Construction and Performance Studies of a Pseudo-Orthogonal Code for Fiber Optic Cdma Lan

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Abstract*— A pseudo-orthogonal prime sequence code and a modified prime sequence code using the elements of Galoi's Field (GF) for a particular prime number have been developed. Bit-error rate performances using Gaussian approximation technique have been made. The capacities of the prime sequence codes are determined. Detailed simulation results on the performance of the codes are presented. The codes are useful for medium access in fiber optic CDMA LAN.

***Index Terms*— Galoi's Field, Prime Code, OOC , CDMA, FO-CDMA , S/CDMA.**

I. INTRODUCTION

Fiber Optical code division multiple-access (FO-CDMA) that allows multiple users to share a common optical channel simultaneously and asynchronously, is considered as one of the most promising technologies for the next generation broadband access network [1−3]. Synchronous Code Division Multiple Access (S/CDMA) schemes for fiber optic networks was proposed in [2]. The design of code sequences for both asynchronous and synchronous CDMA networks using optical processing have been the major challenges to many of the researchers since last few decades. Optical orthogonal codes (OOCs) [3-5] are used in direct detection optical CDMA (OCDMA) because they can be easily implemented with intensity modulation and direct detection (IM/DD) optical techniques. In bipolar (electrical) environment where the representation of binary '1' and '0' by positive and negative pulses is possible , the cross-correlation between any two code words in a code set can be made to zero. Thus perfect orthogonal codes are valid only in bipolar environment. In Optical environment binary '1' is represented by the existence of light pulse and '0' by no pulse because nonexistence of negative light pulse in present scenario. Thus perfect orthogonality can not be achieved in an unipolar (optical) environment. By optical pseudo-orthogonal code we refer to the code set of 0, 1 values that maintain the minimum value of cross correlation among them. A new method of generating pseudo-orthogonal prime sequence code and its modified form are presented. The construction of the code is much simpler than OOC and the number of code words in a code set can be enhanced by simply choosing higher value of prime number. The spectral efficiency as well as code utilization efficiency and bit error rate (BER) performance of the code are shown to be superior.

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The paper is organized as follows. The principles of code constructions are presented in section II and III. The section IV describes the analysis and performance studies of the codes. The work is concluded in section V.

II. PRIME CODES

Prime codes are {0,1} valued code which achieve low cross correlation by minimizing coincidence between ones, so that good orthogonality can be exhibited in positive systems [1, 5]. A set of code sequences of length $N=P^2$ where P is a prime number are constructed as follows-

A particular prime number P is chosen, the value of which depends on the number of code words required to be generated. Then the elements of Galois Field GF (P) ={0, 1, …j., P-1} is generated. Now, in order to construct each element $s_{x,j}$ of a prime sequence $S_x=(s_{x,0},s_{x,1}\ldots s_{x,j}\ldots s_{x,p-1})$ each element j from GF(P) is multiplied by x, where x is an element from GF(P) and then reducing modulo P. i.e by the operation modulo-P multiplication. Now, we convert each prime sequence S_x into binary code $C_x=(c_{x,0},c_{x,1},\ldots c_{x,j}\ldots c_{x,p^2-1})$ by using the following mapping-

$$c_{x\,i} = \begin{cases} 1, \text{for } i = s_{xj} + jP & j = 0, 1, \ldots, P-1, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

The prime code for P=5 is given in Table-I.

TABLE-I PRIME CODES FOR P=5

x	i 0 1 2 3 4	New Sequence	Equivalent Binary Sequence
0	0 0 0 0 0	S0	C0=10000 10000 10000 10000 10000
1	0 1 2 3 4	S1	C1=10000 01000 00100 00010 00001
2	0 2 4 1 3	S2	C2=10000 00100 00001 01000 00010
3	0 3 1 4 2	S3	C3=10000 00010 01000 00001 00100
4	0 4 3 2 1	S4	C4=10000 00001 00010 00100 01000

III. NEW METHOD OF GENERATION OF PRIME SEQUENCE CODES

A. Construction of the Code

Given a prime number P then the element of GF(P) can be left(right) circular rotated up to (P-1) times .Taking these shifted elements and following the procedure described below , a new code can be generated which has completely different chip combination from original prime sequence code. The construction method of these codes can be described as follows- First, an appropriate prime number p is chosen. Then we start with Galois Field GF(P) = { 0, 1, …j…., P-1}.

