Evaluating Volatility Forecasting Performance Measure with Generalized ARCH Models

Hemanth Kumar P., S. Basavaraj Patil

Abstract: Volatility Forecasting is an interesting and challenging problem in current financial instruments. There are many financial risks and rewards directly associated with volatility. Hence forecasting volatility becomes the most discussed topic in finance. In this research we apply various univariate conditional heteroskedasticity models for forecasting volatility. The various extensions of the standard Generalized ARCH models such as SGarch, CSGarch, Egarch, IGarch and GJRGarch are used for forecasting. Volatility values are forecasted for 10 days in advance and values are compared with the actual values. Mean square error is computed between Garch forecast and actual values for the 10 days. The model with lowest MSE values over 10 forecasted periods is selected as the best performing model. The forecasted results for 10 days show that GJRGarch ranks top in the accuracy of forecasting and IGarch at the bottom. GJRGarch outperforms all other univariate ARCH models.

Index Terms: SGARCH, CSGARCH, EGARCH, IGARCH

I. INTRODUCTION

Stock market momentum is unpredictable, has no definite regular pattern also dependent on many macroeconomic indicators. Investment in stocks are majorly dependent on stability, the stocks that are stable have more value compared to unstable ones. But the profit associated with stable stocks is limited and needs more time to increase the value. Highly unstable stocks are risky but yield more profit in a short duration. Volatility of the stocks is the key parameter in investing on stocks. High Volatility results in instability of stock market, hampering of trade, increase in rate of commodities and decease in value of currencies. The growth of Stock markets and finance are directly connected with volatility. The volatility issue has gained a lot of attention among bankers, finance regulators, trade experts and researchers. Hence volatility has become one of the most challenging and popular research areas in finance.

II. BACKGROUND

In this paper the author has done a critical assessment of forecasting techniques and evaluation of superior techniques. The author summarizes through his results that Garch techniques are best suited for forecasting Volatility. He arrives at the conclusion after 1000's of observations of Volatility forecasts on different Stock indices such as S&P 500, Nasdaq and Dow jones Index. The author also compares other alternate statistical forecasting techniques with Garch results and states that quality of volatility forecast are better with Garch when compared to alternate techniques [1]. The traditional statistical model lacks the use of stochastic process as used by Garch techniques. Garch has various sub models to overcome the irregularities in forecasting, The author uses EGARCH, IGARCH, TGARCH, GJR-GARCH, NGARCH, AVGARCH and APARCH models for forecasting errors. Few researchers use distributions such as generalized error, Student-t, exponential, normal and normal inverse Gaussian distributions. The usage of different distributions defines the uniqueness of Garch Models. EGARCH, TGARCH, NAGARCH and AVGARCH results are almost same except that TGARCH performs better than the listed types [2].

In another study of Garch models and their application with financial instruments, this paper tests the forecasting performance of four GARCH techniques such as GARCH, EGARCH, GJR and APARCH. These techniques were used with Normal, Student-t and Skewed Student-t distributions. The research explores different possibilities for improving the forecast results by changing conditional variance and distributions. The techniques were experimented on FTSE and DAX 30; these are the two major European stock indices. Daily closing prices are studied for 15 years to improve forecasting accuracies. The results suggest that improvements on estimation are achieved with asymmetric GARCH. The performance of GJR and APARCH give better forecasts than other Garch techniques [3]. In another study, the author compares ARCH-type models with different conditional variances and their performance. The models are compared with DM exchange rate data and IBM return data. The author claims that they find no evidence that a GARCH(1,1) is outperformed by more sophisticated models in our analysis of exchange rates, whereas the GARCH(1,1) is clearly inferior to models that can accommodate a leverage effect in our analysis of IBM returns. The models are compared with the test for superior predictive ability (SPA) and the reality check for data snooping (RC) [4].

This paper investigates the forecasting performance of the Garch models with 9 different error distributions. The experimentation is done on Standard Poor's 500 Index data. The author has considers daily returns of the underlying for comparison. This paper utilizes the theory of realized variance to build a volatility measurement intra-day data, it is observed that a leptokurtic error distribution leads,

To significant improvements in variance forecasts compared to using the normal distribution. The result applies for daily, weekly as well as monthly forecasts [5].

