

Bistability of Cavity Magnonics System with Magnon Kerr Effect



Debabrata Ganthya, Arumay Parai, Paresh Chandra Jana

Abstract: In this study a comprehensive theory is developed for a hybrid cavity magnonics system consisting of a microwave cavity strongly coupled to spin excitations or magnons in a single-crystal yttrium iron garnet (YIG) sample with the magnons exhibiting nonlinear Kerr effect caused by magnetocrystalline anisotropy in YIG. The system dynamics is analysed in Hamilton Langevin formulation and it is shown that the magnon frequency shift due to Kerr nonlinearity is bistable with a upper and a lower branch of cavity magnon polaritons (CMP). Further by analytically and graphically studying different conditions imposed on the bistability equation it is demonstrated that the bistability is controllable by tuning the system parameters involved.

Keywords: Kerr effect caused by magnetocrystalline anisotropy in YIG.

I. INTRODUCTION

Hybrid quantum systems have attracted significant attraction lately due to their potential application in quantum information processing and quantum communication [1,2]. Among them, cavity magnonics systems have attracted considerable attention which consist of collective spin excitations in a singlecrystal yttrium iron garnet (YIG) sample coupled to cavity photons [3–10]. The quasiparticles arising from such systems are called cavity magnon polaritons (CMP) [11,12]. The higher spin density of YIG material enables it to be completely polarized under Curie Temperature (559K) [13]. Also it has been discovered that if the coupling between YIG magnons and cavity photons sufficiently strong, the hybrid system exhibits a low damping rate [3–8]. Many fascinating phenomena have been observed in cavity magnonics systems such as magnon Kerr effect [14–16], magnon dark modes [17], optical manipulation of the system [18], bidirectional microwave-optical conversion [19], cavity spintronics [20,21], synchronized spin-photon coupling [22], cooperative polariton dynamics [23] etc. Besides magnons can be coupled to myriads of different quantum systems like superconducting qubit [24,25], optical whispering gallery modes [26–32] to construct hybrid systems with high applicative potential.

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Recently it has been experimentally demonstrated that cavity magnonics systems with nonlinear magnon Kerr effect have bistability property [14] which can be useful in constructing quantum communication and information processing devices. In this article we develop a comprehensive theoretical model to explain the magnon frequency shift in a cavity magnonics system with magnon Kerr non linearity and study the bistability property of the system. The paper has been organised in the following way:

- in section 2, we present our system which comprises a microwave cavity coupled to magnons in a YIG sample magnetized by a static magnetic field with the magnons exhibiting nonlinear Kerr effect caused by magnetocrystalline anisotropy in YIG [33,34] and formulate the system Hamiltonian.
- in section 3, we calculate the dynamics of the system in Hamilton-Langevin formalism and derive the equation of bistability of magnon frequency shift.
- in section 4 we present the plots of magnon frequency shift as a function of drive power and magnon energy and graphically demonstrate the bistability of CMP.

II. THE SYSTEM HAMILTONIAN

As shown in the schematic diagram the hybrid system comprises a YIG sphere that is coupled to a rectangular three dimensional microwave cavity through the magnetic field of cavity mode. The corresponding Hamiltonian is

$$H_s = \omega_c a^\dagger a - \gamma B_0 S_z + D_x S_x^2 + D_y S_y^2 + D_z S_z^2 + g_s (S^+ + S^-)(a^\dagger + a) \quad (1)$$

Where ω_c is the cavity mode frequency, a and $a^\dagger$ are annihilation and Creation operators corresponding to the cavity mode, γ is the gyromagnetic ratio of YIG sample, B_0 is the magnetic field inside cavity, S_x, S_y, S_z are Macrospin operators and $S^\pm$ is defined as $S^\pm = S_x \pm i S_y$. $D_j^2 j^2$ are the nonlinear terms responsible for Kerr type non linearity ($j = x, y, z$), originating from magnetocrystalline anisotropy in YIG [2-44,45]. The non linear coefficients can be derived as

$$D_j = \beta_j \frac{j \mu_0 K_{an} \gamma^2}{M^2 V_m} \quad (2)$$

Where μ_0 is the permeability of free space, K_{an} is the first order anisotropy constant of YIG, M is the saturation magnetization, V_m is the volume of YIG sample and $\beta_x, \beta_y, \beta_z$ are 3/2, 9/8, 1/2 respectively. The YIG sphere is pumped by microwave field of Rabi frequency Ω_s and frequency ω_d . The interaction Hamiltonian can be formulated as following.