Now the elements of GF(P) are rotated (left circular rotation) up to (P-1) times. Take every rotation and construct each element $s_{x,j,}$ of new sequence S_x=($s_{x,0}$,$s_{x,1}$..$s_{x,j}$,,,,,$s_{x,P-1}$) by multiplying each rotated value of GF(P) element (i.e j) by x also an element of GF(P) and then reducing modulo P. We then convert each new sequence S_x into binary sequence code C_x=($c_{x,0}$,$c_{x,1}$,……$c_{x,j}$…..c_{x,P^2-1}.) by using the same mapping as in (1).

The prime code for P=5 and left circular rotating the element of GF(P) once is given in(Table-II).

TABLE- II
PRIME CODE GENERATION: ELEMENT OF GF (5) HAS BEEN ROTATED ONCE

x	i 1 2 3 4 0	New Sequence	Equivalent Binary Sequence
0	0 0 0 0 0	S0	C0=10000 10000 10000 10000 10000
1	1 2 3 4 0	S1	C1=01000 00100 00010 00001 10000
2	2 4 1 3 0	S2	C2=00100 00001 01000 00010 10000
3	3 1 4 2 0	S3	C3=00010 01000 00001 00100 10000
4	4 3 2 1 0	S4	C4=00001 00010 00100 01000 10000

The prime code generated in this alternative method as seen in Table-II, each code word has different combination of 1's and 0's as compared to the codes shown in Table-I, except the first code word which is same for both the cases. Thus if the number of cyclic rotation is increased from 0 to (P-1) , every rotation will generate the same code but the order of the binary sequence will differ from rotation to rotation.

B. Modified Sequence from the Prime Code

In this work, the newly generated code has been time shifted to create modified sequence as described below. First each of the original P prime sequence S_x are taken as seed, from which a group of new code can be generated. Then we left rotate the CDMA code sequence C0 to obtain the code sequence of the first group i.e x={0}. C0 can be left rotated p-1 times to generate other P-1 code words of the first group. For other P-1 group i.e x= {1,…..,P-1}, the element of the corresponding prime sequence Sx, are left rotated (P-1)times to create new prime sequence S $_{x.t}$={s_{xt0},sx_{t1},………$s_{xt(P-1)}$ },where t represents the number of times sx has been left rotated. Finally each prime sequence Sx.t is then mapped into a code sequence

$C_{x.\,t}$ = ($c_{x.t0}$,…..,$c_{x.t\,(N-1)}$) according to

$$c_{xt\,i} = \begin{cases} 1, \text{for } i = s_{xti} + jP & j = 0,1,\ldots,P-1 \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

The modified codes from the prime codes of Table II are shown in Table-III.