Volatility Forecasting is an interesting challenging topic in current financial instruments as it is directly associated with profits. There are many risks and rewards directly associated

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with volatility. Hence forecasting volatility becomes the most dispensable topic in finance. The GARCH distributions play an important role in the risk measurement and option pricing. The min motive of this paper is to measure the performance of GARCH techniques for forecasting volatility by using different distribution models. We have used nine variations in distribution models that are used to forecast the volatility of a stock entity. The different GARCH distribution models observed in this paper are Std, Norm, SNorm, GED, SSTD, SGED, NIG, GHYP and JSU. Volatility is forecasted for 10 days in advance and values are compared with the actual values to find out the best distribution model for volatility forecast. From the results obtained it has been observed that GARCH with GED distribution models has outperformed all models [6-7].

A) Data Description:

In this paper we use 10 years of stock market data for volatility forecasting. SP 500 index data is used as input in this paper, end of the day closing prices are used for volatility calculations. The S&P 500 is an American stock market index based on the market capitalizations of 500 large companies. It is the most valued and followed index of American Stock Market among other indices. The SP 500 index is used as the input dataset, the 10 years of data set results in 2518 samples of data set. Out of 2518 data samples 2418 samples are used for training to forecast future 30 samples. The data used in this paper **are** collected between 5th December 2005 **and** 4th December 2015. In SP 500 stock Index one data sample is generated every day except on Saturday and Sunday. The closing price plot for 10 years is shown in the Figure 1.

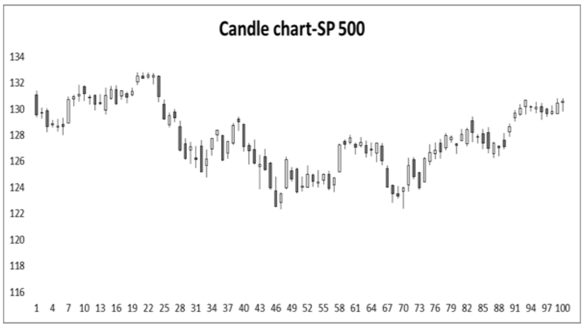

Fig.1 Candle Chart Indication OHLC of SP 500

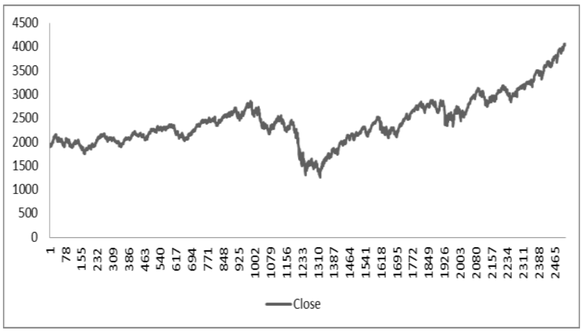

Fig.2 Closing Prices of SP 500

III. METHODOLOGY

The method followed to forecast volatility with Garch models is explained below in the steps. The pictorial representation of methodology is shown in Fig.2.

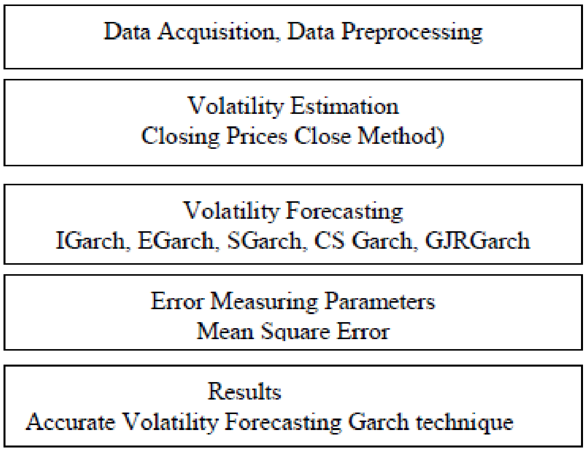

Fig.3 Pictorial Representation of Methodology

A) Data Acquisition

SP 500 index closing price data is downloaded from Yahoo Finance for Volatility forecasts requires analysis of large ample of data. 10 years closing prices are used in this experiment. The data consists of 2517 samples and the end of the day closing prices are used for volatility calculations.