$$H_d = \Omega_s (S^+ + S^-)(e^{i\omega_d t} + e^{-i\omega_d t}) \quad (3)$$



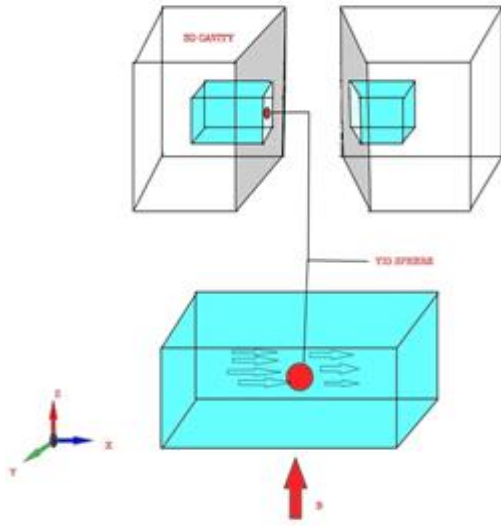


Figure 1: Upper section: The schematic diagram of YIG sphere coupled with 3D microwave cavity. Lower section: Simulated magnetic field distribution of the Fundamental cavity mode.

In addition a probe field of frequency ω_p is applied to the input port of the cavity:

$$H_P = \epsilon_p(a^\dagger + a)(e^{i\omega_p t} + e^{-i\omega_p t}) \quad (4)$$

Where ϵ_p is the coupling strength between probe field and the cavity. Therefore the total Hamiltonian $H = H_s + H_d + H_p$ will be :

$$H = \omega_c a^\dagger a - \gamma B_0 S_z + D_x S_x^2 + D_y S_y^2 + D_z S_z^2 + g_s(S^+ + S^-)(a^\dagger + a) + \Omega_s(S^+ + S^-)(e^{i\omega_d t} + e^{-i\omega_d t}) + \epsilon_p(a^\dagger + a)(e^{i\omega_p t} + e^{-i\omega_p t}) \quad (5)$$

Now using Holstein-Primakoff Transformation

$$\begin{aligned} S^+ &= \sqrt{2S - b^\dagger b} b \\ S^- &= b^\dagger \sqrt{2S - b^\dagger b} \\ S_z &= S - b^\dagger b \end{aligned} \quad (6)$$

we can transform the macrospin operators to magnon operators (b and $b^\dagger$ are the magnon annihilation and creation operators). Under the condition of very large Spin i.e. $\langle b^\dagger b \rangle / (2S) \ll 1$, we can write

$$\begin{aligned} S^+ &= \sqrt{2S(1 - b^\dagger b / (2S))} b \\ &= \sqrt{2S}(1 - b^\dagger b / 4S) b \\ S^- &= b^\dagger \sqrt{2S(1 - b^\dagger b / (2S))} \\ &= b^\dagger \sqrt{2S}(1 - b^\dagger b / 4S) \\ S_z &= S - b^\dagger b \end{aligned} \quad (7)$$

rearranging equation(4) and putting $D_x = D_y = 0$ we get,

$$\begin{aligned} H &= \omega_c a^\dagger a - \gamma B_0 S + \gamma B_0 b^\dagger b + D_z(s - b^\dagger b)(s - b^\dagger b) + g_s S^+(a^\dagger + a) \\ &+ g_s S^-(a^\dagger + a) + \Omega_s S^+(e^{i\omega_d t} + e^{-i\omega_d t}) \\ &+ \Omega_s S^-(e^{i\omega_d t} + e^{-i\omega_d t}) + \epsilon_p(a^\dagger + a)(e^{i\omega_p t} + e^{-i\omega_p t}) \end{aligned} \quad (8)$$

