# Performance Comparison of Uplink Cognitive Cellular Network under Rayleigh and Nakagami-*m* Fading Environments

**M. Nazimuzzaman, Himadri S. Saha, Md. Imdadul Islam, M. R. Amin**

***Abstract*—In this paper we consider a mobile cellular network where two types of users: primary user (PU) and cognitive user (CU) share the entire spectrum of the base station (BS). Opportunistic scheduling scheme of CU is widely used to alleviate interference between CU and PU users. Recent literature deals with such networks under Rayleigh fading environment. The objective of this paper is to determine the performance of such networks under Nakagami-m fading environment and to compare the results with the results for the Rayleigh fading model. The paper shows the comparison of average bit error rate (BER) and mean channel capacity of target transmission rate taking outage probability as a parameter. It is found that for comparatively lower value of the outage probability the Nakagami-m fading has higher BER than the corresponding Rayleigh fading case whereas for higher values of the outage probability the situation becomes reverse. It is further observed that the channel capacity under Nakagami-m fading environment is better than the Rayleigh fading environment. The paper depicts the real-time performance with some explanations.***

***Index Terms*— PU, CU, average BER, mean channel capacity, opportunistic spectrum access, target transmission rate.***

## I. INTRODUCTION

Cognitive radio (CR) [1] is becoming one of the most promising transmission technologies for efficient radio spectrum utilization. Basically, there are two types of CR operations: opportunistic spectrum access (OSA) and spectrum sharing (SS). OSA is a sensing-based technology, which allows a secondary user (SU) in the CR network to opportunistically access the frequency band originally allocated to a primary user (PU) when the PU transmission is detected to be inactive. Spectrum sharing has been regarded as an important enabling function for CRs, where multiple cognitive users (CUs) can share the same spectrum with the PU on condition that CUs cannot cause harmful interference to the PU [2], [3]. It is to be noted here that in multiuser cognitive systems, different CUs experience different channel conditions for a given time slot. Spectrum utilization can be improved significantly by making it possible for a SU (who is not been serviced) to access a spectrum hole unoccupied by the PU at the right location and the time in question [4]. By opportunistic scheduling the CU, by exploiting the fluctuations of the fading channels, the spectrum utilization can be improved [1].

Some recent studies have added a further dimension to the CR protocols which allows both PU and SU to transmit simultaneously in the same frequency spectrum. In these protocols the cognitive users are assumed to be willing to collaboratively relay the PU’s information [5]. This is the opportunistic spectrum access. Capacity for opportunistic spectrum access in the absence of channel fading is analyzed in [6] and [7] but in those cases no fading effects were considered. For spectrum sharing, system capacity is analyzed under Nakagami-*m* and Rayleigh fading channel in [8].

For non-spectrum sharing environments, there have been many studies on characterizing the multiuser diversity gains [9]-[12]. Downlink multiuser diversity in a single cell is analyzed for a large number of users in [12] and [13]. The uplink performance analysis considering the Rayleigh fading environment has been studied in [14].

In this paper, we show a comparison of this opportunistic scheduling of CR network under Rayleigh fading and Nakagami-*m* fading channels.

The paper is organized as follows. The system model is discussed in Sec. II. Section III describes the results of the investigation and finally Sec. IV concludes the paper.

## II. SYSTEM MODEL

We consider an interference–limited uplink cognitive cellular network in which multiple CUs share the same base station (BS) with the PU. Let us denote the transmit powers of the PU and the CU by $P_0$ and $P$ respectively. We consider two types of channel models: Rayleigh and Nakagami-*m* channel models. Each of these channels is considered to be independent and identically distributed (iid). Let us denote the channel gains from the *i*-th PU and the CU to the BS by $G_0$ and $G_i$ respectively. The transmit powers of the CUs are assumed to be the same and therefore, the CU corresponding to the minimum channel gain is selected for transmission.

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In order to protect the PU from harmful interference, the transmit power of the selected CU should satisfy the following outage probability requirement of the PU [14]:

$$\Pr\left[\ln(1+\gamma_0) \le R_0\right] \le \zeta_0, \tag{1}$$

where $\gamma_0 = P_0 G_0 / P \min_i G_i$ is the received signal-to-interference ratio (SIR) of the PU. The quantities $R_0$ and $\zeta_0$ denote respectively the target transmission rate and the outage probability of the PU.

If $P_{\max}$ be the maximum transmit power of the CU, then the transmit power $P$ of the selected CU satisfies the following relation [14]:

$$P \le P_{\max} = \frac{NP_0}{\delta}, \tag{2}$$

where

$$\delta = \frac{e^{R_0} - 1}{\dfrac{1}{1-\zeta_0} - 1}, \tag{3}$$

and $N$ is the number of the CUs in the system.

