Effect of Waveguide Parameters on the Growth Rates in a Solid Beam Driven Plasma Loaded Backward Wave Oscillator

Dilip Kumar Sarker, Md. Mortuza Ali, Diponkar Kundu, Pallab Kanti Podder, Md. Galib Hasan

***Abstract*—this paper contains results of analytical investigation of a solid beam driven plasma loaded backward wave oscillator. Here, an instability leading to microwave generation involves a process of three-wave interaction. The theory of approximate cubic dispersion equation valid near resonance for annular beam driven vacuum backward wave oscillator (BWO), was derived earlier. In this paper, by extending and modifying this theory is used for investigating the effect of variation of SWS size parameters on the oscillation frequency and growth rates for solid beam driven plasma loaded BWO.**

***Index Terms*— BWO, SWS, instability, plasma-loaded, dispersion, corrugated structure.**

I. INTRODUCTION

In this paper, the effects of waveguide parameters on the oscillation frequency and temporal and spatial growth rates of a plasma loaded BWO with sinusoidally corrugated slow wave structure having very smaller corrugation depth driven by a solid intense relativistic electron beam have been investigated analytically. The analytical study is based on the approximate linear theory of absolute instability derived for a vacuum BWO with an annular electron beam [1]. Here, this theory is extended and modified for a plasma-loaded BWO with a solid electron beam. In the previous works, most of the researchers have devoted their interests for efficiency and resonance enhancement, power enhancement and frequency shifting of microwave emission in plasma filled BWO [2-5]. Most of them carried out their analysis keeping the structure parameters constant. Some of the researchers conducted investigations on the slow-wave instability by numerical analysis [6-10]. Some researchers work on absolute instability phenomena comprising plasma-loaded BWO with annular electron beam [11, 12]. So far to the author's knowledge, the effect of structure-size parameter variation on the temporal and spatial growth rates of a solid beam driven plasma loaded BWO by absolute instability analysis has not been investigated in the previous works. So, the effect of variation of BWO structure parameters together with plasma density on the temporal and spatial growth rates is studied here by absolute instability analysis. In this analysis, the analytical solution of the dispersion relation is obtained from the existence of a saddle point in the complex k-plane, where one can find two equal roots of complex wave number, k for some complex frequency, ω with positive imaginary part (i.e., $\omega_i>0$). At the saddle point, the values of complex k and ω give the oscillation frequency and temporal and spatial growth rates of electromagnetic radiation. However, the present analysis is confined for the case of TM_{01} mode only.

Section II of this paper contains formulation of the analytical dispersion relation. Section III describes the analytical results of the analysis. Discussion and conclusions are given in section IV.

II. FORMULATION

To derive the analytical dispersion relation, a BWO system model as shown in Fig. 1 is considered. It consists of a sinusoidally corrugated-wall structure, having very smaller corrugation depth, h (i.e., $h<<R_0$) according to the relation:

$$R(z) = R_0\,[1+a\cos(k_0 z)] \qquad (1)$$

Where, R (z) = the inner surface radius of the structure; $a=h/R_0$; $k_0=2\pi/z_0$; z_0=period of corrugation of the structure inner wall.

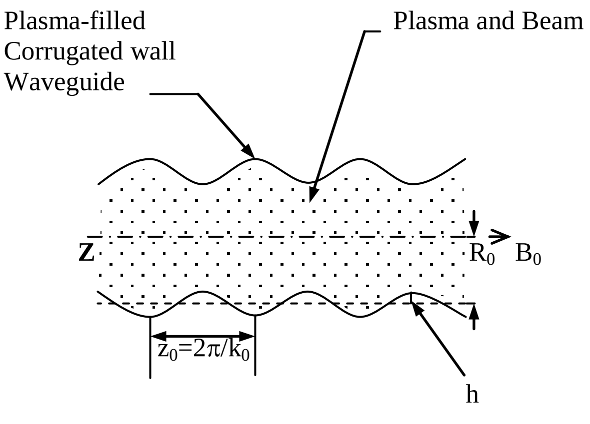

Fig.1: Plasma loaded sinusoidally corrugated slow wave structure and electron beam model.

