

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

Hamed Ghasemian, Qasim Zeeshan

Abstract: Failure Mode and Effect Analysis (FMEA) is a mitigative risk management tool which prevents probable failures in the system and provides the foundation for policies and remedial measures to tackle them. In this article, a method called Fuzzy Risk Priority Ranking (FRPR) is proposed based on fuzzy if-then rules and determination of fuzzy rule-based Risk Priority Number (RPN). The different combination modes of risk factors (i.e. severity (S), occurrence (O), and detection (D)) are prioritized between 1 and 1000. Comparing between FRPR and RPN approaches, and an illustrative example of an aeronautical gas turbine system the merits of the proposed method are explained.

Keywords: Failure Mode and Effect Analysis, Fuzzy rule-based RPN, Fuzzy Risk Priority Ranking

I. INTRODUCTION

The emergence of a failure is a phenomenon that can make a disorder in any complex system and results in a delay in production (Linton, 2003). Therefore, for confronting the different failures which may occur, the experts take the proper measures in different steps like designing, manufacturing, and operation (Stamatis, 1995). The common FMEA process, which has been employed since the 1960s, surveys over different kinds of failure modes in the system by prioritizing them, and then, based on the obtained rating and recognition of the critical components, the concept of Reliability Centered Maintenance (RCM) is offered. After lapse of a definite period and the renewed analysis of the failures that have occurred, the effectiveness of the maintenance policies is evaluated (Sharma et al, 2005).

1.1. FMEA Procedure

The first step to exert FMEA is categorizing the system into three levels: Main system, Subsystems, and Components, as shown in Figure 1 (adapted from Liu, 2011). In this categorization, the occurrence of a failure in a component can affect the higher levels or other subsystems. In the next step, the probable failure modes of the system are listed, and each of the considered risk factors are evaluated separately regarding each failure. The number of risk factors executable on each failure can be so high, but three of them are of greater importance, and a number between 1 and 10 is allocated to each of risk factors depending on the criticality of the failure mode.

Revised Version Manuscript Received on February 28, 2017.

Hamed Ghasemian, Ph.D. Scholar, Department of Mechanical Engineering, Eastern Mediterranean University, North Cyprus, E-mail: ghasemian.hamed@gmail.com

Dr. Qasim Zeeshan, Associate Professor, Department of Mechanical Engineering, Eastern Mediterranean University, North Cyprus.

These factors are severity (S), occurrence (O), and detection (D). In Tables 1, 2, and 3, the basis for scoring of risk factors is explained.

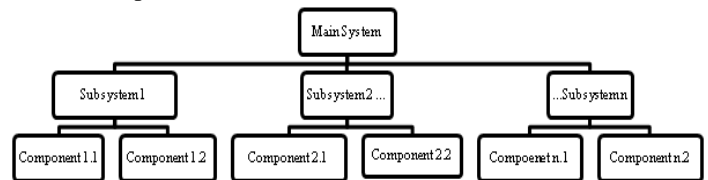


Figure 1. System Hierarchical Structure

Table 1. Severity rating criteria of a failure in FMEA (Ford Motor Company, 1988; Sankar et al, 2001; Xu et al, 2002; Chang, 2009; Chin et al, 2009; Liu et al, 2012)

Rating	Failure Effect	Severity of effect
10	Dangerous without warning	Very high severity ranking when a probable failure mode affects system operation without warning
9	Dangerous with warning	Very high severity ranking when a probable failure mode affects system operation with warning
8	Very high	System inoperable with destructive failure without safety
7	High	System inoperable with equipment damage
6	Moderate	System inoperable with minor damage
5	Low	System inoperable without damage
4	Very low	System operable with significant degradation of performance
3	Minor	System operable with some degradation of performance
2	Very minor	System operable with minimal interference
1	None	No effect

Table 2. Occurrence rating criteria of a failure in FMEA (Ford Motor Company, 1988; Sankar et al, 2001; Xu et al, 2002; Chang, 2009; Chin et al, 2009; Liu et al, 2012)

Rating	Occurrence Probability	Failure Probability
10	Nearly Certain	>0.5
9	Very High	0.16666666
8	High	0.125
7	Moderately High	0.05
6	Moderate	0.0125
5	Low	0.0025

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

4	Very Low	0.0005
3	Remote	0.000066
2	Very Remote	0.0000066
1	Nearly impossible	0.00000066

Table 3. Detection rating criteria of a failure in FMEA (Ford Motor Company, 1988; Sankar et al, 2001; Xu et al, 2002; Chang, 2009; Chin et al, 2009; Liu et al, 2012)

Rating	Detection	Likelihood of Detection by Control Mechanism
10	Absolute uncertainty	Control mechanism cannot detect potential cause of failure mode
9	Very remote	Very remote chance the control mechanism will detect potential cause of failure mode
8	Remote	Remote chance the control mechanism will detect potential cause of failure mode
7	Very low	Very low chance the control mechanism will detect potential cause of failure mode
6	Low	Low chance the control mechanism will detect potential cause of failure mode
5	Moderate	Moderate chance the control mechanism will detect potential cause of failure mode
4	Moderately high	Moderately high chance the control mechanism will detect potential cause of failure mode
3	High	High chance the control mechanism will detect potential cause of failure mode
2	Very high	Very high chance the control mechanism will detect potential cause of failure mode
1	Almost Certain	Control mechanism will almost certainly detect a potential cause of failure mode

Ultimately, by describing the following formula, the concept of Risk Priority Number (RPN) will be computed (Su et al, 2014; Maria et al, 2013; IEEE 493, 2007; Šolc, 2012):

$$RPN = S \times O \times D \quad (1)$$

Where S is severity, O is occurrence, and D is detection of the system failure mode.

The output of FMEA process can be summarized as in Table 4. In this table, other than notification of the failure mode, failure cause and effect will be evaluated and compared. The RPN obtained before and after holding maintenance policy will determine the quality of confronting the failure.

Table 4. FMEA Worksheet

Subsystem	Component	Failure mode analysis			Existing conditions					Feedback results				
		Failure mode	Failure cause	Failure effect	S	O	D	RPN	Failure disposition	S	O	D	RPN	

1.2. Drawbacks of FMEA

Due to numerous criticisms against RPN method, it has not been considered as an ideal approach and has been replaced by alternative methods in FMEA. The most important criticisms are (Sankar & Prabhu, 2001; Puente et al, 2002; Tay & Lim, 2006):

- Different combinations of S, O and D ratings may be led to production of the same value of RPN, but their hidden risk concepts may be different totally. For example, two different failure modes with the values of 5, 7, 2 and 10, 1, 7 for S, O, and D, respectively, will have the same RPN value of 70. However, the hidden risk concepts of the two failure modes may be very different because of the different severities of the failure consequence. In some cases, this may cause a high-risk failure mode being unnoticed.
- RPNs are distributed heavily at the scale from 1 to 1000 and this causes problems in interpreting the

meaning of different RPN values. For example, is the difference between the neighboring RPNs of 1 and 2 the same as or less than the difference between 10 and 20?

