Performance Analysis for Concatenated Coding schemes with Efficient Modulation Techniques

Hala M. A. Mansour, Labib Francis Gergis, Mostafa A. R. Eltokhy, Hoda Z. Said

Abstract— in digital communication systems, channel coding is the method of adding redundancy to the data in order to reduce the frequency of errors or to increase the capacity of a channel. Concatenated codes are the most superior class of codes making achievable channel capacity almost at par with the Shannon limits. Concatenated codes are error correcting codes constructed by combining two or more simple codes through an interleaver in order to obtain powerful coding schemes. In this paper a special construction of concatenated convolutional coding scheme called parallel-serial concatenated convolutional code (P-SCCC) is presented. The upper bound to the bit error probability of the proposed code is evaluated. Results showed that the error performance of this proposed code scheme is better than that of both classical serial and parallel concatenated convolutional codes. The performance of the proposed code has been studied with different types of digital modulation schemes.

Keywords: — Code concatenation, convolutional code, frequency shift keying, phase shift keying, and quadrature amplitude modulation.

I. INTRODUCTION

Code concatenation has been proved to be an effective coding scheme to obtain high coding gain and hence reducing transmission errors in communication systems.

It is initially presented in [1] by Forney as a method of building long codes from shorter codes, which results in decreasing the decoding complexity. Concatenated codes, a simple and powerful coding scheme are defined in [2].

It has been shown to perform near the Shannon capacity limit in an additive white Gaussian noise (AWGN) channel. The initially introduced concatenated codes are parallel combination of two codes joined by an interleaver. Later, new class of concatenated codes was presented, the serial concatenated codes and the hybrid concatenated codes. In this study we concern about serial and parallel concatenation.

Manuscript Received January. 13, 2012.

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Considerable work has been done on the design and the performance of concatenated codes in order to achieve better performance.

The influence of the constituent codes and the interleaver length on the overall performance of the concatenated coding scheme has been studied in [3]. In [4] a method has been proposed for the first time to evaluate the bit error probability of parallel concatenated coding schemes independently from the interleaver used, parallel concatenated convolutional is investigated, the concept of uniform interleaver has been also introduced in this paper, the uniform interleaver permits an easy derivation of the weight enumerating function of the parallel concatenated coding scheme.

Upper bounds to the average maximum-likelihood bit error probability of serially concatenated convolutional coding scheme have been derived in [5]. Design guidelines for outer and inner codes that maximize the interleaver gain (the factor that decreases the bit error probability as a function of the interleaver size) have been introduced. Finally a new low-complexity iterative decoding algorithm that yields performance close to Shannon limit has been illustrated.

Improved upper bounds on the performance of parallel and serial concatenated codes have been illustrated; the influence of the interleaver length and the memory length of the component codes has been investigated in [6].

A modified code construction for parallel and serial concatenated codes has been presented in [7], to behave very much like Forney's classical codes with improved decoding complexity. Also a soft-decision, reliability-based, linear-time decoding algorithm has been introduced.

An improved concatenated code structure which generalizes parallel and serial concatenated convolutional codes has been proposed in [8]. The structure is ideal for designing low-complexity rate-compatible code families with good performance in both the waterfall and error floor regions. Also a design criterion for the generalized class of concatenated convolutional codes has been designed based on the error probability and extrinsic information transfer (EXIT) charts for decoding threshold.

To achieve power and bandwidth requirements for the communication system, powerful coding schemes should be combined with bandwidth efficient modulation techniques. The performance of concatenated codes with spectrally efficient modulation techniques have been studied in [9]-[11].

Concatenated codes are considered in [12]. In this

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scheme the outer code has redundancy and the overall transmission rate is reduced.

The problem of bandwidth expansion has been overcome by two procedures. First, the code rate of inner codes is increased with a special coding design. Second, a two-level concatenated scheme is constructed to compensate the rate loss. This way, the low-complexity concatenated coded modulation schemes can be designed without bandwidth expansion as compared with the uncoded scheme.