TABLE -III
GENERATION OF THE MODIFIED CODES FROM THE PRIME CODES OF TABLE -II

x	i 1 2 3 4 0	Sequence	Code Sequence
0	0 0 0 0 0 4 4 4 4 4 3 3 3 3 3 2 2 2 2 2 1 1 1 1 1	S0.0 S0.1 S0.2 S0.3 S0.4	C0.0= 10000 10000 10000 10000 10000 C0.1= 00001 00001 00001 00001 00001 C0.2= 00010 00010 00010 00010 00010 C0.3= 00100 00100 00100 00100 00100 C0.4= 01000 01000 01000 01000 01000
1	1 2 3 4 0 2 3 4 0 1 0 1 2 3 4 4 0 1 2 3 3 4 0 1 2	S1.0 S1.1 S1.2 S1.3 S1.4	C1.0= 01000 00100 00010 00001 10000 C1.1= 00100 00010 00001 10000 01000 C1.2= 10000 01000 00100 00010 00001 C1.3= 00001 10000 01000 00100 00010 C1.4= 00100 00010 00001 10000 01000
2	2 4 1 3 0 4 1 3 0 2 0 2 4 1 3 3 0 2 4 1 1 3 0 2 4	S2.0 S2.1 S2.2 S2.3 S2.4	C2.0= 00100 00001 01000 00010 10000 C2.1= 00001 01000 00010 10000 00100 C2.2= 10000 00100 00001 01000 00010 C2.3= 00010 10000 00100 00001 01000 C2.4= 10000 00010 10000 00100 00001
3	3 1 4 2 0 1 4 2 0 3 4 2 0 3 1 0 3 1 4 2 2 0 3 1 4	S3.0 S3.1 S3.2 S3.3 S3.4	C3.0= 00010 01000 00001 00100 10000 C3.1= 01000 00001 00100 10000 00010 C3.2= 00001 00100 10000 00010 01000 C3.3= 10000 00010 01000 00001 00100 C3.4= 00100 10000 00010 01000 00001
4	4 3 2 1 0 3 2 1 0 4 0 4 3 2 1 1 0 4 3 2 2 1 0 4 3	S4.0 S4.1 S4.2 S4.3 S4.4	C4.0= 00001 00010 00100 01000 10000 C4.1= 00010 00100 01000 10000 00001 C4.2= 10000 00001 00010 00100 01000 C4.3= 01000 10000 00001 00010 00100 C4.4= 00100 01000 10000 00001 00010

C. Properties of Generated Sequences:

The properties of prime sequence codes generated using new method are summarized as follows – (i) Number of code words in a code set = P; (ii) Code word length = P^2 (iii) Number of 1's in a code word or code weight = P. (iv) The cross-correlation function at chip position j for any pair of code sequence C_x and C_y can be found from discrete state-time position cross correlation function[4]

$$\theta_{C_xC_y}(j\tau) = \sum_{i=0}^{P^2-1} C_x(i\tau).\,C_y(i\tau - j\tau) \quad (3)$$

with 0≤ j≤ (P^2 -1), where x, y, i, and j are integer and τ is the chip width.

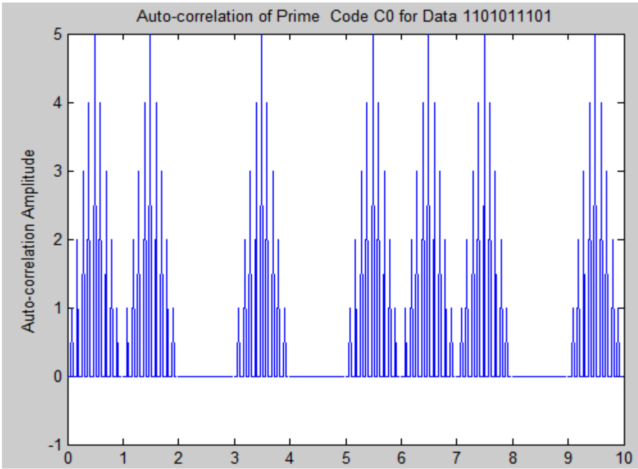

Fig-1: Auto-correlation of Generated Code C0 of Table-II.

The figure above shows the auto-correlation peaks of code C0 in Table-II for the data sequence 1101011101 where the peak is 5. Similarly, for any arbitrary value of the of the prime number P the auto-correlation peak is P.

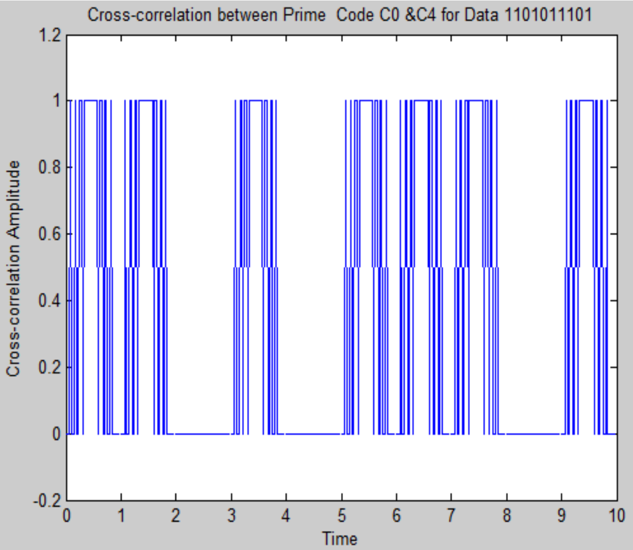

Fig-2: Cross-correlation between Code C0 & C4 in Table-II.