1.3. Literature Review of Fuzzy FMEA

The common fuzzy approach can be described as a general method substituting older ones for risk analysis. There are several reasons why this approach is evaluated as better than the previous one (Bozdog, 2015). Firstly, it can handle both precise and imprecise information in a consistent manner. Second, it allows combination of probability of failures occurrence, severity and detestability in a more pragmatic manner (Sharma et al, 2005). Finally, the risk assessment function can be varied according to the specific system under consideration (Liu et al, 2013). In Table 5, recent developments of fuzzy approaches are mentioned.



Table5. Classification of Fuzzy Approaches

FMEA Fuzzy Approach	Approach Category	Literature
Fuzzy DEA	Mathematical programming	Garcia et al (2005), Chang and Sun (2009), Chin et al (2009)
Fuzzy rule-based system	Artificial Intelligence	Bowles and Peláez (1995), Puente et al (2002), Pillay and Wang (2003), Yang et al (2008), Gargama and Chaturvedi (2011)
Fuzzy ART		Keskin and Ozkan (2009)
Fuzzy cognitive map (FCM)		Peláez and Bowles (1996)
Fuzzy AHP	Integrated approach	Abdelgawad and Fayek (2010)

In Fuzzy Data Envelopment Analysis (Fuzzy DEA) approach, risk factors (S, O and D as inputs) were modeled as fuzzy sets; where crisp values (from 1 to 10) were assigned to inputs. Fuzzy rule-based approach used for prioritizing failures in a system uses linguistic variables to describe S, O, D and fuzzy risk number. The relationships between the risk number and inputs were characterized by fuzzy if-then rules which were developed from experts' knowledge and expertise. Fuzzy Adaptive Resonance Theory (Fuzzy ART) was applied to evaluate RPN, where S, O, and D values were evaluated separately for each input. Fuzzy Cognitive Map (FCM) is a diagram to represent the causality of failures with failure node and casual relation path. The path was described by using linguistic variables (e.g. some, always, and often). In Fuzzy Analytic Hierarchy Process (Fuzzy AHP), S was referred to as impact (I) and had three dimensions: cost impact (CI), time impact (TI) and scope/quality impact (SI). Fuzzy AHP was conducted to aggregate CI, TI, and SI into a single variable entitled aggregated impact (AI).

II. FUZZY LOGIC AND FUZZY RPR APPROACH

Fuzzy logic is based upon definition of fuzzy sets consisting of elements in a bounded range, which membership function specifies the set elements; and a value called membership degree within the unit interval [0, 1] is assigned to each element. If the given element does not belong to the set, then the assigned value is 0. If the element belongs to the set, then membership degree is 1 and if the value lies within the interval (0, 1), then the element only partially belongs to the set. Fuzzy numbers are special cases of fuzzy sets. A fuzzy number is a convex fuzzy set characterized by a given interval of real numbers, each with a membership degree between 0 and 1. The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are respectively defined as the following functions (fuzzy sets A_1 and A_2 in order respectively), where for brevity triangular and trapezoidal fuzzy numbers are often denoted as (a,b,d) and (a,b,c,d). Obviously, triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers with $b = c$. The method proposed in this article can be regarded as a kind of the development for fuzzy rule-based approach, because in this method, at two steps the fuzzy logic controllers (as shown in Figure 2) based on the Tables 5 & 6 will determine the fuzzy rule-

based RPN and after that a number between 1 and 1000 is allocated to failure modes for prioritizing them.

$$\text{Triangular membership functions} = \mu(x) = \begin{cases} (x-a) / (b-a), & a \leq x \leq b, \\ (d-x) / (d-b), & b \leq x \leq d, \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Triangular membership functions} = \mu(x) = \begin{cases} (x-a) / (b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ (d-x) / (d-c), & c \leq x \leq d, \\ 0, & \text{otherwise} \end{cases}$$

If we consider all the possible states of S, O, and D, and determine one "if-then" based rule for each of states, 1000 rules are produced finally. This is based on the importance of the states: $O = 10$, $D = 10$ and $S = 10$ are placed on the first rank and $O = 1$, $D = 1$ and $S = 1$ will be placed on the 1000th rank. In a general state, the two main steps of the process are as following flowchart:



Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

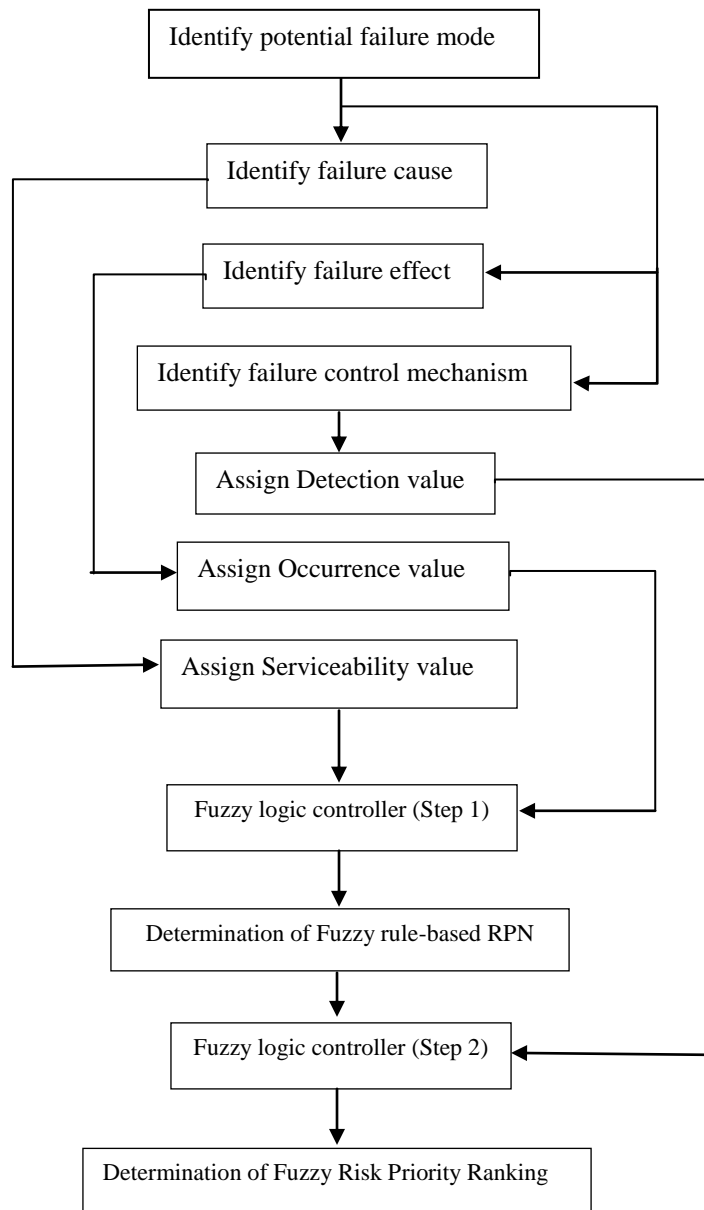


Figure 2. FRPR Process Flowchart

Step 1- Based on the combination of S and O values (each risk factor from 1 to 10), and according to the rules in Table 5 a fuzzy number is exploited (Shaout & Trivedi, 2013). This step is as the first stage of multi-stage fuzzy architecture which the related input membership functions and the generated surface of logic controller are shown in Figures 3 and 4.