In this paper a special construction for concatenated convolutional coding scheme called parallel-serial concatenated convolutional code (P-SCCC) is introduced, this code is a combination of both parallel and serial concatenation. The bit error probability performance of the P-SCCC is evaluated in AWGN channel and its performance is compared with that of parallel and serial concatenated convolutional code. P-SCCC is combined with quadrature amplitude modulation (QAM), phase shift keying (PSK), and Minimum shift keying (MSK) scheme as a special case of Frequency shift keying (FSK) type.

When comparing M-ary QAM and M-ary PSK techniques we find that as M increases ($M > 4$) the distance between signal points in M-ary PSK constellation becomes smaller than that of M-ary QAM constellation as a result, the error performance of M-ary QAM becomes better than that of M-ary PSK for $M > 4$ [14]. However, because QAM is not a constant envelop modulation technique, the superior error performance of M-ary QAM can't be realized except if the channel is free from nonlinearities.

MSK has the advantage of being both power and bandwidth efficient modulation technique and in addition, it has constant envelop waveform so it can be effectively employed in systems which use nonlinear high power amplifiers.

This paper is organized as follows, in section II; a brief review on concatenated codes is presented. Construction of parallel-serial concatenated code is presented in section III. Section IV evaluates the bit error probability of P-SCCC. A review on PSK, QAM and FSK modulation techniques are introduced in Section V. Section VI introduces the performance of P-SCCC with each of the previously mentioned modulation techniques. The conclusions are discussed in section VII.

II. CONCATENATED CODES

Consider a liner (n, k) code C with code rate $R_c = k/n$, the input-output weight enumerating function (IOWEF) of the code C is defined as

$$A(W,X) = \sum_{w=0}^{k}\sum_{x=0}^{n} A_{w,x} W^w X^x = \sum_{w=0}^{k} W^w A(w,X) \qquad (1)$$

where $A_{w,x}$ is the input-output weight enumerating coefficients (IOWEC) of the code C and is defined as the number of codewords of the code having output weight x generated by information words of weight w, $A(w,X)$ is the conditional weight enumerating function (CWEF) of the code C and it can be defined as

$$A(w,X) = \sum_{x=0}^{n} A_{w,x}\ X^x \qquad (2)$$

The bit error probability $P_b(e)$ of the code C under maximum-likelihood (ML) decoding for binary phase shift keying (BPSK) modulation in an AWGN channel can be upper-bounded as [5]

$$P_b(e) \le \frac{1}{2}\sum_{x=1}^{n}\sum_{w=1}^{k}\frac{w}{k} A^{C}_{w,x}\ erfc\left(\sqrt{\frac{x R_c E_b}{N_0}}\right)$$
(3)

where E_b/N_o is the signal to noise ratio (SNR) per bit.

$$\frac{1}{2} erfc(z) < e^{z^2} \qquad (4)$$

We can obtain a more compact form of (1) as [5]

$$P_b(e) < \sum_{w=1}^{k}\frac{w}{k}[A(w,X)]_{X=e^{-R_c E_b / N_0}}$$
(5)

Where $A(w,X)$ is the conditional weight enumerating function (CWEF) of the code C.

A. Serial concatenated convolutional code

Serial concatenated convolutional code SCCC, that shown in Fig. 1, consists of two constituent codes, the outer code C_o has rate $R_o = k/p$ and the inner code C_i has rate $R_i = p/n$, the resulting SCCC, C_s has rate $R_s = k/n$. The two constituent codes are joined by an interleaver (π) with length N.

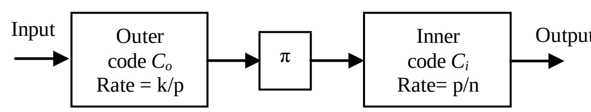

Fig. 1: Serial Concatenated Convolutional Code.