We analyze the performances of the considered system for both the channel models, viz: Rayleigh and Nakagami-*m* models in terms of the mean capacity and the average BER of the selected CU and obtain expressions for both of them in order to investigate the effects of the main channel parameters on the system performance.

The received SIR of the selected CU can be expressed as $\gamma_{\max} = P_{\max} \min_i G / P_0 G_0$, where $P = P_{\max}$ is adopted in order to enhance the system performance. Let us consider the random variable $Z = Y / X$, where $X = P_0 G_0$ and $Y = P_{\max} \min_i G$, then the probability density function (pdf) of the received SIR, $\gamma_{\max}$, of the selected CU is [14 ]

$$f_{\gamma_{\max}}(z) = \int_0^\infty x\, f_Y(xz)\, f_X(x)\, dx, \tag{4}$$

where $f_X(x)$ and $f_Y(xz)$ are respectively the pdfs of the PU signal power and the maximum power of the CU signal.

Let us now derive the expressions for the mean capacity and the average BER of the selected CU for the two types of fading channels: Rayleigh fading channel and Nakagami-*m* fading channel environments separately.

**Rayleigh Channel Model**

As we know that if the envelope of the signal follows the Rayleigh distribution, then the signal power follows the exponential distribution, we write down the pdfs of the signal powers of PU and CU in normalized forms for the Rayleigh channel as [14]:

$$f_X(x) = \frac{1}{P_0} e^{-x/P_0}, \tag{5}$$

and

$$f_Y(xz) = \frac{N}{P_{\max}} e^{-Nxz/P_{\max}}. \tag{6}$$

In this case, the pdf of the received SIR of the selected CU is obtained by substituting the expressions for $f_X(x)$ and $f_Y(xz)$ from Eqs. (5) and (6) into Eq. (4):

$$f_{\gamma_{\max} Rayleigh}(z) = \int_0^\infty x \frac{N}{P_{\max}} e^{-Nxz/P_{\max}} \frac{1}{P_0} e^{-x/P_0}\, dx. \tag{7}$$

Using the expression (7) for the pdf of the received SIR of the selected CU, the mean capacity of the scheduled CU can be expressed as

$$\begin{aligned} C_{Rayleigh} &= E\left[\ln(1+\gamma_{\max})\right] \\ &= \int_0^\infty \ln(1+z)\, f_{\gamma_{\max} Rayleigh}(z)\, dz \\ &= \int_0^\infty \ln(1+z) \\ &\quad \times \int_0^\infty x \frac{N}{P_{\max}} e^{-Nxz/P_{\max}} \frac{1}{P_0} e^{-x/P_0}\, dx\, dz. \end{aligned} \tag{8}$$

Similarly, the average BER of the selected CU in the Rayleigh channel model is obtained as

$$\begin{aligned} P_{b Rayleigh} &= \int_0^\infty P(e|\gamma_{\max})\, f_{\gamma_{\max} Rayleigh}(z)\, dz \\ &= \int_0^\infty P(e|\gamma_{\max}) \\ &\quad \times \int_0^\infty x \frac{N}{P_{\max}} e^{-Nxz/P_{\max}} \frac{1}{P_0} e^{-x/P_0}\, dx\, dz. \end{aligned} \tag{9}$$

**Nakagami-m Channel Model**

In the Nakagami-*m* channel model, the pdfs of the signal powers of the PU and the CU in normalized form are given respectively by the Gamma distribution:

$$f_X(x) = \frac{m^m x^{m-1}}{P_0^m\, \Gamma(m)} e^{-mx/P_0}, \tag{10}$$

and

$$f_Y(xz) = \frac{N^m m^m (xz)^{m-1}}{P_{\max}^m\, \Gamma(m)} e^{-Nmxz/P_{\max}}, \tag{11}$$

where $m$ is the Nakagami-*m* parameter and is the ratio of the line-of-sight (LOS) signal power and the multipath component signal power.