Now substituting equation (6) into equation (7) we derive:

$$\begin{aligned} H &= \omega_c a^\dagger a - \gamma B_0 S + \gamma B_0 b^\dagger b + D_z(S^2 - S b^\dagger b - b^\dagger b S + b^\dagger b b^\dagger b) + \\ &g_s \sqrt{2S}(1 - b^\dagger b / 4S) b(a^\dagger + a) + g_s b^\dagger(1 - b^\dagger b / 4S)(a^\dagger + a) + \\ &\Omega_s \sqrt{2S}(1 - b^\dagger b / 4S) b(e^{i\omega_d t} + e^{-i\omega_d t}) + \Omega_s \sqrt{2S}(1 - b^\dagger b / 4S) b^\dagger \\ &(e^{i\omega_d t} + e^{-i\omega_d t}) + \epsilon_p(a^\dagger + a)(e^{i\omega_p t} + e^{-i\omega_p t}) \end{aligned} \quad (9)$$

Now applying rotating-wave approximation (RWA) and neglecting the fast oscillating terms the above Hamiltonian can be written as:

$$\begin{aligned} H &= \omega_c a^\dagger a + \omega_m b^\dagger b + K b^\dagger b b^\dagger b + g_m(1 - b^\dagger b / 4S)(a^\dagger b + a b^\dagger) \\ &+ \Omega_d(1 - b^\dagger b / 4S)(b^\dagger e^{-i\omega_d t} + b e^{i\omega_d t}) + \epsilon_p(a^\dagger e^{-i\omega_p t} + a e^{i\omega_p t}) \end{aligned} \quad (10)$$

Where

$$\omega_m = \frac{\gamma B_0 + 13\mu_0 \rho_s s K_{an} \gamma^2}{8M^2} \quad (11)$$

is the angular frequency of magnon mode, where ρ_s is the net spin density of

YIG, s is the $K = \frac{-13\mu_0 K_{an} \gamma^2}{16M^2 V_m}$ microspin, is the the kerr nonlinear coefficient, $\sqrt{g_m} = \sqrt{2S} g_s$ is the collectively enhanced magnon-photon coupling strength and $\Omega_d = 2S\Omega_s$ is the spin normalised Rabi frequency.

Let,

$$b^\dagger b = \langle b^\dagger b \rangle + \delta b^\dagger b \quad (12)$$

$$a^\dagger a = \langle a^\dagger a \rangle + \delta a^\dagger a$$

Therefore,

$$\begin{aligned} b^\dagger b b^\dagger b &= (\langle b^\dagger b \rangle + \delta b^\dagger b)(\langle b^\dagger b \rangle + \delta b^\dagger b) \\ &= (\langle b^\dagger b \rangle)^2 + \langle b^\dagger b \rangle \delta b^\dagger b + \delta b^\dagger b \langle b^\dagger b \rangle + (\delta b^\dagger b)^2 \end{aligned} \quad (13)$$

Now neglecting the square of the fluctuation part the value of $b^\dagger b b^\dagger b$ will be,

$$b^\dagger b b^\dagger b = (\langle b^\dagger b \rangle)^2 + 2\langle b^\dagger b \rangle \delta b^\dagger b \quad (14)$$

From equation (11) we put $\delta b^\dagger b = b^\dagger b - \langle b^\dagger b \rangle$ into equation (13) and derive

$$\begin{aligned} b^\dagger b b^\dagger b &= (\langle b^\dagger b \rangle)^2 + 2\langle b^\dagger b \rangle(b^\dagger b - \langle b^\dagger b \rangle) \\ &= (\langle b^\dagger b \rangle)^2 + 2\langle b^\dagger b \rangle b^\dagger b - 2\langle b^\dagger b \rangle^2 \\ &= -\langle b^\dagger b \rangle^2 + 2\langle b^\dagger b \rangle b^\dagger b \end{aligned} \quad (15)$$

Applying the mean field approximation to the Hamiltonian in equation (9),

$$\begin{aligned} H &= \omega_c a^\dagger a + \omega_m b^\dagger b - K \langle b^\dagger b \rangle^2 + 2K \langle b^\dagger b \rangle b^\dagger b \\ &+ \Omega_d(1 - \langle b^\dagger b \rangle / 4S)(b^\dagger e^{-i\omega_d t} + b e^{i\omega_d t}) + \epsilon_p(a^\dagger e^{-i\omega_p t} + a e^{i\omega_p t}) \\ &+ g_m(1 - \langle b^\dagger b \rangle / 4S)(b a^\dagger + a^\dagger b) \end{aligned} \quad (16)$$