The structure filled completely and uniformly with plasma of density, N_p and an electron beam of density N_b. The beam is moving along the waveguide axis with a velocity, v_b relative to the background plasma with the guidance of a strong and infinite magnetic field, $\mathbf{B}_0$. The numerical dispersion relation of this system, D(k, ω) = 0,

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Dilip Kumar Sarker, Department of Electrical and Electronic Engineering, Pabna Science & Technology University, Pabna-6600, Bangladesh. Email: dks_ms@yahoo.com)

Md. Mortuza Ali, Department of Electrical and Electronic Engineering, Rajshahi University of Engineering & Technology, Rajshahi, Bangladesh. Email: mmali.ruet@gmail.com)

Diponkar Kundu, Department of Electrical and Electronic Engineering, Pabna Science & Technology University, Pabna-6600, Bangladesh. Email: d.kundu.eee@gmail.com)

Pallab Kanti Podder, Department of Information and Communication Engineering, Pabna Science & Technology University, Pabna-6600, Bangladesh. Email: pallab_ice@yahoo.com)

Md. Galib Hasan Department of Electrical and Electronic Engineering, Pabna Science & Technology University, Pabna-6600, Bangladesh. (Mobile Email: eeemgh@yahoo.com)

where D is the value of the determinant of a square matrix with elements D_{mn}, and k and ω are respectively the wave number and frequency.

The approximate dispersion relation for the resonance interaction of the zeroth beam harmonic with the electromagnetic first slow harmonic can be expressed as,

$$\begin{bmatrix} D_{-1-1} & D_{-10} \\ D_{0-1} & D_{00} \end{bmatrix} = 0 \qquad (2)$$

The matrix elements of the above relation are:

$$\left.\begin{aligned} D_{-1-1} &= J_0(X_{-1}) \\ D_{00} &= J_0(X_0) \\ D_{-10} &= \left(1+\frac{k_0 k}{\frac{\omega^2}{c^2}-k^2}\right)\frac{a}{2}X_0 J_0'(X_0) \\ D_{0-1} &= \left(1-\frac{k_0 k_{-1}}{\frac{\omega^2}{c^2}-k_{-1}^2}\right)\frac{a}{2}X_{-1} J_0'(X_{-1}) \end{aligned}\right\} \qquad (3)$$

Here,

$$\frac{X_n^2}{R_0^2} = \left(\frac{\omega^2}{c^2}-k_n^2\right)\left(1-\frac{\omega_p^2}{\omega^2}\right) - \left[\left(\frac{\omega^2}{c^2}-k^2\right)\frac{\omega_b^2}{\gamma^3(\omega-k_n v_b)^2}\right]\delta_{n,0} \qquad (4)$$

where, c, ω_p, ω_b and v_b are light velocity, plasma frequency, beam frequency and light velocity respectively, and $\delta_{n,0} = 0$ unless n = -1; $k_n = (k + nk_0)$.

The oscillation frequency ω_q and hence the wave number k_q can be obtained by solving eq.(2) with $\omega_b = 0$.

The cubic equation describing the frequency and wave number perturbations of the three waves involved in the resonance interaction is obtained from the dispersion relation stated in eq. (2) as,

$$(\delta\omega - v_b\delta k)^2(\delta\omega - v_g\delta k) = \Delta \qquad (5)$$

where,

$$\Delta = -\frac{\omega_b^2\gamma^{-3}\beta_1 a^2\lambda_{-1}^2\left(\frac{\omega_q^2}{c^2}-k_q^2\right)}{8\lambda_0\left[\frac{\omega_q}{c^2}-\frac{\omega_p^2(k_q-k_0)^2}{\omega_q^3}\right]} \times \frac{J_0'(\lambda_{-1})}{J_0(\lambda_0)}\left[1+\lambda_0\frac{J_0''(\lambda_0)}{J_0'(\lambda_0)}\right]$$