The resulting pdf of the received SIR of the selected CU in the Nakagami-*m* channel model is thus

$$\begin{aligned} f_{\gamma_{\max} Nakagami-m}(z) &= \int_0^\infty x \frac{N^m m^m (xz)^{m-1}}{P_{\max}^m\, \Gamma(m)} e^{-Nmxz/P_{\max}} \\ &\quad \times \frac{m^m x^{m-1}}{P_0^m\, \Gamma(m)} e^{-x/P_0}\, dx. \end{aligned} \tag{12}$$

The expression for the mean capacity of the scheduled CU in the Nakagami-*m* channel model is obtained as

$$
\begin{aligned}
C_{Nakagami-m} &= E\left[\ln\left(1+\gamma_{\max}\right)\right] \\
&= \int_0^\infty \ln(1+z)\, f_{\gamma_{\max} Nakagami-m}(z)\, dz \\
&= \int_0^\infty \ln(1+z) \int_0^\infty x \frac{N^m m^m (xz)^{m-1}}{P_{\max}^m\, \Gamma(m)} e^{-Nmxz/P_{\max}} \\
&\times \frac{m^m x^{m-1}}{P_0^m\, \Gamma(m)} e^{-mx/P_0}\, dx\, dz.
\end{aligned}
\tag{13}
$$

Similarly, the expression for the average BER of the scheduled CU in this Nakagami-*m* channel model can be written as

$$
\begin{aligned}
P_{b\,Nakagami-m} &= \int_0^\infty P(e|\gamma_{\max})\, f_{\gamma_{\max} Nakagami-m}(z)\, dz \\
&= \int_0^\infty P(e|\gamma_{\max}) \int_0^\infty x \frac{N^m m^m (xz)^{m-1}}{P_{\max}^m\, \Gamma(m)} \\
&\times e^{-Nmxz/P_{\max}} \frac{m^m x^{m-1}}{P_0^m\, \Gamma(m)} e^{-mx/P_0}\, dx\, dz.
\end{aligned}
\tag{14}
$$

## III. RESULTS

This section deals with the relative performance of Rayleigh and Nakagami-*m* fading ($m$ = 4) channel for dual mode service (CU and PU users) models of a cellular network. Let us observe the variation of average BER and mean capacity of the selected CU against target transmission rate taking outage probability as a parameter. For the simplicity of presentation, we have considered only binary phase shift keying (BPSK) modulation in this investigation.

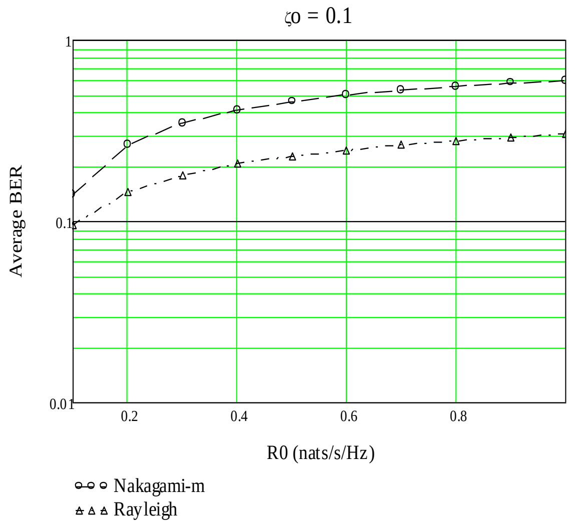


**Fig. 1:** Comparision of upper bound on the average BER of the selected CU versus the target transmission rate $R_0$ for target outage probablitites $\zeta_0$=0.1 of the PU for Nakagami-*m* and Rayleigh fading case.

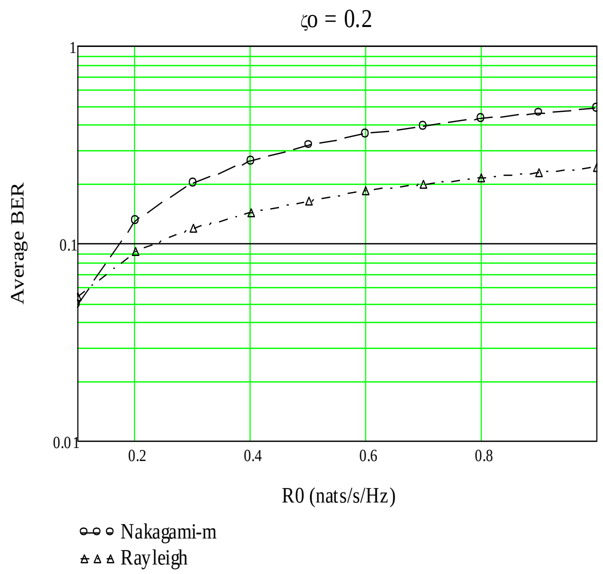


**Fig. 2:** Comparision of upper bound on the average BER of the selected CU versus the target transmission rate $R_0$ for target outage probablitites $\zeta_0$=0.2 of the PU for Nakagami-*m* and Rayleigh fading case.