Table 5. Fuzzy Rules based on Severity and Occurrence Values

		The Occurrence value									
		10	9	8	7	6	5	4	3	2	1
The Severity value	10	10.00	9.569	9.093	8.616	8.140	7.664	7.187	6.711	6.235	5.758
	9	9.440	8.964	8.488	8.011	7.535	7.059	6.582	6.106	5.630	5.153
	8	8.835	8.359	7.883	7.406	6.930	6.454	5.977	5.501	5.025	4.548
	7	8.230	7.754	7.278	6.801	6.325	5.849	5.372	4.896	4.420	3.943
	6	7.625	7.149	6.673	6.196	5.720	5.244	4.767	4.291	3.815	3.338
	5	7.021	6.544	6.068	5.592	5.115	4.639	4.163	3.686	3.210	2.734

4	6.416	5.939	5.463	4.987	4.510	4.034	3.558	3.081	2.605	2.129
3	5.811	5.334	4.858	4.382	3.905	3.429	2.953	2.476	2.000	1.524
2	5.206	4.729	4.253	3.777	3.300	2.824	2.348	1.871	1.395	0.919
1	4.6011	4.1247	3.6484	3.1721	2.6957	2.2194	1.7431	1.2667	0.7904	0.3141

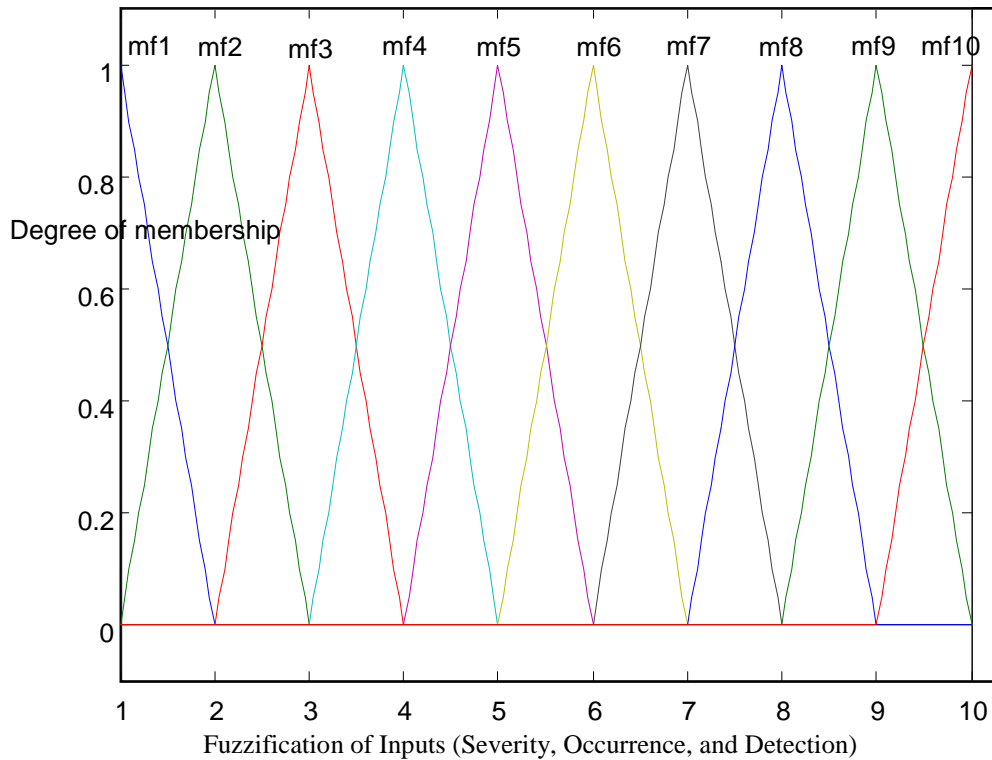


Figure3. Membership Functions of Inputs

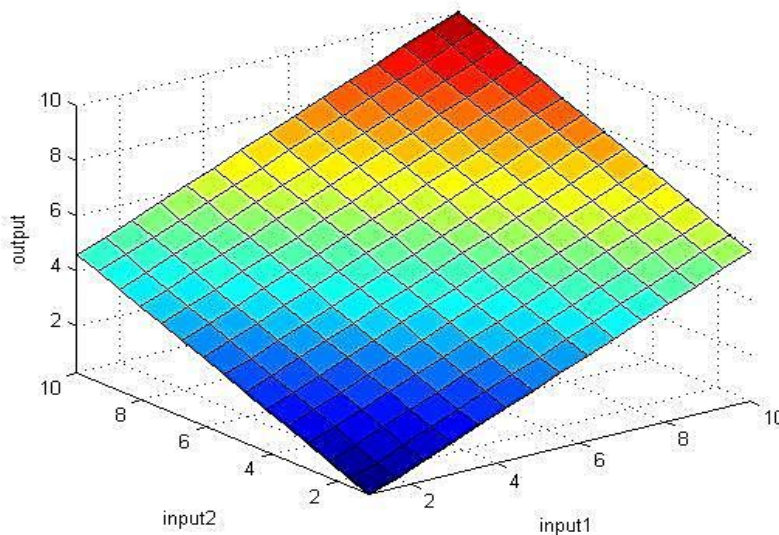


Figure 4. The Generated Surface at Each of Fuzzy Control Stages

The output number of first stage is defined based on one hundred triangular membership functions (Mamdani, 1977; Wang et al, 2009) which for each of functions, a unique fuzzy set is determined (The related MATLAB program is mentioned in Appendix A).

Step 2- In this step, Based on the combination of the number drawn in previous step and D value (from 1 to 10), the fuzzy rule-based RPN of failure mode is determined (according to the rules in Table 6).