Assuming uniform interleaver, an input word with Hamming weight l will be permuted through the interleaver into an output word has also Hamming weight l with equal probability $1/\binom{N}{l}$.

The conditional weight enumerating function of the serial concatenated convolutional code can be obtained as [5]

$$A^{C_s}(w,X) = \sum_{l=0}^{N}\frac{A^{C_o}_{w,l} \times A^{C_i}(l,X)}{\binom{N}{l}} \qquad (6)$$

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Where $A_{w,l}^{C_o}$ is the IOWEC of the outer code C_o and $A^{C_i}(l,X)$ is the CWEF of the inner code C_i.

B. Parallel concatenated convolutional code

The structure of parallel concatenated convolutional code (PCCC) is shown in Fig.2. Two codes, C_1 with rate $R_{C_1}=k/n_1$, and C2 with rate $R_{C_2}=k/n_2$ are connected by an interleaver with length N resulting in an overall PCCC, C_p with rate $R_P=R_{C_1}R_{C_2}/(R_{C_1}+R_{C_2})$. The uniform interleaver with length N will permute a given input sequence of length N and weight w randomly to every possible permutation of it with uniform probability $1/\binom{N}{w}$.

The output code word length n = n_1 + n_2 and the overall code rate is $R_p=k/n$.

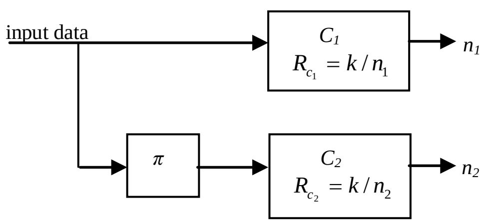

Fig. 2: Parallel concatenated convolutional code.

CWEF of the parallel concatenated convolutional code can be obtained as [3]

$$A^{C_P}(w,X)=\frac{A^{C_1}(w,X)\times A^{C_2}(w,X)}{\binom{N}{w}} \tag{7}$$

The upper bound to the bit error probability of parallel concatenated convolutional code can be expressed as [3]

$$P_b(e)\le\sum_{w=1}^{N}\frac{w}{N}W^w A^{C_P}(w,X)\Big|_{W=X=e^{-R_c E_b/N_0}} \tag{8}$$

III. PARALLEL-SERIAL CONCATENATED CONVOLUTIONAL CODE

The construction block diagram of parallel-serial concatenated coding scheme is illustrated in Fig.3. The overall coding scheme is a parallel combination of two codes C_{S_1} and C_{S_2} joined by an interleaver of length N. Each of C_{S_1} and C_{S_2} is a serial concatenation of two codes. This scheme is first proposed in [13] using block codes as constituent codes. In this paper, parallel-serial concatenated coding scheme is applied using convolutional codes as constituent codes to form a parallel-serial concatenated convolutional code (P-SCCC).

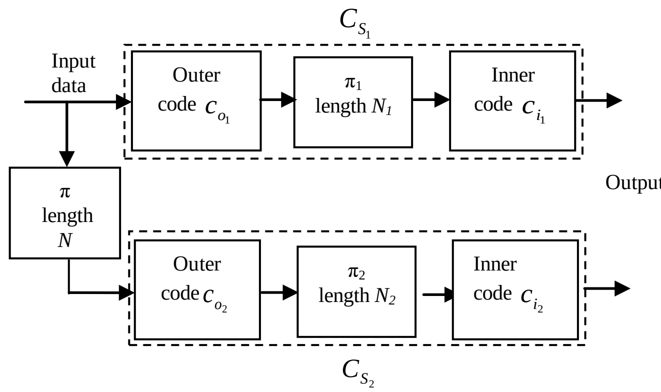

Fig. 3: Parallel-serial concatenated convolutional code.