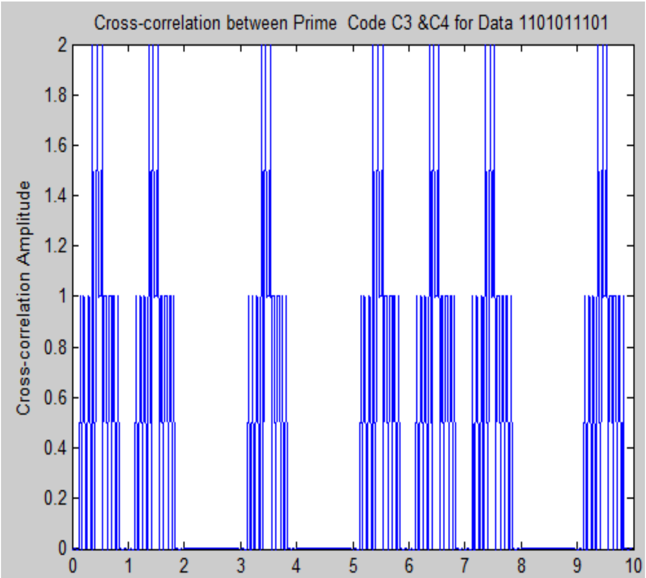

Fig-3: Cross-correlation between Code C0 & C4 in Table-II.

The Fig-2 and Fig-3 shows that the cross-correlation value between any two code can be at most 2.

The properties of generated modified prime sequence codes are as follows – (i) Number of code words in a code set = P^2; (ii) Code word length = P^2 (iii) Number of 1's in a code word or code weight = P. (iv) Auto correlation at last chip position = P as shown in fig-4.. The cross correlation at last chip position can be evaluated using the discrete time position cross-correlation function given by [4] with the chip position j=P^2-

$$\theta_{C_{x.s} C_{y.t}((P^2-1)\tau)} = \sum_{i=0}^{P^2-1} C_{x.s}(i\tau) . C_{y.t}(i\tau - (P^2-1)\tau)) \quad (4)$$

Where i is integer, τ is chip width and s and t are the number of times the elements of S_x and S_y has been left rotated respectively.

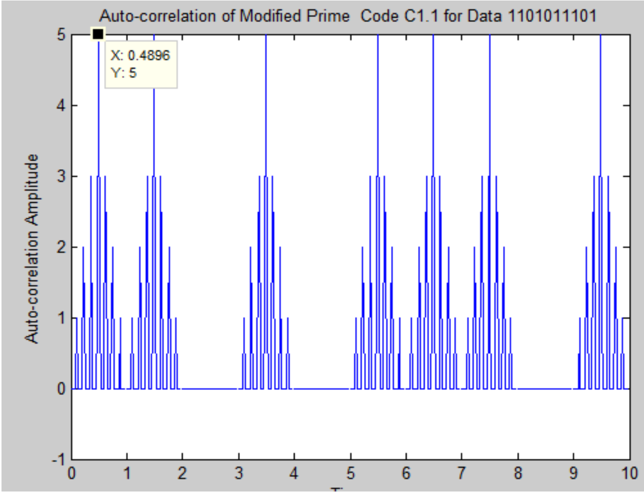

Fig-4: Auto-correlation of Modified Sequence C1.1 in Table-III.