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

The rules and configuration of inputs and output step and just the names of inputs are varied in this step. membership functions of this step are same as the previous

Table 6. Fuzzy Rules Based on Output Number of Table 5 and Detection Value

		The Detection value									
		10	9	8	7	6	5	4	3	2	1
The output value of step 1	10	10.00	9.569	9.093	8.616	8.140	7.664	7.187	6.711	6.235	5.758
	9	9.440	8.964	8.488	8.011	7.535	7.059	6.582	6.106	5.630	5.153
	8	8.835	8.359	7.883	7.406	6.930	6.454	5.977	5.501	5.025	4.548
	7	8.230	7.754	7.278	6.801	6.325	5.849	5.372	4.896	4.420	3.943
	6	7.625	7.149	6.673	6.196	5.720	5.244	4.767	4.291	3.815	3.338
	5	7.021	6.544	6.068	5.592	5.115	4.639	4.163	3.686	3.210	2.734
	4	6.416	5.939	5.463	4.987	4.510	4.034	3.558	3.081	2.605	2.129
	3	5.811	5.334	4.858	4.382	3.905	3.429	2.953	2.476	2.000	1.524
	2	5.206	4.729	4.253	3.777	3.300	2.824	2.348	1.871	1.395	0.919
	1	4.6011	4.1247	3.6484	3.1721	2.6957	2.2194	1.7431	1.2667	0.7904	0.3141

In Table 7, for some of example combinations of risk factors (S, O, and D) values, the related fuzzy rule-based RPN and FRPR are calculated and assigned.

Table7. Example Ratings of Risk Factors Combinations

Severity	Occurrence	Detection	Fuzzy rule-based risk No.	FRPR
10	10	10	9.808880107	1
10	9	10	9.618478706	2
10	8	9	8.893619909	15
10	9	8	8.935737586	13
10	10	7	8.741746294	18
10	9	7	8.588867197	21
10	8	7	7.932777249	54
10	3	10	8.026794035	50
10	5	9	8.188573777	43
10	4	9	7.455828221	94
5	10	9	7.667647059	74
7	10	8	7.921502455	57
7	6	10	7.788996764	67
7	5	10	7.233394495	128
10	7	3	5.870752688	340
5	10	8	7.418463074	99
5	8	10	7.627292737	78
8	6	3	4.743396226	576
8	8	2	4.233838384	667
10	6	1	4.903219666	542
3	10	7	6.420000000	244
2	10	8	6.357142857	258
7	6	2	4.124698795	683
9	3	1	3.000128480	818
10	1	2	3.785671493	734
2	8	7	5.344537815	452
2	10	6	5.449765258	427
4	5	4	3.800131291	731
8	1	2	2.854545455	842
8	2	1	2.654590818	868
1	10	3	3.823181258	729
2	2	10	5.168339307	487
3	7	3	3.250170526	796
7	1	2	2.594714555	872
7	2	1	2.545854484	877

1	7	5	3.520963690	765
1	6	5	2.730701754	856
4	2	4	2.436188877	886
5	2	3	2.242035657	906
5	2	1	1.510287870	957
1	3	5	2.163789869	913
1	4	4	1.547564531	955
1	5	2	1.481630864	959
1	4	2	0.802136656	987
3	1	2	1.004668578	983
1	2	3	1.147539328	977
1	1	3	1.150000000	976
2	2	1	0.560218603	995
1	2	1	0.336337307	999
1	1	1	0.336299633	1000

The advantage of this method over the RPN approach is more usefulness in the case of the unification of RPNs between two or more different failures, because in this method, the exclusive rankings are determined for each combination of S, O, and D numbers. Furthermore, the low necessity of mathematical calculations and the decrement in uncertainty level of results are other merits of the method.

**III. AN ILLUSTRATIVE EXAMPLE
(AERONAUTICAL GAS TURBINE)**

Aeronautical gas turbines have a very high power to weight ratio and are lighter and smaller than internal combustion engines of the same power. Though they are mechanically simpler than reciprocating engines, and their characteristics of high speed and high temperature operation require high

precision components and exotic materials making them more expensive to manufacture. The reliability modeling of the aeronautical gas turbine is conducted by dividing the whole working process into different functional components, each of which fulfills its respective functional diagram is designed (as shown in Figure 5). The gas turbine obtains its power by utilizing the energy of burnt gases and air which are at high pressure and temperature by expanding through the several fixed vanes and moving blades. The working of gas turbine is described thermodynamically by the Brayton cycle, which ambient air is compressed isentropically, combustion occurs at nearly constant pressure and expansion over the turbine occurs isentropically and finally gases are exhausted toward outside.

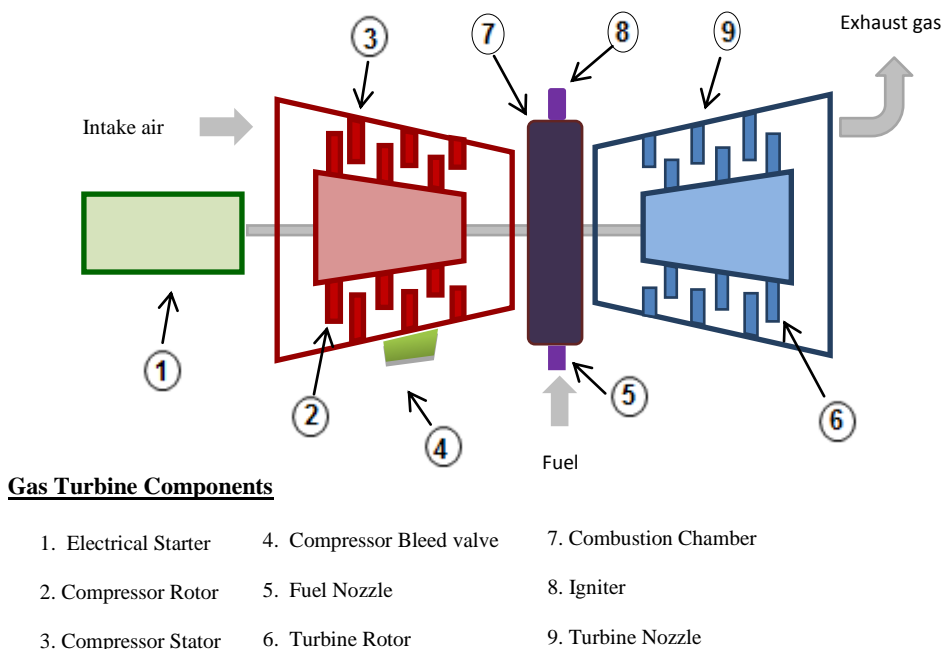


Figure 5. The Schematic of Gas Turbine System

Components

In Table 8, the typical failure modes of gas turbine are listed (based upon Meher & Gabriles, 1995; Carter, 2005; Mazur et al, 2005; Yang et al, 2011; Kazempour Liacy et al, 2011; Maktouf & Saï, 2015; Gulnar et al, 2015) and for each

failure mode, the failure cause and effect are determined and the values of risk factors and RPN

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

are provided as well. Finally according to the procedure RPN and FRPR are calculated and determined. mentioned before, for each of rows the fuzzy rule-based