Now we consider the approximation

$(\langle b^\dagger b \rangle / 4S) \ll 1$ i.e. $(1 - \langle b^\dagger b \rangle / 4S) = 1$ and $\langle b^\dagger b \rangle^2 = 0$ and rewrite the total Hamiltonian as follows:

$$H = \omega_c a^\dagger a + (\omega_m + \Delta_m) b^\dagger b + g_m (a^\dagger b + ab^\dagger) + \Omega_d (b^\dagger e^{-i\omega_d t} + b e^{i\omega_d t}) + \epsilon (a^\dagger e^{-i\omega_p t} + a e^{i\omega_p t}) \quad (17)$$

Where, $\Delta_m = 2K\langle b^\dagger b \rangle$ is the magnon frequency shift.

III. THEORETICAL DYNAMICS

To understand the dynamics of the system we apply the Hamilton-Langevin formulation [35] and write down the quantum Langevin equations,

$$\begin{aligned} \frac{da}{dt} &= -i(\omega_c - iK_c)a - ig_m b - i\epsilon_p e^{-i\omega_p t} + \sqrt{2K_c} a_{in} \\ \frac{db}{dt} &= -i(\omega_m + \Delta_m - i\gamma_m)b - ig_m a - i\Omega_d e^{-i\omega_d t} + \sqrt{2\gamma_m} b_{in} \end{aligned} \quad (18)$$

Where K_c and γ_m are the damping rates of the cavity mode and the magnon respectively, whereas a_{in} and b_{in} are the input noise operators corresponding to cavity and magnon modes with their average value $\langle a_{in} \rangle = 0$ and $\langle b_{in} \rangle = 0$. Now Let us consider,

$$= \langle a \rangle + \delta_a \quad (19)$$

$$a = \langle b \rangle + \delta_b$$

Where δ_a and δ_b are the fluctuation parts and $\langle a \rangle$ and $\langle b \rangle$ are the expectation values of operator a and b . Under the above assumption, equation (18) will transform as

$$\begin{aligned} \frac{d\langle a \rangle}{dt} &= -i(\omega_c - iK_c)\langle a \rangle - ig_m \langle b \rangle - i\epsilon_p e^{-i\omega_p t} \\ \frac{d\langle b \rangle}{dt} &= -i(\omega_m + \Delta_m - i\gamma_m)\langle b \rangle - ig_m \langle a \rangle - i\Omega_d e^{-i\omega_d t} \end{aligned} \quad (20)$$

The drive field is much stronger than the Probe field i.e. $\Omega_d \gg \epsilon_p$. Therefore we can treat the Probe field as a perturbation field. Hence we consider the following ansatz

$$\begin{aligned} \langle a \rangle &= A_0 e^{-i\omega_d t} + A_1 e^{-i\omega_p t} \\ \langle b \rangle &= B_0 e^{-i\omega_d t} + B_1 e^{-i\omega_p t} \end{aligned} \quad (21)$$

Differentiating equation (21) we have,

$$\begin{aligned} \frac{d\langle a \rangle}{dt} &= -i\omega_d A_0 e^{-i\omega_d t} - i\omega_p A_1 e^{-i\omega_p t} + e^{-i\omega_d t} \frac{dA_0}{dt} + e^{-i\omega_p t} \frac{dA_1}{dt} \\ \frac{d\langle b \rangle}{dt} &= -i\omega_d B_0 e^{-i\omega_d t} - i\omega_p B_1 e^{-i\omega_p t} + e^{-i\omega_d t} \frac{dB_0}{dt} + e^{-i\omega_p t} \frac{dB_1}{dt} \end{aligned} \quad (22)$$

substituting the values $\langle a \rangle$ and $\langle b \rangle$ from equation (21) in equation (20) We get,

$$\begin{aligned} \frac{d\langle a \rangle}{dt} &= -(\omega_c - iK_c)(A_0 e^{-i\omega_d t} + A_1 e^{-i\omega_p t}) - ig_m(B_0 e^{-i\omega_d t} + B_1 e^{-i\omega_p t}) - i\epsilon_p e^{-i\omega_p t} \\ \frac{d\langle b \rangle}{dt} &= -i(\omega_m + \Delta_m - i\gamma_m)(B_0 e^{-i\omega_d t} + B_1 e^{-i\omega_p t}) - ig_m(A_0 e^{-i\omega_d t} + A_1 e^{-i\omega_p t}) - i\Omega_d e^{-i\omega_d t} \end{aligned} \quad (23)$$