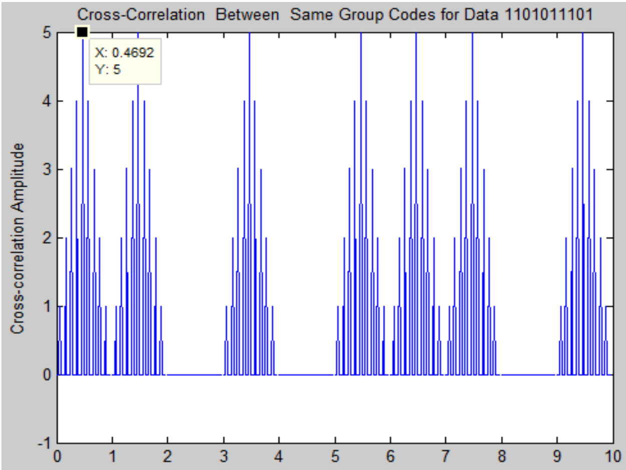

Fig-5: Cross-correlation between Same Group Codes in Modified Sequence.

We can say from the above correlation property that the value of cross correlation at the last chip position for any two code words depends on whether the codes are from the same group or not. The cross correlation is zero for the codes of same group and 1 for different groups codes as shown in Fig-5 and Fig-6.

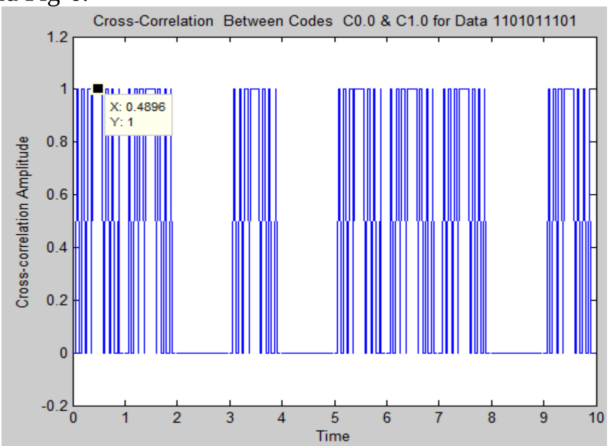

Fig-6: Cross-correlation between Codes C0.0 & C1.0 in Modified Sequence .

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Also, the auto-correlation at the last chip position for code sequences can be as high as P as shown in Fig-4. The value of cross correlation at any other chip position can be as high as P as shown in Fig-5, provided that the codes are from same group otherwise if they originates from different group as shown in Fig-7, then the value is at most two.

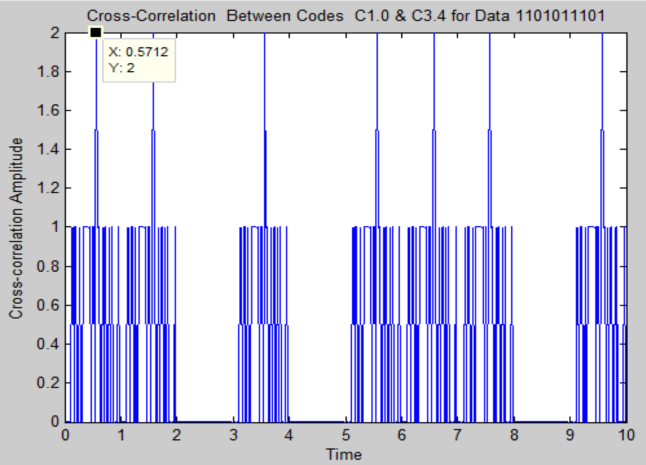

Fig-7: Cross-correlation between Codes C1.0 & C3.4 in Modified Sequence.

Though the value of cross correlation is as high as the auto correlation peak for two modified prime sequence codes, but they always occurs either delayed or ahead of auto correlation peak. So if the receiver is synchronized to the expected position of the auto correlation peaks, then it can be easily distinguished from the adjacent cross correlation peaks. Thus by the aid of synchronization the codes are till pseudo-orthogonal and can be used as a signature sequence for synchronous code division multiple access (S/CDMA) scheme with optical signal processing.

IV. PERFORMANCE STUDIES OF THE CODES

The total number of subscriber that can be accommodated by Optical Code Division Multiple Access (OCDMA) network depends on the number of codes available for a given prime number [6]. The newly generated code can produce P code words which can be assigned to P subscribers. These code words are applicable in asynchronous OCDMA systems. The modified version of the newly generated code can generate P^2 number of code words for applications in synchronous OCDMA (abbreviated S/CDMA) systems. Hence, total number of subscriber that a modified code set can accommodate is P^2. The number of possible subscriber versus the prime number has been plotted in Fig .8 for both the new code and its modified version. It is seen that the modified code accommodates greater number of users. As an example for P=5, prime code can accommodate 5 users where the modified version of this code can accommodate 25 users.

