the arc is assumed as unit, then $I = \frac{t^3}{12}$

The total M acting on each element will be as follows:

$$M = M_E + M_C + H_C y + V_C x \tag{3}$$

Where, M_C, H_C, and V_C are the internal forces at the crown of the arc (Fig.1), and M_E is the moment produced by external forces at the element. By summing up the equation (2) on elements at left hand side of the crown of the arc, we can write:

$$\sum\nolimits_M \partial\alpha = \sum \frac{M_E \Delta S}{EI} + \sum \frac{M_C \Delta S}{EI} + \sum \frac{H_C y \Delta S}{EI} + \sum \frac{V_C X \Delta S}{EI} \tag{4}$$

Similarly this can be done on all elements at left hand side of the arc and the amount of rotation produced by moment can be computed at crown of the arc. It should be noted that the concentric loads acting on each element are because of the distributed load of water and hydrodynamic load of earthquake that have been multiplied in length of external arc of the element and acts on the middle of external surface of the element.

If the rotation produced by thermal gradient (Δf) and also the reaction of abutments are added to equation (4), then the total rotation produced at the crown of the arc can be computed as follows:

$$\sum \partial\alpha = \sum\nolimits_M \partial\alpha + \sum\nolimits_{\Delta f} \partial\alpha + \partial_a \alpha \tag{5}$$

In similar way the displacements in x and y directions can be computed at the crown of the arc considering the effect of left side elements as follows [3]:

$$\sum \partial x = \sum\nolimits_M \partial x + \sum\nolimits_T \partial x + \sum\nolimits_S \partial x + \sum\nolimits_f \partial x + \sum\nolimits_{\Delta f} \partial x + y\partial_a \alpha + \partial_a S \; Cos\alpha + \partial_a n \; Sin\alpha \tag{6}$$

$$\sum \partial y = \sum\nolimits_M \partial y + \sum\nolimits_T \partial y + \sum\nolimits_S \partial y + \sum\nolimits_f \partial y + \sum\nolimits_{\Delta f} \partial y + x\partial_a \alpha + \partial_a S \; Sin\alpha + \partial_a n \; Cos\alpha \tag{7}$$

in which M, T, S, f, and Δf represent effects of moment, tangential force, shear force, constant temperature and thermal gradient to produce deflections respectively. Subscript a represent the effect of reactions of elastic abutments of dam which can be computed using Fredrick Wogutt method [3]. This effect depends on thickness of dam in abutments, angle of dam with the base, modulus of elasticity of abutments, the ratio of dam cross section to base cross section at the intersection point, and Poison's ratio of the base.

Similar procedure can be used for right hand side of the arc. Using the compatibility equations of (1) and simplifying them, a set of equations with the internal forces of the arc at the crown as unknowns can be obtained as follows.

$$A_\alpha H_C + B_\alpha V_C + C_\alpha M_C + D_\alpha = 0 \tag{8}$$

$$A_x H_C + B_x V_C + C_x M_C + D_x = 0 \tag{9}$$

$$A_y H_C + B_y V_C + C_y M_C + D_y = 0 \tag{10}$$

In which A, B, and C are the coefficients of forces M_C, V_C, H_C and α considers the effect of rotation, the indices x and y show the effects in displacements in x and y directions, and D parameters are the coefficients for external forces of the arc.

After finding these unknowns from equations (5) to (10), the displacements produced in all elements can be obtained. For this purpose, it is enough to sum up from the crown of the arc to the element under consideration. Hence three displacement components will be obtained for all elements of the arc. If the thickness of the arc is constant Integration can be used in place of summing up, and the problem will be simpler; for this purpose U.S.B.R [1] has produced tables and simple relationships, which can be used instead of integration. Using U.S.B.R. tables and having the above-mentioned parameters for a unit load, the displacements can be computed for total load under consideration. In reference [4] some more simplifying assumptions are presented to analyze the arcs.