Table 8. Scoring of Failure Modes in the Gas Turbine System

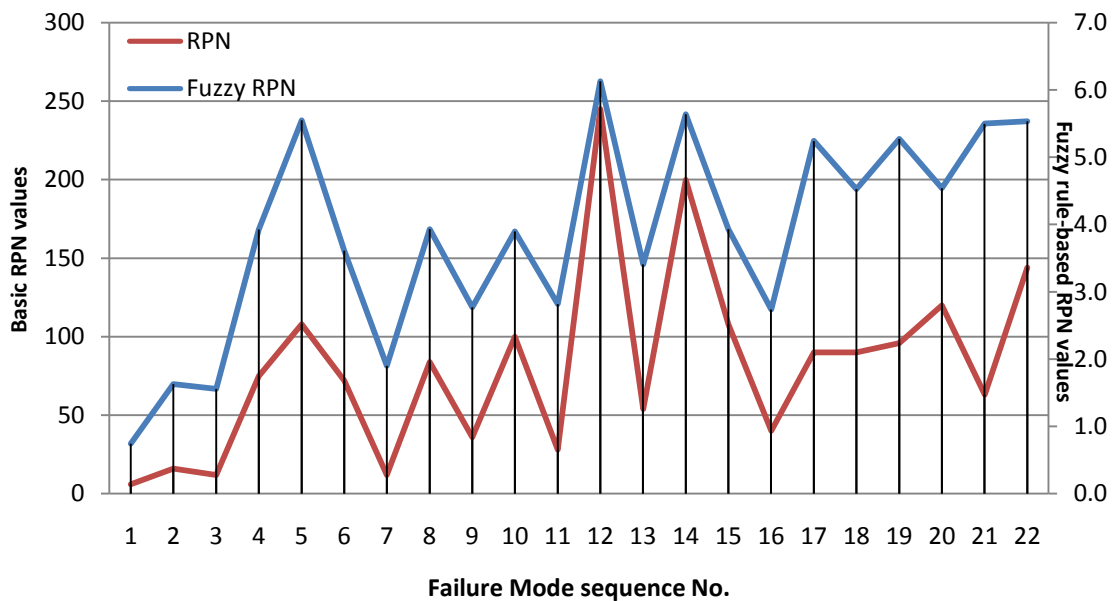
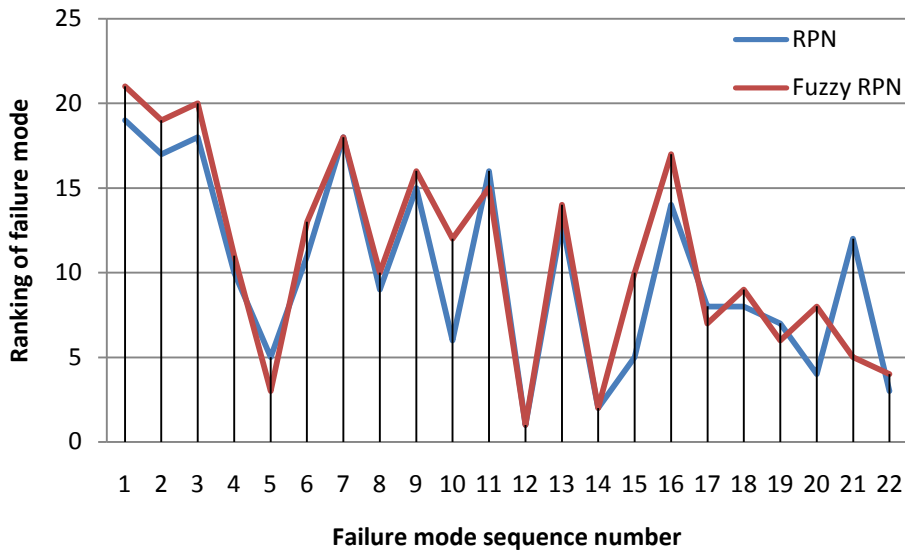
Component	Failure mode	Failure Cause	Failure effect	S	O	D	RPN	Fuzzy rule-based RPN	FRPR	Ranking by RPN	Ranking by Fuzzy RPN
Starter	No operation	No electrical power	No engine start	1	3	2	6	0.74337304289	989	19	21
	Under-speed	Induction mechanism failure	Engine is unable to reach idle speed	2	2	4	16	1.62640248595	942	17	19
	Over-speed	Drive shaft sheared	No engine start and burn of starter windings	4	1	3	12	1.55766704576	943	18	20
Compressor rotor	Vibration	Defective bearings	Oscillated structure, speed indicator fluctuation	5	3	5	75	3.92843253729	715	10	11
	Shaft locked	Rubbing of rotor blades with compressor casing	Engine coast-down lower than limits	9	2	6	108	5.55023474178	403	5	3
	Deformation	Foreign object damage	Vortex creation & stall	6	6	2	72	3.61250000000	747	11	13
Compressor stator	Stall	Ice formation on engine inlet	Increase in temperature plus speed indicator hang-up or drop-off	6	2	1	12	1.89861680619	938	18	18
		Binding of variable stator vanes		7	4	3	84	3.93293537032	702	9	10
		Foreign object damage		6	3	2	36	2.76940677966	866	15	16
Compressor bleed valve	Valve stuck open	Low compressor discharge pressure	Slow acceleration	5	5	4	100	3.90030015008	699	6	12
	Valve stuck closed	Internal spool failure	Stall during deceleration	7	2	2	28	2.81666666667	857	16	15
Combustion chamber	Hot spot	Gas temperature exceeding limits	Burning of combustion liner, Reduction of combustion efficiency	7	5	7	245	6.13000000000	291	1	1
	Gas leakage	Cracking of cases	Reduction of output power	6	3	3	54	3.40814362391	805	13	14
Fuel nozzle	Flame-out	Nozzle cloggage	Unwanted engine shut-down, drastic reduction of output power	8	5	5	200	5.63988657845	397	2	2
	Instability of flame pattern	Irregular fuel-to-air ratio		6	6	3	108	3.93293537032	665	5	10
Igniter	Eroded tips	Material removal by excessive discharge	Weak ignition while starting	5	4	2	40	2.73855932203	874	14	17
Turbine rotor	Shaft seized	Rubbing of rotor blades with turbine casing	Reduction of turbine speed	9	2	5	90	5.24285714286	495	8	7
	vibration	Defective bearings	Oscillated structure, speed indicator fluctuation	6	3	5	90	4.52666666667	650	8	9
	Deformation	Improper material and heat treatment	Drastic low power	8	2	6	96	5.27211796247	471	7	6
	Corrosion	Impurities in high-temperature gas		6	4	5	120	4.54000000000	596	4	8

Fracture	Loss of coating by thermal and centrifugal stresses	9	1	7	63	5.50000000000	368	12	5		
Turbine nozzle	Burnt vanes	Gas over-temperature	Turbulence in gas stream	8	3	6	144	5.53361169102	418	3	4

Table9. Comparison of Results in RPN and FRPR Approaches

	RPN approach	Fuzzy rule-based RPN
Max value	245	6.13000
Min value	6	0.74337

Range	239	5.38663
Average	81.77273	3.85461
Standard Deviation	58.70026	1.49467



Figures 6 & 7. Comparative graphs of RPN and FRPR values

The results show that the number of criteria for prioritization in FRPR approach is higher than that in RPN method, and it leads to a more precise distribution of failure modes in rankings. Also, as resulted in Table 9 and Figures 6 and 7 positioning of failure modes in 1000 possible ratings gives a better sense of criticality than a survey over RPNs with possibility of unification.