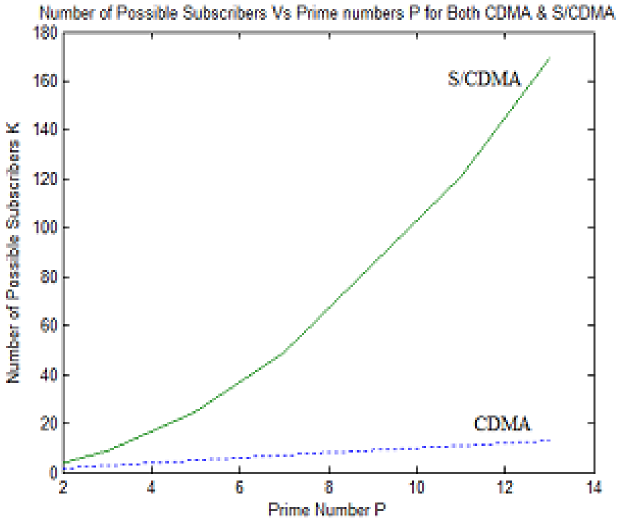

Fig-8: Number of possible subscriber versus prime number P for both newly generated code and its modified version

The probability of error using Gaussian approximation [6] is denoted by Pe|G (to emphasize the Gaussian approximation) as a function of SNR for CDMA is given by

$$Pe|G = \varphi\left(\frac{-\sqrt{SNR}}{2}\right) = \varphi\left(\frac{-P}{\sqrt{1.16\,(K-1)}}\right) \qquad (5)$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}\,dy$ is the unit normal cumulative distribution function. The The probability of error (5) using Gaussian approximation for CDMA, against number of simultaneous user has been plotted in Fig. 9 for different prime numbers P. As an example for P= 31(i.e N=961), the newly generated code can accommodate 31 subscriber and 23 simultaneous users with a probability of error less than the order of 10^{-10}, whereas for 23 simultaneous user if P=17 is chosen then the error rate increases to the order of 10^{-4}. So for a fixed number of simultaneous users larger the value of the prime number P smaller the error probability.

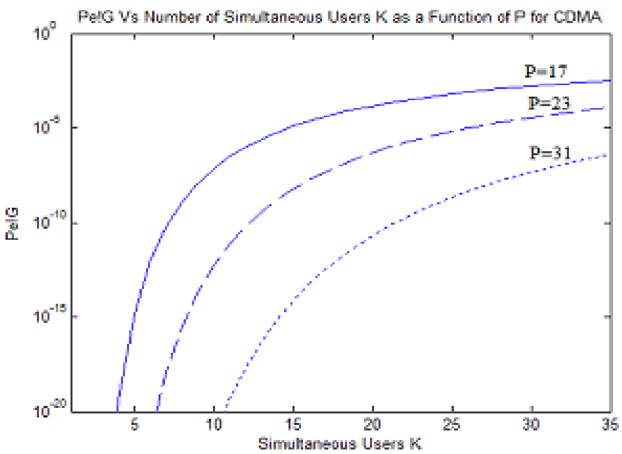

Fig-9: The probability of error and the number of simultaneous users as a function of P for newly generated prime codes using Gaussian approximation.

The probability of error using Gaussian approximation [6] for S/CDMA that uses the modified version of the code is given by

$$Pe|G = Pe|G = \varphi\left(\frac{-P}{\sqrt{(K-1)}}\right) \quad (6)$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}$ dy is the unit normal cumulative distribution function.

From Fig.10, we see that the probability of error for modified code increases as the number of simultaneous subscriber (K) increases. As an example, for P=31when the number of simultaneous users K=10, the error probability is almost zero as is observed from the graph that the error curve in this case coincide with the horizontal axis. But for the same case (i.e. P=31) when the number of simultaneous users K= 32, then the error probability exists with the value equals to 1.29 x 10^{-8}.