IV. NUMERICAL ANALYSIS

In our analysis we use C_{o_1} and C_{o_2} as two identical rate *1/2* systematic feedback convolutional codes with generator matrix $G(D)=[1\ (1+D^2)/(1+D+D^2)]$ and C_{i_1} and C_{i_2} as two identical rate *2/3* systematic feedback convolutional codes with generator matrix

$$G(D)=\begin{bmatrix}1 & 0 & 1/(D^2+D+1)\\ 0 & 1 & (D^2+1)/D^2+D+1)\end{bmatrix}. \tag{9}$$

The input output weight enumerating function (IOWEF) of the outer code C_{o_1} is

$$\begin{aligned}A^{C_{o_1}}(W,L)={}&\mathrm{W2}(6L^4+3L^6)+W^3(7L^2+13L^4+7L^6)\\&+W^4(5L^2+16L^4+3L^6+3L^8)+W^5(3L^2\\&+10L^4+12L^6+2L^8)+W^6(L^2+14L^4\\&+12L^6)\ +W^7(7L^4+2L^6)+W^9L^6.\end{aligned} \tag{10}$$

and the IOWEF of the inner code C_{i_1} is

$$\begin{aligned}A^{C_{i_1}}(L,X)={}&5LX^2+L^2(9X^3+13X^4+12X^5+3X^6)\\&+L^3(5X^3+7X^4+30X^5+33X^6+14X^7\\&+3X^8)+L^4(21X^5+54X^6+41X^7+32X^8\\&+7X^9)+L^5(3X^5+37X^6+54X^7+62X^8\\&+38X^9+7X^{10})+L^6(8X^6+30X^7+56X^8\\&+69X^9+27X^{10}+8X^{11})+L^7(6X^7+22X^8\\&+55X^9+47X^{10}+22X^{11}+4X^{12})+L^8(3X^8\\&+16X^9+29X^{10}+28X^{11}+19X^{12})\\&+L^9(4X^{10}+8X^{11}+14X^{12}+6X^{13}+2X^{14})\\&+L^{10}(2X^{12}+3X^{13}+X^{14}+X^{15}).\end{aligned} \tag{11}$$

From the IOWEF of C_{o_1} and C_{i_1} we can get $A_{w,l}^{C_{o_1}}$ and $A^{C_{i_1}}(l,X)$ then substituting into (6) to get the CWEF of the serial concatenated code C_{S_1}. After some calculations the CWEF of C_{S_1}, $A^{C_{S_1}}(w,X)$ is obtained as

$$A^{C_{S_1}}(2,X) = 1.8X^5 + 5.46X^6 + 6.7X^7 + 8.74X^8 + 7.4X^9 + 2.89X^{10} + 0.85X^{11}.$$

$$A^{C_{S_1}}(3,X) = 2.25X^3 + 3.25X^4 + 6.9X^5 + 12.75X^6 + 15.1X^7 + 19.94X^8 + 17.25X^9 + 6.75X^{10} + 2X^{11}.$$

$$A^{C_{S_1}}(4,X) = 1.6X^3 + 2.32X^4 + 6.94X^5 + 13.7X^6 + 12.6X^7 + 13.3X^8 + 7.4X^9 + 2.89X^{10} + 0.86X^{11}$$

$$A^{C_{S_1}}(5,X) = 0.96X^3 + 1.39X^4 + 4.29X^5 + 11.42X^6 + 18.72X^7 + 19.94X^8 + 7.4X^9 + 2.89X^{10} + 0.86X^{11}.$$

$$A^{C_{S_1}}(6,X) = 0.32X^3 + 0.46X^4 + 4.63X^5 + 14.3X^6 + 21X^7 + 30.44X^8 + 29.2X^9 + 11.57X^{10} + 3.4X^{11}.$$

$$A^{C_{S_1}}(7,X) = 2.1X^5 + 5.97X^6 + 6.24X^7 + 7.2X^8 + 4.9X^9 + 1.93X^{10} + 0.75X^{11}.$$

$$A^{C_{S_1}}(8,X) = 0.$$

$$A^{C_{S_1}}(9,X) = 0.29X^6 + X^7 + 2X^8 + 2.4X^9 + 0.96X^{10} + 0.29X^{11}. \quad (12)$$