Now comparing the coefficient $e^{-i\omega_d t}$ in the first two equations in equation (22) and (23), We get,

$$-i(\omega_c - iK_c)A_0 - ig_m B_0 = -i\omega_d A_0 + \frac{dA_0}{dt} \quad (24)$$

or,

$$-i(\omega_c - \omega_d - K_c)A_0 - ig_m B_0 = \frac{dA_0}{dt} \quad (25)$$

In the steady state condition, $\frac{dA_0}{dt} = 0$

Therefore,

$$-i(\omega_c - \omega_d - iK_c)A_0 = -ig_m B_0$$

or,

$$A_0 = -\frac{g_m B_0}{\delta_c - iK_c} \quad (26)$$

Where $(\omega_c - \omega_d) = \delta_c$ is the frequency detuning of cavity mode.

Now comparing the co-efficient of $e^{-i\omega_d t}$ in the last two equations in equation (22) and (23) we get,

$$-i(\omega_m + \Delta_m - i\gamma_m)B_0 - ig_m A_0 - i\Omega_d = -i\omega_d B_0 + \frac{dB_0}{dt} \quad (27)$$

or,

$$-i(\omega_m + \Delta_m - i\gamma_m - \omega_d)B_0 - ig_m A_0 - i\Omega_d = \frac{dB_0}{dt} \quad (28)$$

In the case of steady state, $\frac{dB_0}{dt} = 0$

Therefore,

$$-i(\omega_m + \Delta_m - i\gamma_m - \omega_d)B_0 - ig_m A_0 - i\Omega_d = 0$$

or,

$$(\omega_m + \Delta_m - i\gamma_m - \omega_d)B_0 + g_m A_0 + \Omega_d = 0$$

Let $(\omega_m - \omega_d) = \delta_m$ be the frequency detuning of the magnon mode.

Therefore,

$$(\delta_m + \Delta_m - i\gamma_m)B_0 + g_m A_0 + \Omega_d = 0$$

Now putting the value $A_0 = -\frac{g_m B_0}{\delta_c - iK_c}$

$$(\delta_m + \Delta_m - i\gamma_m)B_0 - \frac{g_m^2 B_0}{(\delta_c - iK_c)} + \Omega_d = 0 \quad (29)$$

or,

$$(\delta_m + \Delta_m - i\gamma_m)B_0 - \frac{g_m^2 B_0 (\delta_c + iK_c)}{(\delta_c^2 + K_c^2)} + \Omega_d = 0 \quad (30)$$

Now let $\frac{g_m^2}{\delta_c^2 + K_c^2} = \eta$,

Therefore the above equation will be reformed,

$$(\delta_m + \Delta_m - i\gamma_m - \eta\delta_c - i\eta K_c)B_0 + \Omega_d = 0 \quad (31)$$

Let us define the following:

$$\delta'_m = \delta_m - \eta\delta_c \text{ and } \gamma'_m = \gamma_m + \eta K_c$$

Hence the above equation will transform as

$$(\delta'_m + \Delta_m - i\gamma'_m)B_0 + \Omega_d = 0 \quad (32)$$

Taking Complex conjugate of equation (31), we get

$$(\delta'_m + \Delta_m + i\gamma'_m)B_0 + \Omega_d = 0 \quad (33)$$

Multiplying equation (31) and equation (32)

$$[(\delta'_m + \Delta_m)^2 + \gamma'^2_m]B_0^2 - |\Omega_d|^2 = 0 \quad (34)$$

Since, the magnon frequency shift $\Delta_m = 2K\langle b^\dagger b \rangle$ and the drive field being far greater than the probe field $B_0 e^{-i\omega_d t} \gg B_1 e^{-i\omega_p t}$ we can deduce

$$\langle b^\dagger b \rangle = |B_0|^2 \text{ and therefore } \Delta_m = 2K|B_0|^2$$

Substituting $|B_0|^2 = \frac{\Delta_m}{2K}$ in the equation (33)

$$[(\delta'_m + \Delta_m)^2 + \gamma_m'^2] \Delta_m - 2K|\Omega_d|^2 = 0 \quad (35)$$

Let, $CP_d = 2K|\Omega_d|^2$ where P_d is the drive field power and C is the coupling strength coefficient between the cavity and the magnon mode.