$$\beta_1 = \left[1+\frac{k_0 k_q}{\frac{\omega_q^2}{c^2}-k_q^2}\right]\left[1-\frac{k_0(k_q-k_0)}{\frac{\omega_q^2}{c^2}-(k_q-k_0)^2}\right]; \qquad a = \frac{h}{R_0}$$

$$\lambda_{-1}^2 = \left[\frac{\omega_q^2}{c^2}-(k_q-k_0)^2\right]\left[1-\frac{\omega_p^2}{\omega_q^2}\right]R_0^2$$

$$\frac{J_0'(\lambda_{-1})}{J_0(\lambda_0)}\left[1+\lambda_0\frac{J_0''(\lambda_0)}{J_0'(\lambda_0)}\right] = \begin{cases} 1 \text{ for } \lambda_0 \text{ is imaginary} \\ -1 \text{ for } \lambda_0 \text{ is real} \end{cases}$$

$$\lambda_n = X_n(\omega_q, k_q, X_b = 0)$$

III. ANALYTICAL SOLUTION OF THE DISPERSION RELATION

In Figs. 2 and 3, the temporal and spatial growth rates for different structure average radius with the increase in plasma density have been shown. In these observations, keeping the corrugation period, z_0 and the corrugation amplitude, h constant the variation of temporal as well as spatial growth rates at the saddle point for different values of structure average radius, R_0 are plotted. From the figures it is seen that temporal growth rate decreases and spatial growth rate increases with the increase in plasma density. These figures also reveal that the temporal growth rate increases and the spatial growth rate decreases with the increase in structure average radius.

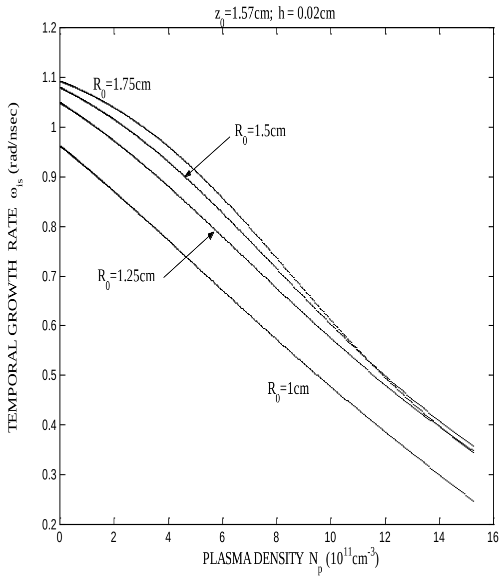

Fig.2: Variation of temporal growth rates with plasma density and structure average radius.

The effect of plasma density and structure average radius on the oscillation frequency is shown in Fig. 4. In this observation, the variation of oscillation frequency for various structure average radiuses, R_0 are plotted keeping the corrugation period, z_0 and the corrugation depth, h constant. From the figure it is seen that the oscillation frequency increases with plasma density and decreases with the structure average radius.

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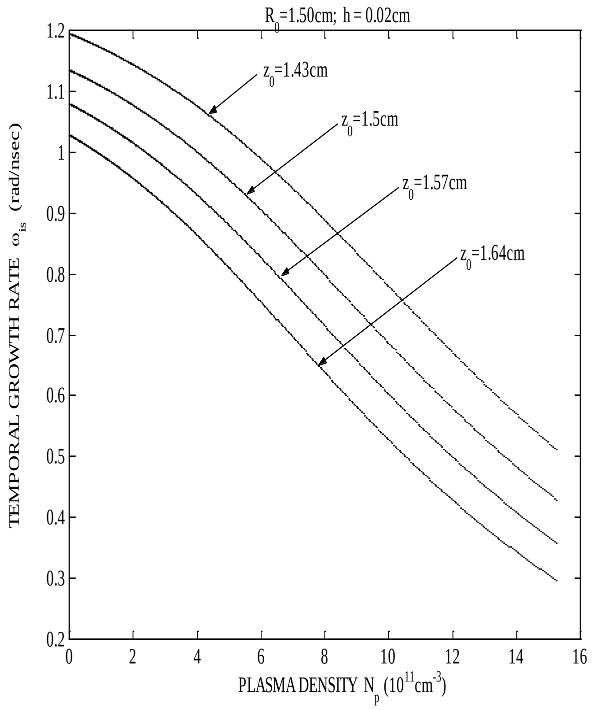

Fig.3: Variation of spatial growth rates with plasma density and structure average radius.