Since the two outer codes and the two inner codes are identical, the CWEF of C_{S_2} will be the same as that of C_{S_1}. Using (7) we can get the CWEF of the parallel-serial concatenated convolutional code, $A^{C_{PS}}(w,X)$ as

$$A^{C_{PS}}(w,X) = \frac{[A^{C_{S_1}}(w,X)]^2}{\binom{N}{w}} \quad (13)$$

After getting $A^{C_{PS}}(w,X)$ and substituting into (8), the upper bound to bit error probability of the P-SCCC could be derived. The bit error probability performance of the P-SCCC is illustrated in Fig. 4, and is compared to that of both PCCC and SCCC. It is shown from results that the P-SCCC has better performance than both codes at high SNRs.

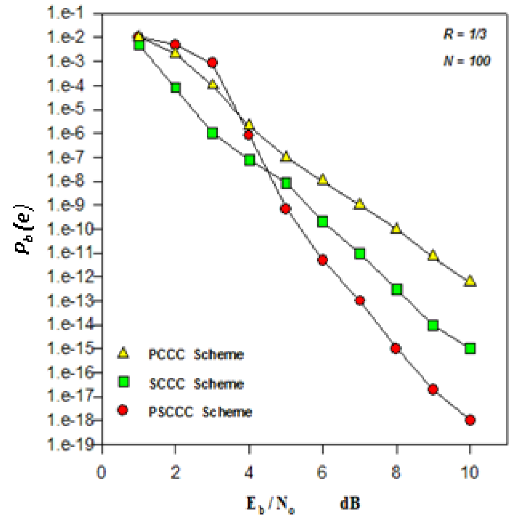

Fig. 4: Error probability performance of P-SCCC, PCCC and SCCC.

V. REVIEW ON DIGITAL MODULATION TECHNIQUES

A. M-ary Phase Sift Keying Modulation (MPSK)

In *M*-ary phase shift keying (MPSK), the phase of the carrier takes one of *M* possible values, $\theta i = 2\pi i/M$, where $i = 0,1,\ldots, M-1$. *M*-ary PSK signal set is defined as [14]

$$S_i(t) = \sqrt{\frac{2E_s}{T}}\cos(2\pi f_c t + \theta_i) \qquad 0 \le t \le T$$

(14)

where E_s is the signal energy per symbol and f_c is the carrier frequency. For M-ary PSK modulation, as M increases, the bandwidth efficiency increases but power requirements become excessive.

B. M-ary Quadrature Amplitude Modulation (MQAM)

Quadrature amplitude modulation is a class of non-constant envelope modulation in which both amplitude and phase of the transmitted signal are changed at the same time. The general form of *M*−ary QAM is defined by the transmitted signal [14]

$$S_i(t) = \sqrt{\frac{2E_0}{T}}\, a_i \cos(2\pi f_c t) + \sqrt{\frac{2E_0}{T}}\, b_i \sin(2\pi f_c t) \qquad 0 \le t \le T$$

(15)

where E_0 is the energy of the signal with lowest amplitude, a_i and b_i is a pair of independent integers chosen in accordance with the location of the message point.

In applications where bandwidth efficiency is more important requirement than the constant envelope property of the modulation technique, QAM can achieve higher bandwidth efficiency than MPSK with the same average signal power.