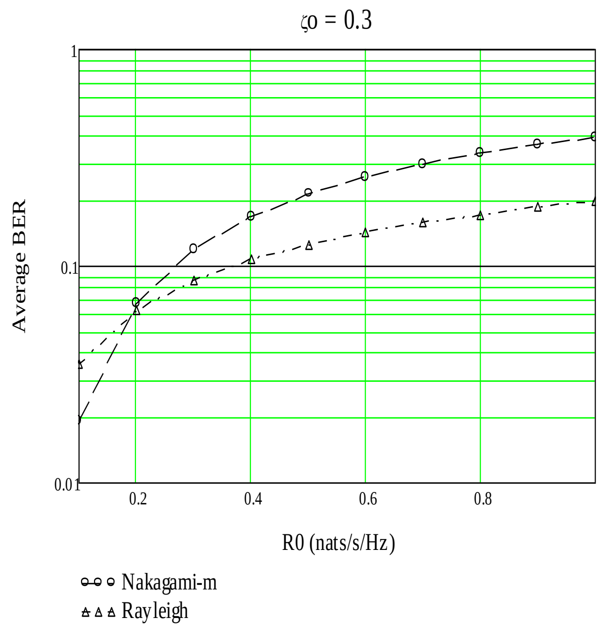


**Fig. 3:** Comparision of upper bound on the average BER of the selected CU versus the target transmission rate $R_0$ for target outage probablitites $\zeta_0$=0.3 of the PU for Nakagami-*m* and Rayleigh fading case.

For the numerical appreciation of our results, we have integrated numerically Eqs. (8), (9) and Eqs. (13), (14) for the expressions of the mean capacity and average BER for the scheduled CU for Rayleigh and Nakagami-*m* channel models respectively. The results are displayed in terms of graphs in Fig. (1)-(6).

It is observed from Figs. (1)-(3) that the average BER decreases with increase in $\zeta_0$ but BER increases with $R_0$ for both fading case. The BER for Nakagami-*m* case is greater than that of Rayleigh case for $\zeta_0$ = 0.1 and 0.2. For $\zeta_0 \geq 0.3$, the BER of the Nakagami-*m* fading is found to be lower for $R_o \leq 0.2$ (nats/s/Hz). But at the higher values of $R_0$, the situation is reversed.

Figures 4-6 compare the mean channel capacity $C$ for two fading cases. It is observed that the mean capacity of the selected CU versus the target transmission rate ($R_0$) curves for the two fading cases intersects at $R_0$=0.2,0.4 and 0.64 at $\zeta_0$ =0.1 ,0.2 and 0.3.The channel capacity of the Nakagami-*m* fading case is found to be much higher than that of the Rayleigh fading case for $R_0 \leq 0.6$ at $\zeta_0 \geq 0.3$. For $R_o > 0.6$, the capacities of both the fading cases are very close to each other as is visualized from Figs. 4-6.

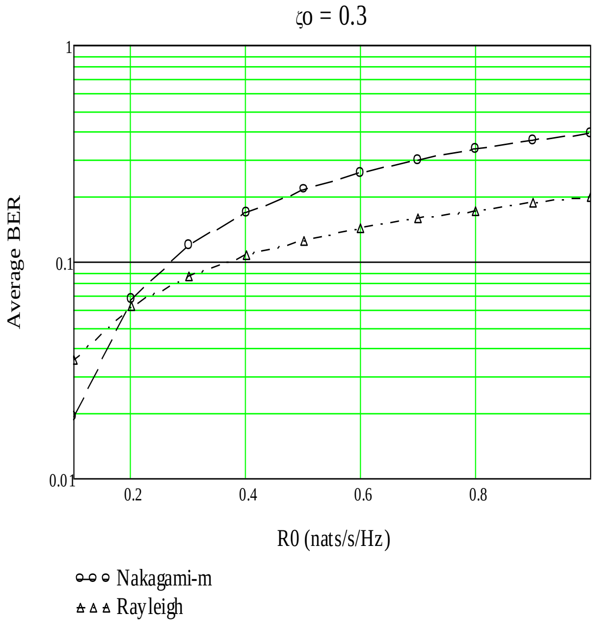


**Fig. 3:** Comparision of upper bound on the average BER of the selected CU versus the target transmission rate $R_0$ for target outage probablitites $\zeta_0$=0.3 of the PU for Nakagami-*m* and Rayleigh fading case.