Hence final equation indicating the bistable behaviour of magnon frequency shift of the hybrid system is given by

$$[(\delta'_m + \Delta_m)^2 + \gamma_m'^2] \Delta_m - CP_d = 0 \quad (36)$$

The bistability equation is a cubic equation in magnon frequency shift Δ_m and consequently under some specific values of the parameters Δ_m has two turning points which are given by the solutions of the quadratic equation obtained by taking the derivative of equation (35) with respect to Δ_m

$$3\Delta_m^2 + 4\delta'_m \Delta_m + \delta_m'^2 + \gamma_m'^2 = 0 \quad (37)$$

Above equation yields two real roots under the following condition :

$$4\delta_m'^2 - 12\gamma_m'^2 > 0 \quad (38)$$

Hence we can write down the condition for the bistability of magnon frequency shift :

$$\begin{aligned} \delta'_m &> \sqrt{3}\gamma'_m, \quad K < 0 \\ \delta'_m &< -\sqrt{3}\gamma'_m, \quad K > 0 \end{aligned} \quad (39)$$

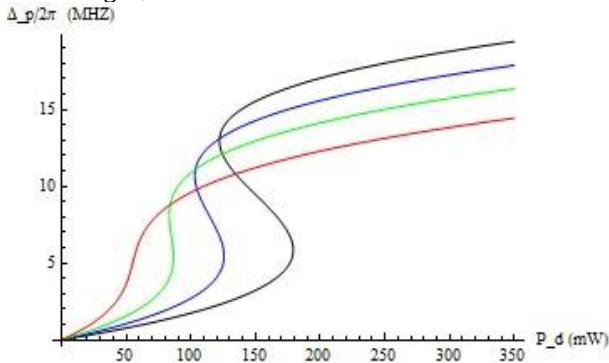
Whereas equation (36) yields only one real solution where the bistability disappears under the condition:

$$4\delta_m'^2 - 12\gamma_m'^2 = 0 \quad (40)$$

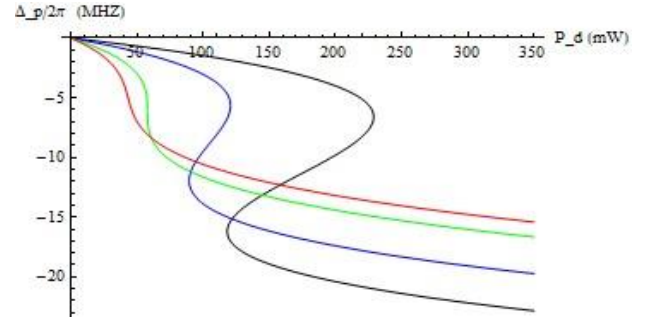
The corresponding driving power is the critical power P_c given by:

$$P_C = \pm \frac{8\sqrt{3}\gamma_m'^3}{9C} \quad (41)$$

C being positive for $K > 0$ and negative for $K < 0$. Equation (36) yields no real solutions under the condition $4\delta_m'^2 - 12\gamma_m'^2 < 0$ and Δ_m increases monotonically with increasing P_d .



(a) $\delta'_m/2\pi = -14.1, -12.1, -10.1$ and -7.5 MHz for black, blue, green and red lines respectively



(b) $\delta'_m/2\pi = 17.1, 13.2, 9.2$ and 7.5 MHz for black, blue, green and red lines respectively

Figure 2: magnon frequency shift Δ_m vs the driving power P_d for different values of effective magnon frequency detuning δ'_m

IV. RESULTS AND DISCUSSIONS

In this section we present the graphical interpretation of the bistability equation derived in the previous section with parameter values close to successfully demonstrated experiments [36] and discuss their implication. We have taken effective magnon damping rate γ'_m to be 53.25 MHz in all scenarios and rest of the parameters are specified later. In the following two subsections we plot the magnon frequency shift Δ_m against the driving power P_d and against the effective magnon frequency detuning δ'_m respectively and observe that the bistable behaviour of the hybrid system can be controlled by the system parameters involved.