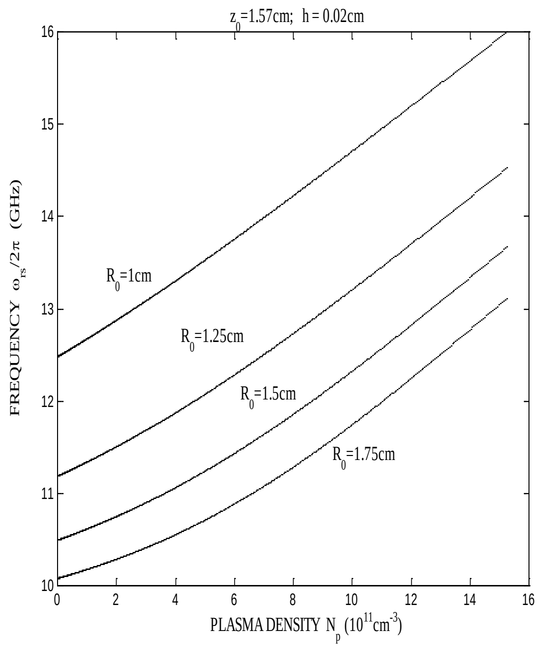

Fig.4: Variation of oscillation frequency with plasma density and structure average radius.

In Figs. 5 and 6, the temporal and spatial growth rates for different structure corrugation periods, z_0 with the increase in plasma density has shown. In this observation, keeping the structure average radius, R_0 and the corrugation depth, h constant the variation of temporal as well as spatial growth rates at the saddle point for different structure corrugation periods are plotted. In the figures, it is observed that temporal growth rate decreases and spatial growth rate increases with plasma density. These figures also show that temporal growth rate decreases and spatial growth rate increases with the structure corrugation period. Fig. 5 also shows that the rate of decrease of spatial growth rate with the increase of structure corrugation period is greater at high plasma densities.

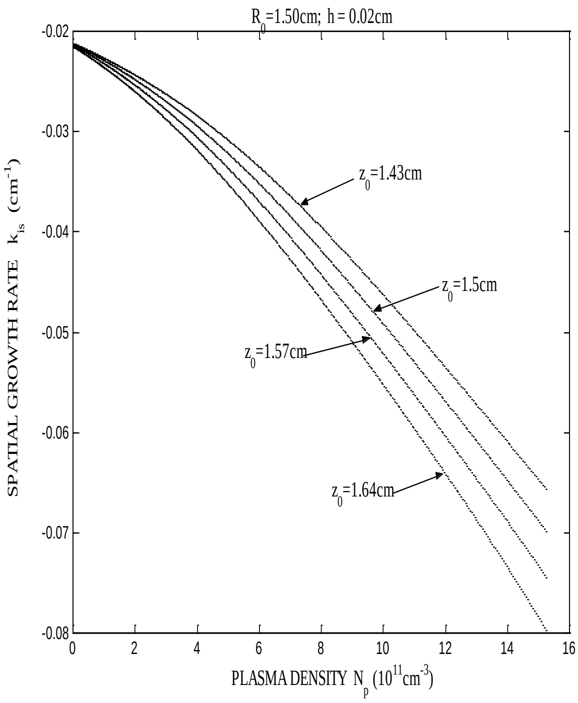

Fig.5: Variation of temporal growth rates with plasma density and Structure corrugation period.