B) Data Preprocessing

The downloaded financial data consists of 6 columns such as Date, Open (O), High(H), Low(L), Close(C), Volume and Adjusted Closing prices. The Open High Low Close data is often referred as OHLC data is represented using candle chart as shown in Figure 2. The historical data has latest data on the top and oldest data at the bottom. It is arranged to contain latest data at the bottom to predict the volatility values for future 30 values. The data is cleaned by eliminating missing values or NA values and imputed with mean or median values. The data is also checked for outliers and removed from the data set.

C) Estimating Volatility

The volatility is estimated from open, high, low and close values of stock data, generally volatility is calculated as the standard deviation or returns of stock data. The volatility is calculated by considering every 10 days as interval of stock data. Volatility estimation using closing is most commonly used in finance for option pricing. This estimation technique is computed using closing prices of the stock data. The plot of estimated volatility for S&P500 using Close price is as shown in Fig.2. The equation 1 indicates the formula used for estimating volatility.

$$\text{Volatility}_{\text{Close}} = \sigma_{cc} = \frac{F}{N-1}\sum_{i=1}^{N}(xi - x)^2 \qquad (1)$$

D) Forecasting of Volatility

The volatility estimated from close prices is forecasted using Garch techniques. In this paper we apply 5 different Garch models in order to forecast accurately.

The Garch models used in this paper are various extensions of the standard Generalized ARCH models. The volatility is forecasted for 10 days in advance.

E) Result Comparison

The results of Garch technique with different models are compared with error measuring parameter Absolute Error. The distribution model with lowest AE value is considered as the most accurate distribution model compared to others. The main aim of this research is to find out a distribution model with lowest error.

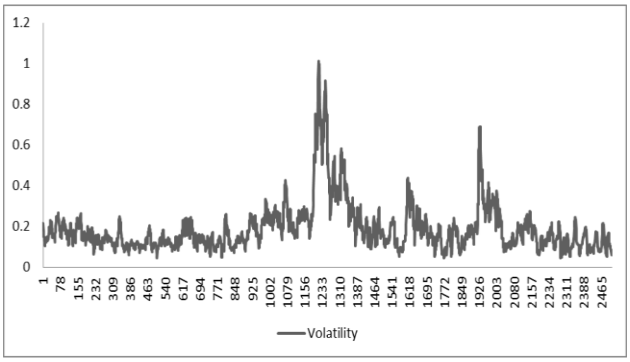

Fig.3 Volatility values plot calculated from closing prices

IV. GENERALIZED ARCH MODELS

The generalized autoregressive restrictive heteroskedasticity (GARCH) model is an econometric technique developed in 1982 by Robert F. Engle, a business analyst and 2003 champ of the Nobel Memorial Prize for Economics. The models aimed to depict volatility in financial market. There are a few types of GARCH techniques based on the distribution type used. The GARCH techniques are extensively used in financial market to forecast volatility.

ARCH models are just utilized as a part of time-series econometrics, so we can only apply this with time series data.

The generalized Autoregressive (p) model uses 'p' lag variables which is written in the form:

$$Y_t = c + \sum_{i=1}^{p} \varphi_i \, Y_{t-i} + \epsilon_t \tag{2}$$

The main principle of Auto regressive models **is** that the target variable is a linear function and is in the same pattern as of its previous lag variables. The relation can be understood with the equation 1, where t is the error term at time 't', where φ1 is constant, t is generally referred as white noise. White noise is considered as standard noise defined as independent with identical distribution with center as 0. In most of the cases Guassian noise is considered with a normal distribution and with 0 mean.

ARCH Model is being replaced by Garch Models that were suggested by Bollerslev and Taylor. These models were independent of each other in conditional variance and linear functions. The linear function was of the form

$$h_t = \alpha_0 \sum_{j=1}^{q} \alpha_j \, \varepsilon_{t-j}^2 \tag{3}$$

$$h_t = \alpha_0 \sum_{j=1}^{q} \alpha_j \, \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \, h_{t-j} \tag{4}$$

The equation 3 represents conditional variance with unconditional autocorrelation property (ε^2). The model with (ε^2) is characterized with exponential slow decays. But for ARCH model the decay rate is too high in financial time series with a lag q as represented in equation 4. ARCH Models with higher degrees are in forecasting compared to lower order degrees.