B. Analysis of the cantilever

At first it is assumed that similar to analysis of the arc, the load is given to cantilevers and after analysis, the displacements of points are obtained and the results are compared for common nodes in two structures. As mentioned before, analysis of a cantilever is similar to a fixed end beam with a combined section. When sharing the loads between arcs and cantilevers, vertical forces of water and the weight of concrete (dead load) are carried by cantilever and the horizontal force of earthquake by arcs. The horizontal force of water is shared between the cantilever and the arc on the base of compatibility of displacements.

The cantilever, which is a statically determinate structure, is analyzed easily using theories of strength of materials. The details can be seen in reference [4].

C. The process of trial load method

As The analysis can be performed in three methods:

Analysis of central cantilever: In this type of analysis, only the analysis of central cantilever and the arcs are considered and the displacements computed in the crown of arcs are compared with the displacements of central cantilever. In this method the compatibility of radial displacements is considered. This method is usually used in symmetric arch dams.

Analysis of radial displacement: It is similar to (a). The difference is that some other cantilevers in neighborhood of the central cantilever are analyzed.

The complete trial load method: This is similar to (b). The difference is that, rather than radial displacements, tangential and rotational displacements are computed as well.

It is obvious that the method (c) will result in more reliable and confident output because all displacements and all nodes of dam are considered. However the method is time consuming and more expensive than other methods, but it will give more economical cross sections for the dam.

III. ANALYSIS OF ARC DAMS USING FINITE ELEMENT METHOD

In this method, three dimensional, isoparametric elements are used to analyze arch dams (similar to element “solid” in SAP software) (Fig. 2).

The relationship between internal forces and nodal displacements can be written in a matrix form as follows:

$$kq = P_b + P + P_0 \qquad (11)$$

Where, k is stiffness matrix, q is the nodal displacement matrix, P is external point load matrix applied in nodes, P_b is the volume forces matrix, and P_0 is the matrix for nodal equivalent forces because of initial strains (such as effect of variation of temperature) which are written as follow:

$$k = \int B^T EBdv \qquad (12)$$

$$P_b = \int F^T bdv \qquad (13)$$

$$P_0 = \int B^T E\varepsilon_0 dv \qquad (14)$$

$$B = dF \qquad (15)$$

In which, B and F are shape functions matrix; d is differential operator to transform displacement function matrix to nodal strain matrix. E is matrix-relating stresses to strains (σ = E. ε) and b is the volume force applied on the element (such as weight of the element) and dv is volumetric differential.

In equation (14) if the initial strain is due to a uniform variation of temperature (Δf) in the element, we will have:

$$\varepsilon_0 = C_f \Delta f \quad (C_f \text{ is the coefficient of temperature drop}) \qquad (16)$$

In the next step, to determine the stuffiness matrix of each element, it is necessary to transform the elements into standard elements with unit dimension, (Fig. 2), which can be done using Jacobian matrix (J). In this case the stiffness matrix is computed from the following relationship:

$$k = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T(\xi,\eta,\zeta) EB(\xi,\eta,\zeta) \left|J(\xi,\eta,\zeta)\right| d\xi d\eta d\zeta \qquad (17)$$

In which (ξ,η,ζ) are the coordinates in the standard coordinate system for elements having eight nodes, matrix B^T is 24× 6, matrix E is 6×6 and matrix B is 6×24 then the matrix k will be 24×24.

To solve the equation (17), the so-called numerical method of Gauss-Newton [5] may be used. Similarly, the nodal volume forces and initial strain for each element is produced and the stiffness matrix of the structure is obtained and then the equation (11) is solved and matrix of nodal displacements is obtained at different points.