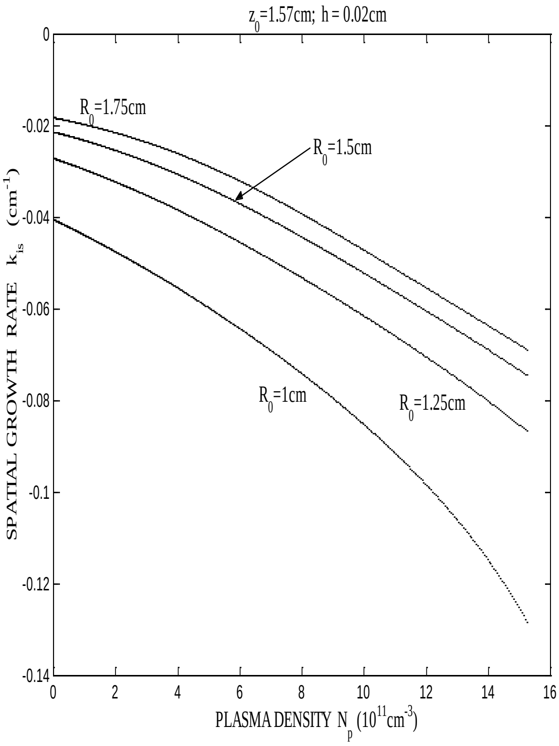

Fig.6: Variation of spatial growth rates with plasma density and structure corrugation period.

The effect of the plasma density and that of the structure corrugation period, z_0 on the oscillation frequency is shown in Fig. 7. In this observation, the structure average radius, R_0 and the corrugation depth, h are kept constant and the variation of oscillation frequency for different values of structure corrugation periods, z_0 are plotted. From the figure it is seen that the oscillation frequency increases with plasma density and decreases with the structure corrugation period. Fig. 7 also shows that the rate of increase of oscillation frequency with the decrease of corrugation period is lower for higher plasma densities.

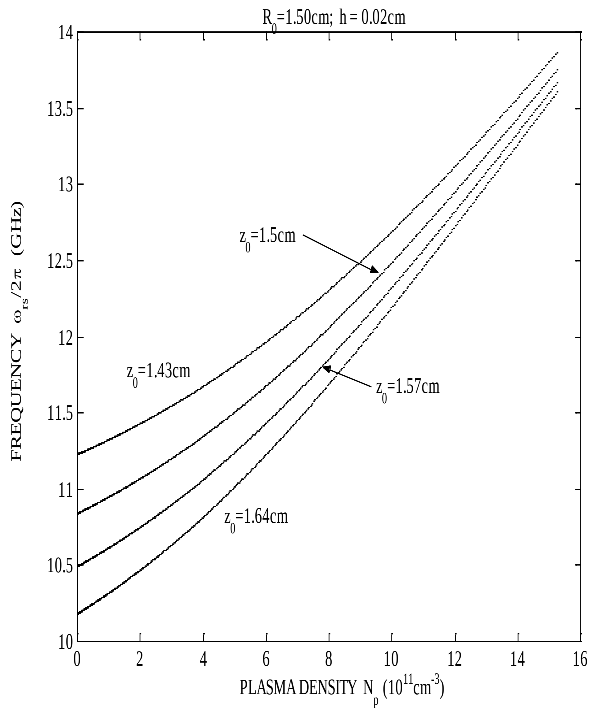

Fig.7: Variation of oscillation frequency with plasma density and structure corrugation period.