The Garch models are vast; there can be many innovations in Garch distribution type used. Based on the applications of distribution types the accuracy in forecasting can be improved. From the understanding of Garch models the conditional distributions of forecasted values have same distribution pattern as that of innovations. The selection of inappropriate distribution in Garch model may lead to under fitting or over fitting errors in forecasting. Here in this research we explore different Garch models with various distributions to check accurate financial forecasts.

From the day Garch has introduced to financial forecasts, many researchers have conducted extensive experiments and developed many Garch based algorithms. These experiments have been done to increase its flexibility and accuracy. The experiments are based on the variations in distribution functions, variances. This research paper will explore various Garch extensions such as SGarch, CSGarch, Egarch, IGarch, GJR Garch.

A) GARCH Models

The autoregressive models are expressed as

$$y_t = \emptyset(L) y_t + x_t \beta + \varepsilon_t \tag{5}$$

Where y is target or dependent variable and x is independent variable respectively, and ε is the error term.

B) IGARCH

The integrated GARCH model **was** proposed by Engle and Bollerslev .**This** model assumes that if the persistence P = 1, the same value is imposed in the estimation. As persistence is equal to 1, the results were limited and cannot be calculated. The Integrated GARCH (IGARCH) is specified as

$$\varepsilon_t = \sigma_t \, Z_t \tag{6}$$

$$\sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \, \sigma_{t-i}^2 \tag{7}$$

As discussed above here in the equation sum of coefficients is limited to 1. The variables can be easily calculated in the various GARCH models by addition of $xt\beta$.

C) EGARCH

The exponential GARCH (EGARCH) may generally be specified as

$$\varepsilon_t = \sigma_t \, Z_t \tag{7}$$

$$ln\sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \, ln \, \sigma_{t-i}^2 \tag{8}$$

This model was developed by Dhamija and Bhalla, it differs from other GARCH Models in variance structure as it uses log of the variance instead of direct variance hence it got the name Exponential GARCH. The model is also defined as

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$$ln\sigma^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \gamma_j \, ln \, \sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i \left(\frac{\varepsilon_{t-i}}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}}\right) \quad (9)$$

D) GJR GARCH

The GJR GARCH model was proposed by Glosten in 1993.The model mainly has asymmetrical conditional variance which generates positive and negative shocks.

The GJR GARCH model is represented by the expression

$$\xi_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_i \, ln \, \sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i \, I_{t-i} \, \varepsilon_{t-i}^2 \quad (10)$$

where I_{t-i} = {$1 \; if \; \beta_{t-i}$ < 0 or 0 if β_{t-i} >= 0}

here γ_i represents the term leverage. The indicator function (I) can have value of 1 for $\varepsilon \leq 0$ and 0 otherwise. Indicator function is uniqueness of GJR Garch, due to this the persistence (p) is dependent on asymmetrical conditional distribution used.

E) SGARCH

The standard GARCH model suggested by Bollerslev in 1986 is most widely used **as** general garch model, the SGarch may be written as

$$\sigma_t^2 = (\omega + \sum_{j=1}^{m} \xi_j \, v_{jt} \quad) + \quad \sum_{j=1}^{q} \alpha_j \, \varepsilon_{t-j}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2 \quad (11)$$

with σ_t^2 denotes the conditional variance, ω represents the intercept and ε_t^2 the residuals. The GARCH order is defined by (q, p) (ARCH, GARCH), with m external pre-lagged regressors v_j. SGarch variance targeting can be calculated by replacing ω by

$$\sigma^2 \, (1 - P) - \sum_{j=1}^{m} \xi_j \, v_j \quad (12)$$

SGarch are best suited for solving financial related problem as model capture volatility clusters that will be further extended or reproduced by persistence parameter P. P for SGARCH models are calculated as,

$$P = \sum_{j=1}^{q} \alpha_j + \sum_{j=1}^{q} \beta_j \quad (13)$$

F) CSGARCH

This model is generally referred **to** as Component Sgarch and it was formulated by Lee and Engle in 1999. This model can decompose the conditional variance into two components such as permanent component and transitory component. These components are designed to investigate the long-run and short-run Volatility movements of underlying financial instruments. If q_t represents permanent component of the conditional variance, the model can then be framed as:

$$\sigma_t^2 = q_t + \sum_{j=1}^{q} \alpha_j (\varepsilon_{t-j}^2 - q_{t-j}) + \sum_{j=1}^{p} \beta_j (\alpha_{t-j}^2 - q_{t-j}) \quad (14)$$

$$q_t = \omega + \rho q_{t-1} + \varphi \, (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \quad (15)$$