IV. CONCLUSION

For prioritization of system failures, Fuzzy Risk Priority Ranking (FRPR) method has been proposed and compared to the conventional Risk Priority Number (RPN) approach. The offered ranking is a development of fuzzy rule-based method,

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

and in view of the 1000 probable combinations of severity (S), occurrence (O), and detection (D) values of different failure modes this method has the capability of prioritization of all combination sets between 1 and 1000 based on the calculated fuzzy rule-based RPN for each of scored sets. Therefore, the higher the effect of a failure on the system indicates the more criticality for the system and the higher ranking allocated to it. Furthermore, this method has the capability of overcoming the shortcomings of conventional RPN method. The proposed method accounts for the uncertainty, and the lack of knowledge and experience of the FMEA team.

REFERENCES

1. Abdelgawad, M., & Fayek, A. R. (2010). Risk management in the construction industry using combined fuzzy FMEA and fuzzy AHP. *Journal of Construction Engineering and Management*, 136, 1028–1036.
2. Bowles, J. B., & Peláez, C. E. (1995). Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis. *Reliability Engineering and System Safety*, 50, 203–213.
3. Bozdogan, E., Asan, U., Soyer, A., Serdarasan, S., (2015). Risk prioritization in Failure Mode and Effect Analysis using interval type-2 fuzzy sets. *Expert Systems with Applications*, 42, 4000–4015.
4. Carter T. J., Common failures in gas turbine blades, *Engineering Failure Analysis* 12 (2005) 237–247.
5. Chang D.S. & Sun K.L.P. (2009) Applying DEA to enhance assessment capability of FMEA. *International Journal of Quality & Reliability Management*. 26. 629–643.
6. Chin, K. S., Wang, Y. M., Ka Kwai Poon, G., & Yang, J. B. (2009). Failure mode and effects analysis using a group-based evidential reasoning approach. *Computers & Operations Research*, 36, 1768–1779.
7. Ford Motor Company. (1988) Potential failure mode and effect analysis (FMEA) reference manual.
8. Garcia, P. A. A., Schirru, R., & Frutuoso Emelo, P. F. (2005). A fuzzy data envelopment analysis approach for FMEA. *Progress in Nuclear Energy*, 46, 359–373.
9. Gargama, H., & Chaturvedi, S. K. (2011). Criticality assessment models for failure mode effects and criticality analysis using fuzzy logic. *IEEE Transactions on Reliability*, 60, 102–110.
10. Gulnar et al, 2015. Model based reasoning approach for automated failure analysis: An industrial gas turbine application. Annual conference of the prognostics and health management society.
11. Kazempour-Liacy H. et al. Failure analysis of a repaired gas turbine nozzle. *Engineering Failure Analysis* 18 (2011) 510–516.
12. Keskin, G. A., & Ozkan, C. (2009). An alternative evaluation of FMEA: Fuzzy ART algorithm. *Quality and Reliability Engineering International*, 25, 647–661.
13. Linton JD. Facing the challenges of service automation: an enabler for e-commerce and productivity gain in traditional services. *IEEE Trans Eng. Manage* 2003;50(4):478–84.
14. Liu H.C., Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory, *Expert Systems with Applications* 38 (2011) 4403–4415.
15. Liu H.C. et al. (2012) Risk evaluation in failure mode and effect analysis with extended VIKOR method under fuzzy environment. *Expert systems with Applications*. 38. 4403-4415.
16. Liu H.C., Liu L., Liu N., (2013). Risk evaluation approaches in failure mode and effect analysis: A literature review. *Expert Systems with Application*, 40, 828–838.
17. Maktouf W., Saï K., An investigation of premature fatigue failures of gas turbine Blade. *Engineering Failure Analysis* 47 (2015) 89–101.
18. Mamdani, E.H., "Applications of fuzzy logic to approximate reasoning using linguistic synthesis," *IEEE Transactions on Computers*, Vol. 26, No. 12, pp. 1182–1191, 1977.
19. Maria jayaprakash A, Senthilvelan T. Failure detection and optimization of sugar mill boiler using FMEA and Taguchi method. *Engineering Failure Analysis* 2013;30:17–26.
20. Mazur Z., A. Luna-Ramírez, J.A. Juárez-Islas, A. Campos-Amezcu, Failure analysis of a gas turbine blade made of Inconel 738LC alloy, *Engineering Failure Analysis* 12(2005) 474–486.
21. Meher Homji C. B. & Gabriles G., 1995. Gas turbine blade failures-Causes, avoidance, and troubleshooting. *Proceeding of the 27th turbomachinery symposium*.

22. Peláez, C. E., & Bowles, J. B. (1996). Using fuzzy cognitive maps as a system model for failure modes and effects analysis. *Information Sciences*, 88, 177–199.
23. Pillay, A., & Wang, J. (2003). Modified failure mode and effects analysis using approximate reasoning. *Reliability Engineering and System Safety*, 79, 69–85.
24. Power systems reliability subcommittee of the power systems engineering committee, IEEE industry applications society, IEEE Std. 493-2007 recom- mended practice for the design of reliable industrial and commercial power systems, IEEE-SA standards board, New York, NY, IEEE 493-2007.
25. Puente, J., Pino, R., Priore, P., & de la Fuente, D. (2002). A decision support system for applying failure mode and effects analysis. *International Journal of Quality and Reliability Management*, 19(2), 137–150.
26. Sankar, N. R., & Prabhu, B. S. (2001). Modified approach for prioritization of failures in a system failure mode and effects analysis. *International Journal of Quality and Reliability Management*, 18(3), 324–335.
27. Shaout A. & Trivedi J., Performance Appraisal System– Using a Multistage Fuzzy Architecture, *International Journal of Computer and Information Technology* (ISSN: 2279 – 0764) Volume 02– Issue 03, May 2013.
28. Sharma, R. K., Kumar, D., & Kumar, P. (2005). Systematic failure mode effect analysis (FMEA) using fuzzy linguistic modeling. *International Journal of Quality & Reliability Management*, 22(9), 986–1004.
29. Šolc M. Applying of Method FMEA (Failure Mode and Effects Analysis) in the logistics process. *Advanced Research in Scientific Areas, Section12, Industrial and Civil Engineering* 2012: 1906–11.
30. Stamatis, D. H. (1995). *Failure mode and effect analysis—FMEA from theory to execution*. New York: ASQC Quality Press.
31. Su C.T., Lin H.C., Teng P.W., Yang T. Improving the reliability of electronic paper display using FMEA and Taguchi methods: a case study. *Microelectron Reliab* 2014;54:1369–77.
32. Tay, K. M., & Lim, C. P. (2006). Fuzzy FMEA with a guided rules reduction system for prioritization of failures. *International Journal of Quality and Reliability Management*, 23(8), 1047–1066.
33. Wang Y.M. et al, Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean, *Expert Systems with Applications* 36 (2009) 1195–1207.
34. Xu K, Tang LC, Xie M, Ho SL, Zhu ML. Fuzzy assessment of FMEA for engine systems. *Reliab Eng Syst Saf* 2002;75:17–29.
35. Yang, Z. L., Bonsall, S., & Wang, J. (2008). Fuzzy rule-based bayesian reasoning approach for prioritization of failures in FMEA. *IEEE Transactions on Reliability*, 57(3), 517–528.