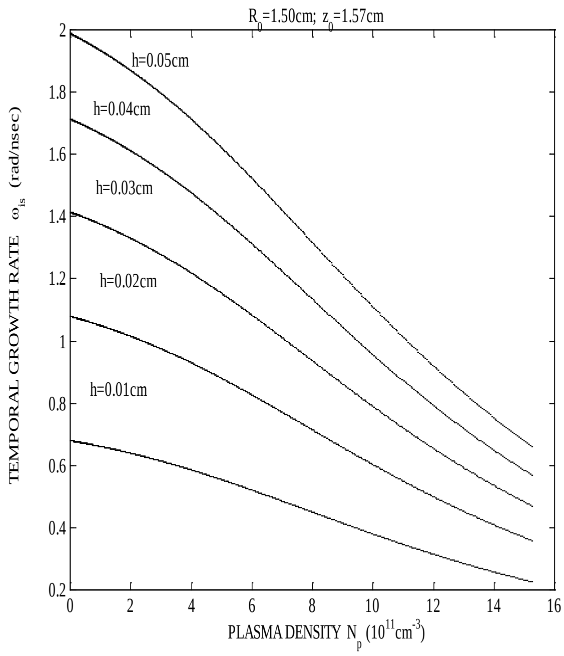

Fig.8: Variation of temporal growth rate with plasma density and structure corrugation depth.

In Figs. 8 and 9, the variation of temporal and spatial growth rates for different structure corrugation depth, h with the increase in plasma density has shown.

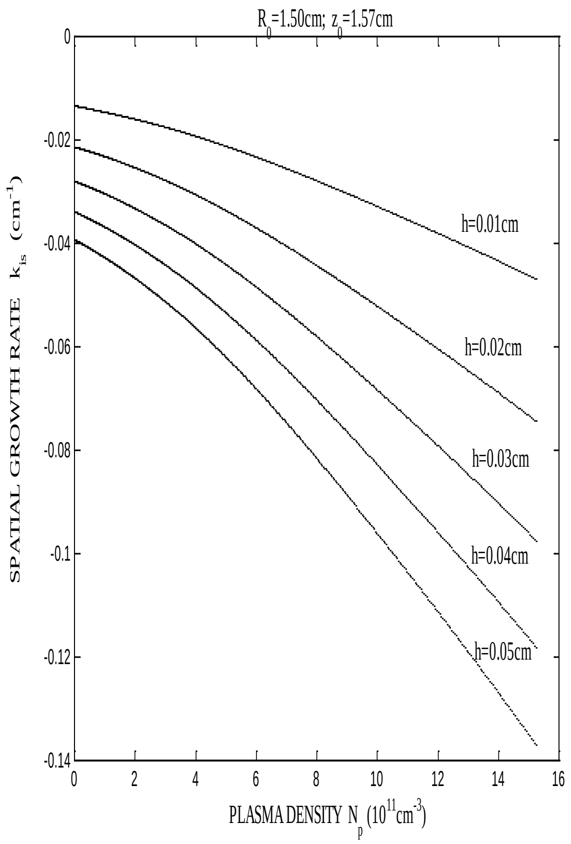

Fig.9: Variation of spatial growth rate with plasma density and structure corrugation depth

In this observation, keeping the structure average radius, R_0 and the corrugation period, z_0 constant the variation of temporal as well as spatial growth rates for different values of structure corrugation depths are plotted. From the figures it is seen that temporal growth rate decreases and spatial growth rate increases with plasma density.

The effect of the plasma density and the structure corrugation depth h on the oscillation frequency is shown in Fig. 10. In this observation, the structure average radius, R_0 and the period of corrugation, z_0 are kept constant and the variation of oscillation frequency for different structure corrugation depths are plotted. From the figure it is observed that the oscillation frequency increases with plasma density. The figure also shows that oscillation frequency decreases with the increase in structure corrugation depth.

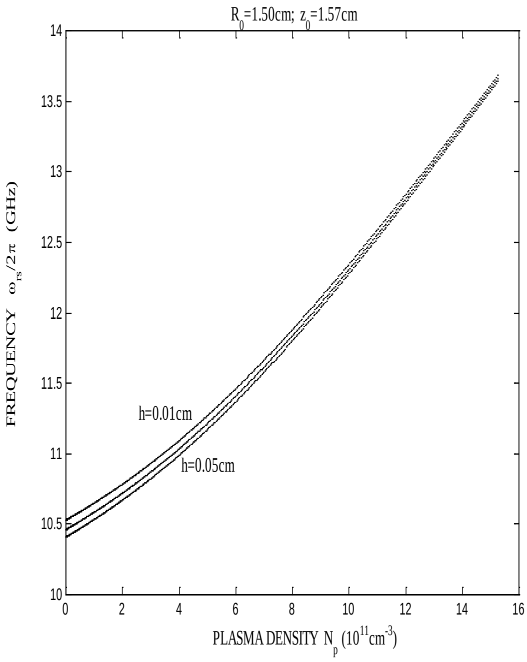

Fig.10: Variation of oscillation frequency with plasma density and structure corrugation depth.