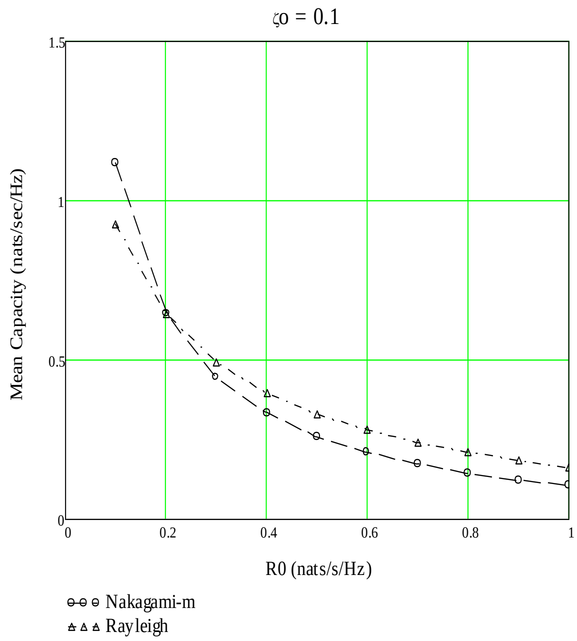


**Fig. 4:** Comparision of mean capacity of selected CU versus the target transmission rate $R_0$ for target outage probablity $\zeta_0$=0.1 for the PU for Nakagami-*m* and Rayleigh fading case.

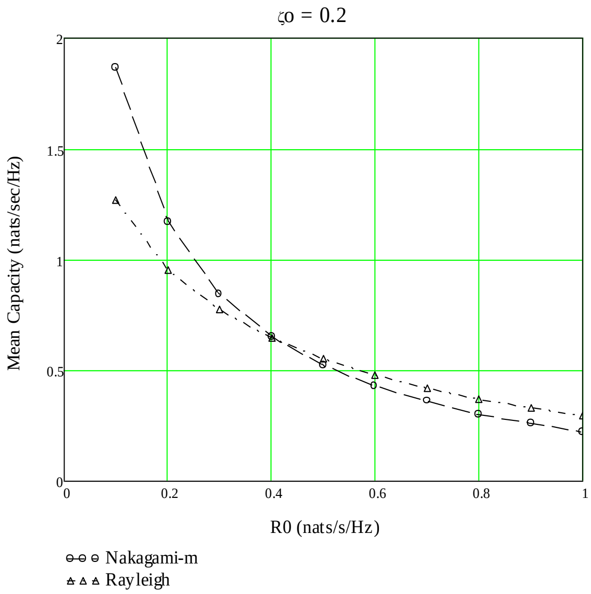


**Fig. 5:** Comparision of mean capacity of selected CU versus the target transmission rate $R_0$ for target outage probablity $\zeta_0$=0.2 for the PU for Nakagami-*m* and Rayleigh fading case.

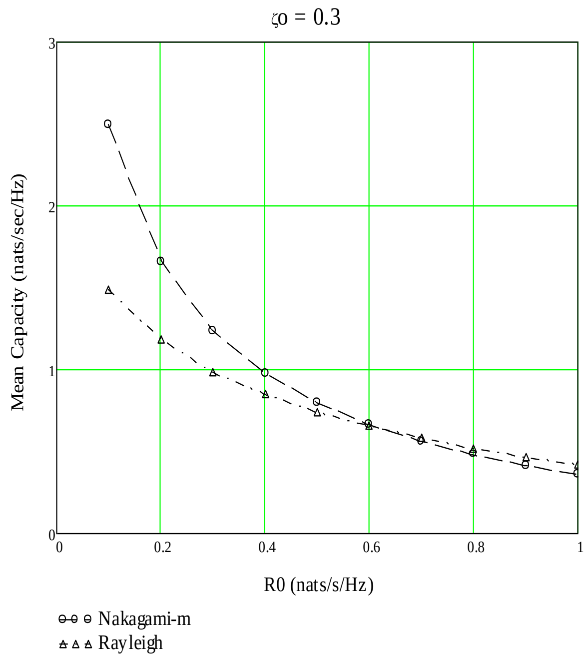


**Fig. 6:** Comparision of mean capacity of selected CU versus the target transmission rate $R_0$ for target outage probablity $\zeta_0$=0.3 for the PU for Nakagami-*m* and Rayleigh fading.

## IV. CONCLUSION

This paper shows that the performance of the network depends on the target transmission rate and the outage probability constraints on the PU for the Nakagami-*m* and Rayleigh fading environments. It is observed that the channel capacity decreases exponentially with the increase in the transmission rate but it increases with increase in the outage probability on the PU for both the fading cases. Situation is found reverse for the average BER case, i.e., the average BER decreases with the increase of the outage probability.

It is further observed that the channel capacity under Nakagami-*m* fading environment is better than that of the Rayleigh fading case at the expense of the average BER. It is to be mentioned here that there is a scope to enhance the analysis of the paper by changing the value of '*m*' in the Nakagami-*m* fading channel for different modulation schemes.

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