Appendix A. Software Model of FRPR Method

As shown in Figure 8, the proposed method is based on two-stage Fuzzy Logic Controller which analysis of each stage is done through the following MATLAB program.

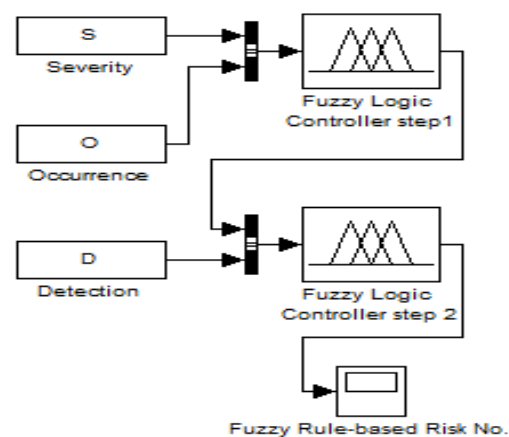


Figure A.1. FRPR Model in Simulink

Fuzzy Program in controller stage1 is as follows: (It is mentionable that Fuzzy Controller stage 2 rules are same as stage 1 and the only difference is the name of inputs, i.e. stage 1 inputs are Severity and Detection and stage 2 inputs are output of stage 1 and Detection)

1. [System]; Name='RPN1'; Type='mamdani'; Version=2.0; NumInputs=2; NumOutputs=1; NumRules=100; AndMethod='min'; OrMethod='max'; ImpMethod='min'; AggMethod='max'; DefuzzMethod='centroid'
2. [Input1]; Name='Severity'; Range=[0 10]; NumMFs=10
3. MF1='1':trimf,[0 1 2]; MF2='2':trimf,[1 2 3]; MF3='3':trimf,[2 3 4]; MF4='4':trimf,[3 4 5]; MF5='5':trimf,[4 5 6]; MF6='6':trimf,[5 6 7]; MF7='7':trimf,[6 7 8]; MF8='8':trimf,[7 8 9]; MF9='9':trimf,[8 9 10]; MF10='10':trimf,[9 10 11]
4. [Input2]; Name='Occurrence'; Range=[0 10]; NumMFs=10
5. MF1='1':trimf,[0 1 2]; MF2='2':trimf,[1 2 3]; MF3='3':trimf,[2 3 4]; MF4='4':trimf,[3 4 5]; MF5='5':trimf,[4 5 6]; MF6='6':trimf,[5 6 7]; MF7='7':trimf,[6 7 8]; MF8='8':trimf,[7 8 9]; MF9='9':trimf,[8 9 10]; MF10='10':trimf,[9 10 11]
6. [Output1]; Name='Failure_effect'; Range=[0 10]; NumMFs=100
7. MF1='10.000':trimf,[9.667 10 10.333]; MF2='9.667':trimf,[9.333 9.667 10]
8. MF3='9.333':trimf,[9 9.333 9.667]; MF4='9.000':trimf,[8.857 9 9.333]
9. MF5='8.857':trimf,[8.714 8.857 9]; MF6='8.714':trimf,[8.571 8.714 8.857]
10. MF7='8.571':trimf,[8.429 8.571 8.714]; MF8='8.429':trimf,[8.286 8.429 8.571]
11. MF9='8.286':trimf,[8.143 8.286 8.429]; MF10='8.143':trimf,[8 8.143 8.286]
12. MF11='8.000':trimf,[7.909 8 8.143]; MF12='7.909':trimf,[7.818 7.909 8]
13. MF13='7.818':trimf,[7.727 7.818 7.909]; MF14='7.727':trimf,[7.636 7.727 7.818]
14. MF15='7.636':trimf,[7.545 7.636 7.727]; MF16='7.545':trimf,[7.455 7.545 7.636]
15. MF17='7.455':trimf,[7.364 7.455 7.545]; MF18='7.364':trimf,[7.273 7.364 7.455]
16. MF19='7.273':trimf,[7.182 7.273 7.364]; MF20='7.182':trimf,[7.091 7.182 7.273]
17. MF21='7.091':trimf,[7 7.091 7.182]; MF22='7.000':trimf,[6.933 7 7.091]
18. MF23='6.933':trimf,[6.867 6.933 7]; MF24='6.867':trimf,[6.8 6.867 6.933]
19. MF25='6.800':trimf,[6.733 6.8 6.867]; MF26='6.733':trimf,[6.667 6.733 6.8]
20. MF27='6.667':trimf,[6.6 6.667 6.733]; MF28='6.600':trimf,[6.533 6.6 6.667]
21. MF29='6.533':trimf,[6.467 6.533 6.6]; MF30='6.467':trimf,[6.4 6.467 6.533]
22. MF31='6.400':trimf,[6.333 6.4 6.467]; MF32='6.333':trimf,[6.267 6.333 6.4]
23. MF33='6.267':trimf,[6.2 6.267 6.333]; MF34='6.200':trimf,[6.133 6.2 6.267]
24. MF35='6.133':trimf,[6.067 6.133 6.2]; MF36='6.067':trimf,[6 6.067 6.133]