IV. PREPARATION OF THE SOFTWARE FOR ANALYSIS OF THE ARCH DAM

The most used software for analysis of arch dams is (ADAP) that has been prepared in U.S.B.R. It analyzes the arch dams using two-dimensional (plane strain) and three-dimensional elements. To compare the above-mentioned methods, three softwares were prepared by the authors using the theories outlined above for analysis of arch dams. For trial load method, because of high volume of repetition of operations, a computer program was prepared in Visual Basic, in which the trial load operations could be performed only for radial displacements and all cantilevers of dam. In the program, the details, which were explained in section 2, have been considered; the other two programs were prepared by FEM. The difference between these two programs was the way of reservation of stiffness matrix and the method of solution of equation (11). The main difficulties in FEM programming are the automatic selection of elements, the construction of general stiffness matrix and lack of memory to save it and solve the equation (11).

To encounter the first difficulty, it is possible to use an automatic mesh generation when the dam is symmetrical. Otherwise, all elements should be defined. For second difficulty, the numerical methods are used. So, in one of the programs, the band matrix and in the other one the frontal method [6] were used. For details of general structure of the program and subroutines the reference [4] can be seen.

V. RESULTS AND DISCUSSION OF A NUMERICAL EXAMPLE

To compare the methods, a symmetrical dam is considered and only one half of the dam is to be analyzed. To analyze the dam, four arcs and four cantilevers have been considered (Fig. 3) and the compatibility of radial displacements has been satisfied by trial load method using the prepared software. The main purpose is to determine the share of hydraulic force portion that is divided between cantilevers and arcs and also amount of the radial displacements in common points. The specifications of the dam are as follows:

- Unit weight of concrete	2400 Kg/m^3
- Unit weight of water	1000 Kg/m^3
- The acceleration of earthquake parallel to axis	0.1 g
- Temperature drop	$f = \frac{340}{t+8}$ ° C (t is section thickness)
- Radial drop of temperature is neglected	$\Delta f = 0$
- The coefficient of temperature drop	C_f = 0.000006 1/ ° C
- Allowable tensile stress	0

- Allowable compressive stress 350 Kg/cm^2

- The modulus of elasticity of dam 210000 Kg/cm^2

- The modulus of elasticity of bed rock 280000 Kg/cm^2

It is necessary the geometrical characteristics of each cantilever and also arcs should be given as data into the software. The geometrical properties of the dam are given in Table 1. The profiles of the dam as well as two cantilevers A and B are shown in Fig. 3.

For geometry of arcs the central angle, external radius, the depth of the arc and also the number of elements should be given. Using these data, the results of trial load method for cantilevers A and B will be as shown in Figs. 4 and 5. In these diagrams the horizontal contributions of water load that are carried by these cantilevers are drawn. In Fig. 6 the contribution of the arc is drawn in height h=24 m. The dam is analyzed by FEM using the software produced by the authors as well.

To compare these two methods, the radial displacements (in y direction) of central cantilever (A) at points of intersection are given in Table 2. The maximum displacements of the dam will be produced in this direction and at this place (cantilever A). (The results are on the base of the assumption that the dam is solid.)

As there is no access to true strains, one cannot judge about the accuracy of these two methods, but by comparison of the results shown in Table 2, the comparability of the results shows the reliability of the methods. Also, it can be concluded from Table 2, that the results of FEM near the base of the dam gives lesser displacements in comparison to trial load method and this is inverse in upper parts. The reason can be found in the structure of the analysis. As described in section 1-2 in trial load method to analyze the arcs, it is assumed as a curved beam with two fixed ends in which case the arc of height h=60 m will have only two fixed ends and the effect of fixity of the base will not affect on this arc. In finite element method, because of constructing the general stiffness of the structure, and interring the fixity of the base by reducing the degrees of freedom of elements, the arc of h = 60 m is fixed from sides as well as down side and so the displacements resulted in this method are lesser in arc in comparison to trial load method.