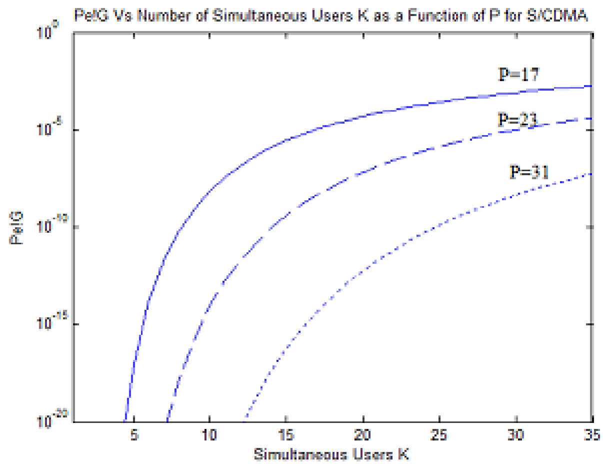

Fig-10: The probability of error and the number of simultaneous users as a function of p for the modified version of newly generated codes using Gaussian approximation.

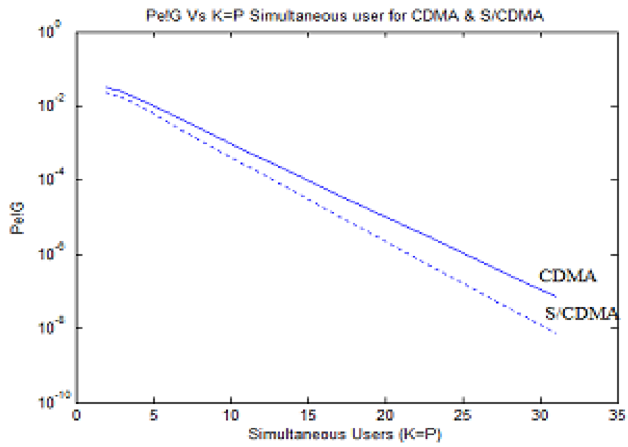

Fig -11: The probability of error vs K= P number of simultaneous user for both CDMA and S/CDMA (using Gaussian approximation)

The probability of error versus K = P simultaneous users for both CDMA and S/CDMA are shown in Fig.11. It can be seen that S/CDMA has an increasingly better performance than CDMA as value of P increases gradually from its minimum value.

The Fig.12 shows the changes in error probability for S/CDMA when the number of simultaneous users (K) is increased gradually for a given prime number(P). Also from the above plot we see that error performance degrades as the number of simultaneous user increases for a given P. Also we have seen that for a given number of simultaneous users (K) as the value of P is increased the error performance improves. So the value of K and the value of P are need to be chosen carefully.

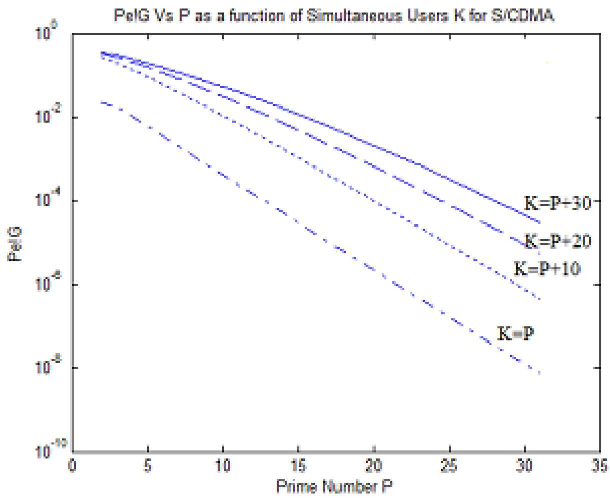

Fig-12: The probability of error vs P as a function of number of simultaneous user.

V. CONCLUSION

An alternative method of generation of prime sequence code and a modified prime sequence code have been developed. The simulation results show that the prime codes can be used in asynchronous optical CDMA where as the modified code can be used in synchronous optical CDMA where multiple users can simultaneously access the optical network with minimum interference.

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