IV. SUMMERY AND DISCUSSIONS

The effects of variation of structure size parameters and plasma density on the temporal and spatial growth rates have been carefully investigated here. The spatial and temporal growth rates are considered as a measure of strength of instability and radiation. So, in order to understand the performance characteristics, experimental requirements and design considerations the factors, spatial and temporal growth rates are very much important for the researchers. From this study one can get information about the parametric and background plasma effects on the spatial and temporal growth rates and oscillation frequency of a backward wave oscillator operating in the X-band frequency range. The developed analytical equation and the predictions of instability phenomena with variable plasma density and structure size parameters may be the new findings of this study and analysis and the presented analytical results may be helpful in designing an efficient slow wave structure for BWO operation in the X-band frequency range. Throughout this work, though a detail investigation about the effect of structure size parameters together with plasma density on the BWO operation have been carried out, yet a method of system optimization technique should be required for selecting the optimum size of the SWS.

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AUTHORS PROFILE

First Author Dilip Kumar Sarker was born in Rajshahi, Bangladesh in 1960. He received the B.Sc engineering degree in electrical and electronic engineering from Dhaka University of Engineering and Technology, Bangladesh in 1995 and the M.Sc engineering degree in electrical and electronic engineering from Rajshahi University of Engineering and Technology, Bangladesh in 2005. He is an Assistant Professor in the department of Electrical & Electronic Engineering of Pabna Science and Technology University. He is now acting as a chairman in the department of Electrical and Electronic Engineering of Pabna Science and Technology University. He is a fellow of the Institution of Engineers Bangladesh (IEB).

Second Author Md. Mortuza Ali was born in Bangladesh in 1957. He received the B.Sc engineering degree in Electrical and Electronic Engineering from Rajshahi University of Engineering and Technology, Bangladesh in 1982 and the M.S. (EE) degree from Niigata University, Japan in 1989. He also received PhD degree from Niigata University, Japan. Now he is a professor in the department of Electrical and Electronic Engineering and Pro Vice-Chancellor of Rajshahi University of Engineering and Technology. He is a fellow of the Institution of Engineers Bangladesh (IEB).

Third Author Diponkar Kundu received B.Sc engineering degree in the department of Electrical and Electronic Engineering department from Khulna University of Engineering and Technology on March 2009. Now he is a lecturer in the department of Electrical and Electronic Engineering of Pabna Science & Technology University, Pabna, Bangladesh. He joined as a Lecturer in the department of Electrical and Electronic Engineering Department of Pabna Science and Technology University on April, 2009.

Fourth Author Pallab Kanti Podder, has completed his B.Sc (Hons) and Masters Degree from the Department of Information & Communication Engineering of Islamic University, Kushtia, Bangladesh. After accomplishing his Masters degree he joined as a lecturer in the Computer Science & Engineering Department of Bangladesh University, Dhaka, Bangladesh. Now he is a lecturer in the Department of Information & Communication Engineering of Pabna Science & Technology University, Pabna, Bangladesh.

Fifth Author Galib Hasan has received his B.Sc engineering degree in the department of Electrical and Electronic Engineering from Khulna University of Engineering and Technology on March 2009. After accomplishing his degree he joined as a lecturer in the Electrical and Electronic Engineering Department of Premier University on September 2009. Now he is a lecturer in the department of Electrical and Electronic Engineering of Pabna Science & Technology University, Pabna, Bangladesh. He joined as a Lecturer in the department of Electrical and Electronic Engineering Department of Pabna Science and Technology University on September, 2010.