Here the intercept is time varying and autoregressive type of Garch model. The main difference between Garch model conditional variance is the transitory component $\alpha_{t-j}^2 - q_{t-j}$ conditional variance.

V. RESULTS

The purpose of this work was to explore an accurate volatility forecasting technique. In this research paper we have used 5 types of Garch models to forecast volatility. Out of 5 forecasts, one technique with minimal error is selected as the accurate forecasting Garch model. The accurate technique is selected based on the error calculation between actual values and forecasted values. Based on 5 Garch model results and their mse values and best model and least accurate Garch model are decided. In Table 1, the actual values are compared with forecasted values. The Volatility values are forecasted for 10 days in increasing order of the days serially. The error is the difference between the actual value and forecasted value (considered the absolute difference). The differences between the actual and forecasted values are tabulated in the Table 2. From the table 2 it is evident that GJR Garch has least values compared to other models. The error values are approximated to 4 decimal points. The forecasted values with 5 models differ by very negligible values hence the errors of all the models almost looks same.

VI. SUMMARY

In this research the paper discuss about the volatility, its importance in finance, challenges associated with volatility forecasts and forecasting applications. The aim of this paper was to explore Garch techniques and their applications in volatility forecasting. Various Garch techniques with different distribution functions are used for volatility forecasts. The various Garch models used are SGarch, CSGarch, Egarch, IGarch and GJRGarch. The Volatility is forecasted for 10 days and compared with real values. The model that has least error rate is considered as accurate technique. From the results the paper concludes that GJRGarch Model is the accurate Garch extension model compared to other Garch models in volatility forecasts.

Table 1 Forecasts Values for 10 Days using Five Different Techniques.

Day_Forecast	Actual_Volatility	CS Garch	S Garch	E Garch	I Garch	GJR GARCH
T+1	0.1146	0.114	0.114	0.114	0.114	0.1139
T+2	0.1143	0.1149	0.1149	0.1148	0.1149	0.1146
T+3	0.1023	0.1157	0.1157	0.1156	0.1157	0.1154
T+4	0.1061	0.1165	0.1166	0.1164	0.1165	0.1161
T+5	0.1063	0.1173	0.1174	0.1171	0.1173	0.1168
T+6	0.0897	0.1181	0.1181	0.1179	0.1181	0.1175
T+7	0.0841	0.1189	0.1189	0.1186	0.1189	0.1181
T+8	0.0793	0.1196	0.1197	0.1193	0.1196	0.1188
T+9	0.0611	0.1204	0.1204	0.12	0.1203	0.1194
T+10	0.0649	0.1211	0.1211	0.1206	0.1211	0.12

Table 2 Consists of Error Rates, MSE Values for Garch Techniques

Error rates	CS Garch	S Garch	E Garch	I Garch	GJRGARCH
T+1	0	0	0	0	**0**
T+2	0	0	0	0	**0**
T+3	0.0002	0.0002	0.0002	0.0002	**0.0002**
T+4	0.0001	0.0001	0.0001	0.0001	**0.0001**
T+5	0.0001	0.0001	0.0001	0.0001	**0.0001**
T+6	0.0008	0.0008	0.0008	0.0008	**0.0008**
T+7	0.0012	0.0012	0.0012	0.0012	**0.0012**
T+8	0.0016	0.0016	0.0016	0.0016	**0.0016**
T+9	0.0035	0.0035	0.0035	0.0035	**0.0034**
T+10	0.0032	0.0032	0.0031	0.0032	**0.003**

From the above tables, forecasts and error rates for 10 days with 5 different Garch techniques, it is evident that GJR Garch forecasts are accurate compared to other techniques.

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