VI. CONCLUSIONS AND RECOMMENDATIONS

a. In spite of the fact that some simplifying assumptions have been considered in trial load method, still arch dams with different geometrical shapes can be analyzed by this method and in spite the progress of other methods such as finite element method, this method can compete with them. The arch dams that have been analyzed by trial load method in the past can be accepted and relied on their stability and economy.

b. The results of the computer program, that has been produced using finite element method and using band matrix, are of high errors. This is due to high number of equations which in solving simultaneous equations, the errors are increased. By investigation in the results of the solved example, it can be concluded that the importance of usual methods of analysis such as Gauss band matrix, Gauss-Seidel [7], and other methods of analysis by finite element, which are discussed in literatures, are classic methods only for education purpose and learning and not for application problem. For accurate analysis, some other special methods are necessary that will be noted in 6-4.

c. The reason for accuracy of trial load method in spite the simplification used, is the independence of equations. In this method the maximum number of simultaneous equations to be solved is 3, which definitely will not increase the error.

d. To increase the accuracy of the analysis by finite element method, the way of saving the data and method of solving the equations should be optimized. For this purpose we used frontal method in programming to reduce the memory needed so the number of elements can be increased and more accurate results can be achieved and by choosing a right method of solving equations, increase of error can be prevented. It is recommended to use other numerical methods such as Skyline [7] or use a combination of different methods to increase the efficiency. For example to reserve the equations the method of Skyline and to solve them the frontal method are recommended [4].

e. As both methods are approximate methods and there was not access to true results, it cannot be judged on the accuracy of the methods, therefore, it is recommended that a laboratory model is made and strain gauges used in the body of dam model to get more accurate and more natural results, then the accuracy of the methods can be studied.

Table 1- Geometric Dimensions of Dam in Meters

Height	Radius	Chord of arc	Thickness (t)
0	108.2	97.5	3
6	106	93	4.5
12	103	88.4	6.2
18	97.7	83.8	8.8
24	92.7	94.2	11.4
30	87.7	74.7	13.6
36	82.7	70.1	15.7
42	78	65.5	17.6
48	73.5	60.3	19.3
54	69	54.8	20.7
60	64.4	47.2	21.8
66	59.5	38.1	22.5
72	54	21.3	22.8

Table 2-Tangential displacement of cantilever 'A'

Height of arc (m)	Trial Load Method (cm)	Finite Element Method (cm)
0	5.5	6.7
24	3.3	4.1
42	2.1	1.8
60	1	0.7

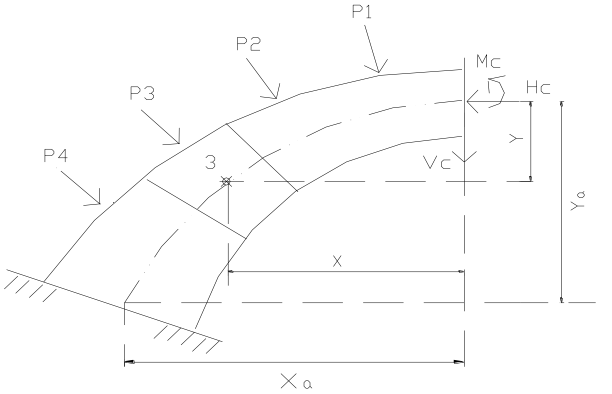

Fig. 1- One half of the selected arc from dam with applied forces

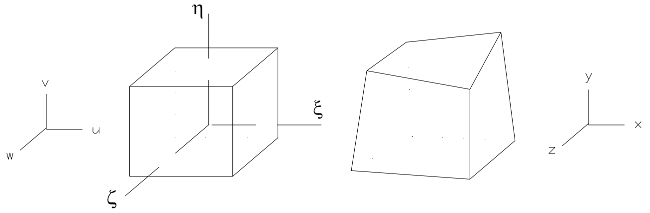

Fig.2- The standard cubic element and iso-parametric element

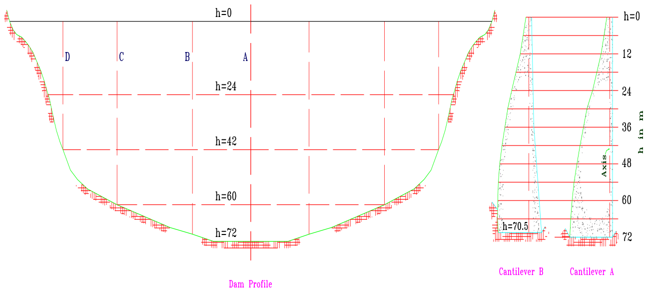

Fig.3- Dam profile and the sections of cantilevers 'A' and 'B'