25. MF37='6.000':trimf,[5.947 6 6.067]; MF38='5.947':trimf,[5.895 5.947 6]
26. MF39='5.895':trimf,[5.842 5.895 5.947]; MF40='5.842':trimf,[5.789 5.842 5.895]
27. MF41='5.789':trimf,[5.737 5.789 5.842]; MF42='5.737':trimf,[5.684 5.737 5.789]
28. MF43='5.684':trimf,[5.632 5.684 5.737]; MF44='5.632':trimf,[5.579 5.632 5.684]
29. MF45='5.579':trimf,[5.526 5.579 5.632]; MF46='5.526':trimf,[5.474 5.526 5.579]
30. MF47='5.474':trimf,[5.421 5.474 5.526]; MF48='5.421':trimf,[5.368 5.421 5.474]
31. MF49='5.368':trimf,[5.316 5.368 5.421]; MF50='5.316':trimf,[5.263 5.316 5.368]
32. MF51='5.263':trimf,[5.211 5.263 5.316]; MF52='5.211':trimf,[5.158 5.211 5.263]
33. MF53='5.158':trimf,[5.105 5.158 5.211]; MF54='5.105':trimf,[5.053 5.105 5.158]
34. MF55='5.053':trimf,[5 5.053 5.105]; MF56='5.000':trimf,[4.941 5 5.053]
35. MF57='4.941':trimf,[4.882 4.941 5]; MF58='4.882':trimf,[4.824 4.882 4.941]
36. MF59='4.824':trimf,[4.765 4.824 4.882]; MF60='4.765':trimf,[4.706 4.765 4.824]
37. MF61='4.706':trimf,[4.647 4.706 4.765]; MF62='4.647':trimf,[4.588 4.647 4.706]
38. MF63='4.588':trimf,[4.529 4.588 4.647]; MF64='4.529':trimf,[4.471 4.529 4.588]
39. MF65='4.471':trimf,[4.412 4.471 4.529]; MF66='4.412':trimf,[4.353 4.412 4.471]
40. MF67='4.353':trimf,[4.294 4.353 4.412]; MF68='4.294':trimf,[4.235 4.294 4.353]
41. MF69='4.235':trimf,[4.176 4.235 4.294]; MF70='4.176':trimf,[4.118 4.176 4.235]
42. MF71='4.118':trimf,[4.059 4.118 4.176]; MF72='4.059':trimf,[4 4.059 4.118]
43. MF73='4.000':trimf,[3.857 4 4.059]; MF74='3.857':trimf,[3.714 3.857 4]
44. MF75='3.714':trimf,[3.571 3.714 3.857]; MF76='3.571':trimf,[3.429 3.571 3.714]
45. MF77='3.429':trimf,[3.286 3.429 3.571]; MF78='3.286':trimf,[3.143 3.286 3.429]
46. MF79='3.143':trimf,[3 3.143 3.286]; MF80='3.000':trimf,[2.909 3 3.143]
47. MF81='2.909':trimf,[2.818 2.909 3]; MF82='2.818':trimf,[2.727 2.818 2.909]
48. MF83='2.727':trimf,[2.636 2.727 2.818]; MF84='2.636':trimf,[2.545 2.636 2.727]
49. MF85='2.545':trimf,[2.455 2.545 2.636]; MF86='2.455':trimf,[2.364 2.455 2.545]
50. MF87='2.364':trimf,[2.273 2.364 2.455]; MF88='2.273':trimf,[2.182 2.273 2.364]
51. MF89='2.182':trimf,[2.091 2.182 2.273]; MF90='2.091':trimf,[2 2.091 2.182]
52. MF91='2.000':trimf,[1.857 2 2.091]; MF92='1.857':trimf,[1.714 1.857 2]
53. MF93='1.714':trimf,[1.571 1.714 1.857]; MF94='1.571':trimf,[1.429 1.571 1.714]

Failure Mode and Effect Analysis (FMEA) of Aeronautical Gas Turbine using the Fuzzy Risk Priority Ranking (FRPR) Approach

54. MF95='1.429':trimf,[1.286 1.429 1.571];
MF96='1.286':trimf,[1.143 1.286 1.429]
55. MF97='1.143':trimf,[1 1.143 1.286];
MF98='1.000':trimf,[0.667 1 1.143]
56. MF99='0.667':trimf,[0.333 0.667 1];
MF100='0.333':trimf,[0 0.333 0.667]
57. [Rules]
58. 10 10, 1 (1) : 1; 10 9, 2 (1) : 1; 9 10, 3 (1) : 1; 10 8, 4 (1) : 1; 10 7, 5 (1) : 1; 9 9, 6 (1) : 1; 9 8, 7 (1) : 1
59. 8 10, 8 (1) : 1; 8 9, 9 (1) : 1; 7 10, 10 (1) : 1; 10 6, 11 (1) : 1; 10 5, 12 (1) : 1; 9 7, 13 (1) : 1
60. 9 6, 14 (1) : 1; 8 8, 15 (1) : 1; 8 7, 16 (1) : 1; 7 9, 17 (1) : 1; 7 8, 18 (1) : 1; 6 10, 19 (1) : 1
61. 6 9, 20 (1) : 1; 5 10, 21 (1) : 1; 10 4, 22 (1) : 1; 10 3, 23 (1) : 1; 9 5, 24 (1) : 1; 9 4, 25 (1) : 1
62. 8 6, 26 (1) : 1; 8 5, 27 (1) : 1; 7 7, 28 (1) : 1; 7 6, 29 (1) : 1; 6 8, 30 (1) : 1; 6 7, 31 (1) : 1
63. 5 9, 32 (1) : 1; 5 8, 33 (1) : 1; 4 10, 34 (1) : 1; 4 9, 35 (1) : 1; 3 10, 36 (1) : 1; 10 2, 37 (1) : 1
64. 10 1, 38 (1) : 1; 9 3, 39 (1) : 1; 9 2, 40 (1) : 1; 8 4, 41 (1) : 1; 8 3, 42 (1) : 1; 7 5, 43 (1) : 1
65. 7 4, 44 (1) : 1; 6 6, 45 (1) : 1; 6 5, 46 (1) : 1; 5 7, 47 (1) : 1; 5 6, 48 (1) : 1; 4 8, 49 (1) : 1
66. 4 7, 50 (1) : 1; 3 9, 51 (1) : 1; 3 8, 52 (1) : 1; 2 10, 53 (1) : 1; 2 9, 54 (1) : 1; 1 10, 55 (1) : 1
67. 9 1, 56 (1) : 1; 8 2, 57 (1) : 1; 8 1, 58 (1) : 1; 7 3, 59 (1) : 1; 7 2, 60 (1) : 1; 6 4, 61 (1) : 1
68. 6 3, 62 (1) : 1; 5 5, 63 (1) : 1; 5 4, 64 (1) : 1; 4 6, 65 (1) : 1; 4 5, 66 (1) : 1; 3 7, 67 (1) : 1
69. 3 6, 68 (1) : 1; 2 8, 69 (1) : 1; 2 7, 70 (1) : 1; 1 9, 71 (1) : 1; 1 8, 72 (1) : 1; 7 1, 73 (1) : 1
70. 6 2, 74 (1) : 1; 5 3, 75 (1) : 1; 4 4, 76 (1) : 1; 3 5, 77 (1) : 1; 2 6, 78 (1) : 1; 1 7, 79 (1) : 1
71. 6 1, 80 (1) : 1; 5 2, 81 (1) : 1; 5 1, 82 (1) : 1; 4 3, 83 (1) : 1; 4 2, 84 (1) : 1; 3 4, 85 (1) : 1
72. 3 3, 86 (1) : 1; 2 5, 87 (1) : 1; 2 4, 88 (1) : 1; 1 6, 89 (1) : 1; 1 5, 90 (1) : 1; 4 1, 91 (1) : 1
73. 3 2, 92 (1) : 1; 3 1, 93 (1) : 1; 2 3, 94 (1) : 1; 2 2, 95 (1) : 1; 1 4, 96 (1) : 1; 1 3, 97 (1) : 1
74. 2 1, 98 (1) : 1; 1 2, 99 (1) : 1; 1 1, 100 (1) : 1