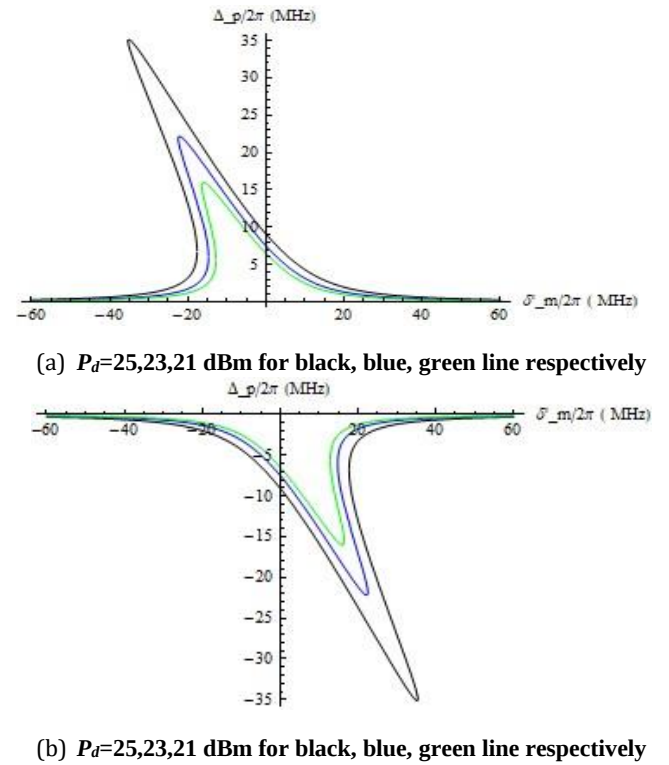


Figure 3: Magnon frequency shift Δ_m vs effective magnon frequency detuning δ'_m for several different values of driving power P_d

1.1 Magnon frequency shift against the Driving power

Fig 2 depicts the dependence of magnon frequency shift Δ_m on the driving power P_d for several different values of effective magnon frequency detuning δ'_m with Fig 2a and 2b representing the $K > 0$ ($c/(2\pi)^3 = 3.15$) and $K < 0$

($c/(2\pi)^3 = -3.15$) case respectively. It is worth noting that the bistable behaviour is controllable by choice of effective magnon frequency detuning δ'_m value. As shown in figures 2a and 2b, the bistable regime is most prominent with two distinct turning points and clear hysteresis loop for higher values of δ'_m (black and blue lines) and gradually disappears for lower values (green and red lines).

1.2 Magnon frequency shift against the Effective Magnon frequency detuning

Fig 3 depicts the dependence of magnon frequency shift Δ_m on effective magnon frequency detuning δ'_m for several different values of driving power P_d with Fig 3a and 3b representing the $K > 0$ ($c/(2\pi)^3 = 3.15$ for black and blue lines $c/(2\pi)^3 = 3.6$ green line) and $K < 0$ ($c/(2\pi)^3 = -3.15$ for black and blue lines $c/(2\pi)^3 = -3.6$ green line) case respectively. It is again worth noting that the bistable behaviour is controllable by choice of driving power P_d value. As it can be seen from figures 2a and 2b the bistability is prominent for higher driving power (black and blue lines) and disappears for lower values of P_d (green lines).

V. CONCLUSION

In this article the bistability of cavity magnon polariton system is theoretically studied by developing a simple model in Hamilton Langevin formulation and the dependence of the magnon frequency shift on various system parameters are demonstrated. It is observed that we can switch easily from upper branch of magnon frequency to lower branch and vice versa by tuning driving power and effective magnon detuning. We also can get rid of the bistable behaviour altogether by controlling the said parameters. The easily tunable bistability of the hybrid cavity magnonics system can have potential applications in quantum memories [37,38], quantum switches [39,40], dissipative phase transition [41,42] and many more related fields.

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