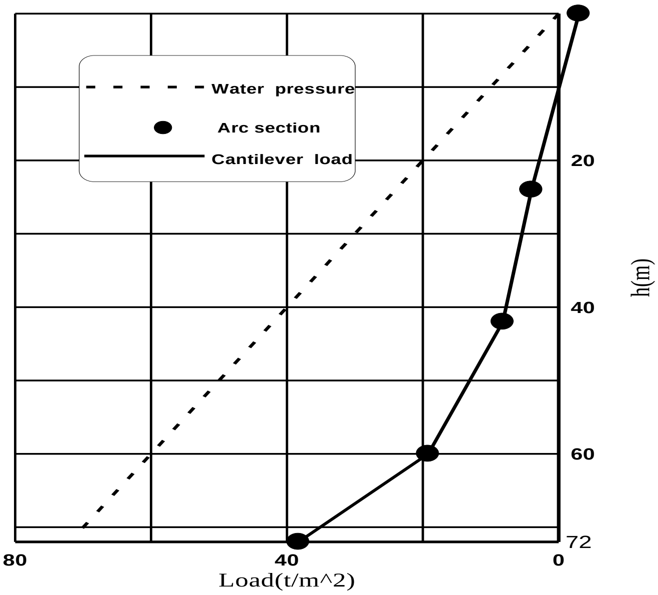

Fig.4- Load share of cantilever 'A'

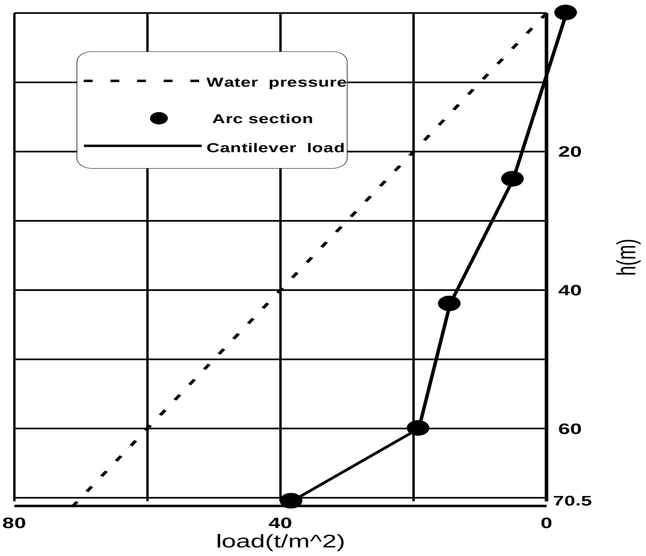

Fig.5- Load share of cantilever 'B'

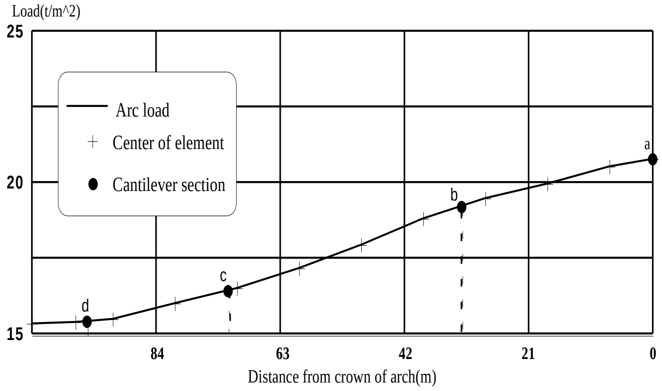

Fig. 6- Load share of arc (h=24 m.)

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