C. Frequency Shift Keying (FSK)

In Binary FSK modulation, signals of two different frequencies of f0 and f1 are used to represent binary 1 and 0. These signals are described as [12]

$$S_0(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_0 t) \qquad 0 \le t \le T_b \tag{16}$$

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_1 t) \qquad 0 \le t \le T_b \tag{17}$$

where T_b is the bit period of the binary data and E_b is the transmitted signal energy per bit. In this form of FSK modulation, the phase of the carrier may change at the beginning of each symbol so it is called discontinuous FSK. Phase discontinuities will tend to introduce a distortion to the FSK signal when amplified through a non-linear amplifier. Continuous phase is effectively achieved with continuous phase frequency shift keying (CPFSK). The general CPFSK signal can be expressed as [15]

$$S(t) = \sqrt{\frac{2E_s}{T}} \exp j\{\theta(t) + \theta_0\} \tag{18}$$

where E_s is the transmitted energy per symbol, T is the symbol duration and θ_0 is the initial phase. The information is contained in the phase $\theta(t)$ which is expressed as [15]

$$\theta(t) = \theta(n) + 2\pi h_i \sum_{i=n-L+1}^{n} u_i q(t - iT);$$

$$nT \le t \le (n+1)T \tag{19}$$

where

$$\theta(n) = \pi h \sum_{i=0}^{n-L} u_i \tag{20}$$

is the accumulated phase, h = r/p is the modulation index (r and p are relatively prime integers), u_i is the M-ary CPM symbol at time interval i. The function $q(t)$ is the phase function and its derivative is the frequency shaping pulse $g(t)$ [16].

$$q(t) = \int_0^t g(t)\, dt \qquad 0 \le t \le LT$$

(21)

where L is the pulse length. The phase function $q(t)$ is a continuous function which has properties given by [17]

$$q(t) = \begin{Bmatrix} 0 & t \le 0 \\ 1/2 & t > LT \end{Bmatrix} \tag{22}$$

Phase function together with modulation index h and input symbols x_n, determine how the phase changes with time. CPFSK has the advantage of being both power and bandwidth efficient modulation technique. Minimum shift keying (MSK) is a special type of CPFSK with h=0.5. In this paper we study the performance of the PSCCC with MSK modulation.

VI. PERFORMANCE OF P-SCCC WITH EFFICIENT MODULATION TECHNIQUES

Performance of P-SCCC has been studied with PSK, QAM and MSK techniques and the results are illustrated in Fig.5. From results we can see that, in addition to its constant envelope property, MSK has better performance with P-SCCC than the other modulation techniques. These two features enable a superior performance for P-SCCC-MSK in applications where nonlinear amplifiers are used.

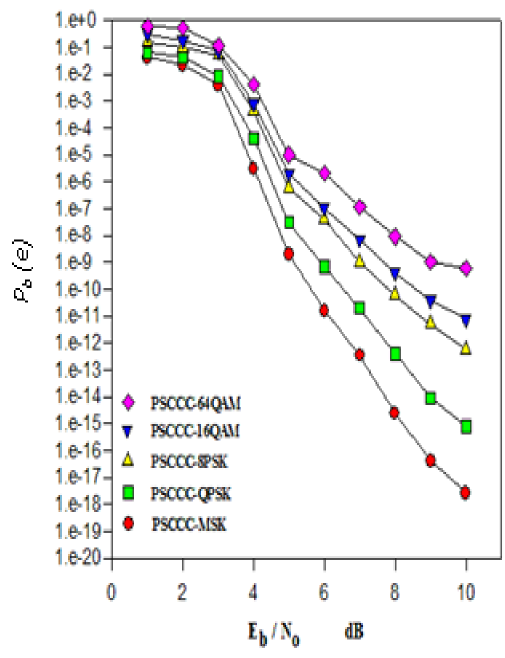

Fig. 5: Performance of P-SCCC with PSK, QAM and MSK techniques.

VII. CONCLUSIONS

A special concatenated convolutional coding scheme called parallel-serial concatenated convolutional code (P-SCCC) has been proposed. The bit error probability performance of the P-SCCC is evaluated and compared with that of PCCC and SCCC and it has been proved that P-SCCC has superior performance than both classical coding schemes at high SNRs. The performances of the P-SCCC with PSK, QAM, and MSK have been studied and results showed that performance of P-SCCC-MSK is better than that of both P-SCCC-PSK and P-SCCC-QAM techniques.

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