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Reliability Data Analysis Procedures for Comparing Failure Rates of the System Using Optimal Truncation of Short Tests

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Abstract—***A test was described for two systems, long term and short term with an exponentially distributed time between failures. The test is intended for checking the ratio $MTBF_l$ /$MTBF_s$ exceeds or equals a prescribed value, versus one that it is less than the prescribed value, by means of long term tests with large average sample number in the earlier system. Our proposed system focus on improving test by using low average sample number in short term which is having the advantage of economy in time requirement and cost. It produces optimum truncated test called binomial Sequential Probability Ratio Test. Criteria are proposed for determining the characteristics of truncated test followed with the discretizing effect of truncation on error probabilities with a view to optimization of its parameters. The search algorithm for truncation apex used in this system achieves closeness to the optimum which depends on successful choice of the initial approximation, search boundaries and on the search step. The enhanced reliability of modern technological systems, combined with the reduced time quotas allotted for creating new system is capable of yielding a highly efficacious test which increases reliability and feasibility of decisions.***

Index Terms— MTBF, Short Truncate Test, Long Term, ADP

I. INTRODUCTION

Quality is the non-functional requirements that support the delivery of the functional requirements, such as robustness or maintainability, the degree to which the software was produced correctly. Reliability is the indirect measure to improve the quality of the software.

Testing Process: Testing is the process of running a system with the intention of finding errors. Testing enhances the integrity of a system by detecting deviations in design and errors in the system.

Truncated test: The ability to search small number of samples from the search limits.

In the previous testing process, the software tested with an exponentially distributed error probabilities which leads to time consuming process(long term).The test is planned for checking whether the ratio $MTBF_s/MTBF_L$ exceeds or equals a prescribed value ,versus the alternative that is lower than the prescribed value. Average sample numbers for this test is relatively large and have a high resolving power.

On testing the software, the failure which involves iterative approximations is considered. Though the test results in low resolving power, the reduced time requirement and the achieved closeness to the optimum which depends on successful choice of the initial approximation and its search boundaries. (that yields highly efficacious tests.) At the same time, reliability of the system that makes greater interest in deciding the low number of failures.

The goal of the project is to develop a methodology for planning short truncated test for improving reliability of the software.

II. TEST METHODOLOGY

Fig 1: shows the test procedure for the proposed system follows that when the software under process fails, it is immediately replaced/restored on considering the MTBF, and on the acceptance and rejection of the null hypothesis H_o based on the ratio of MTBF of short term test to the MTBF of long term test i.e. θ_s/θ_l.

$$H_o: \omega \geq \omega_o,\ Pa(\omega_o) = 1 - e1 \qquad (1)$$

$$H_1: \omega < \omega_o,\ Pa(\omega_1) = e2, \qquad (2)$$

Where $\omega_1 = \omega_o/d,\ d > 1$

Pa (ω), the probability of H_o at a given ω represents the OC curve (≡ **Pa (ω)**) which is replaced with the parameters e1, e2, ω_o, and d which determines the coordinates of (ω_o (1- e1) and (ω_1, e2).

From the figure1 slope **S** is determined between the two oblique boundary lines, which is given by

$$S = \frac{\ln((1+\omega_o)/(d+\omega_o))}{\ln(d(1+\omega_o)/(d+\omega_o))} \qquad (3)$$

The absolute terms for the reject and the accept line is

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determined by long term test.

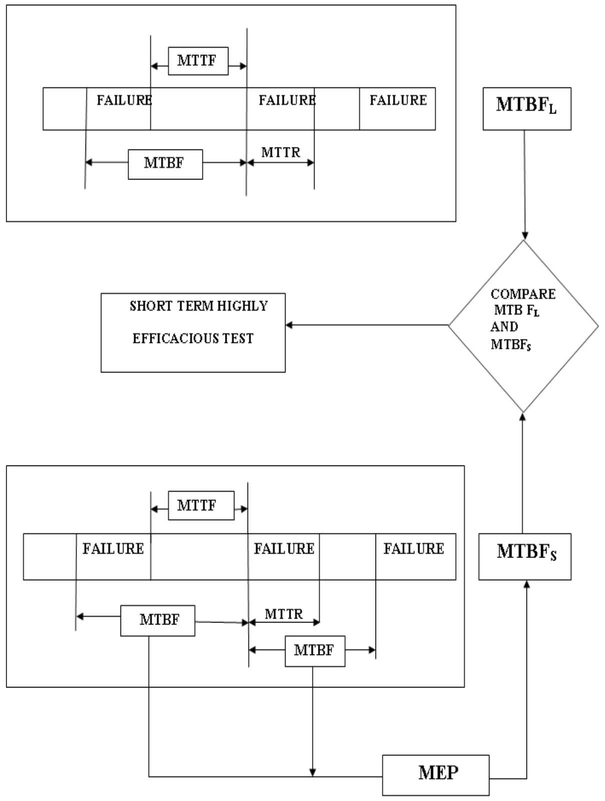

Figure 1: Architecture for Short Term Test

$$A = \frac{\ln (e2^*/ (1-e1^*))}{\ln (d\,(1+ \omega_o)/ (d+ \omega_o))} \quad (4)$$

$$R = \frac{\ln ((1-e2^*)/e1^*)}{\ln (d\,(1+ \omega_o)/ (d+ \omega_o))} \quad (5)$$

Truncation boundaries are determined for the short and long term test by taking the maximum of the failures occurred.

$$B_\iota = (T_\iota \max) \quad (6)$$
$$B_s = (T_s \max) \quad (7)$$

Where T_s = truncation value of RDP (T_ι)

The boundary points for the test is determined by using the parameters S, e1*, e2*, B_ι, B_s, which are chosen to ensure the specific test characteristics e1, e2, ω_o, d, E_{ASN}.
Having the probability 1, the plan point **(T_ι, T_s)** is given by

$$P_{(T_\iota, T_s)} (\omega) = P_{(T_\iota, T_s-1)} (\omega).P_s (\omega) + P_{(T_\iota-1, T_s)} (\omega).\ (1- P_s (\omega)) \quad (8)$$

Where P_s is the probability of failing next during short term test,

$$P_s=1/ (1+\omega) \quad (9)$$

For all the values of **ω**, starting from (0, 0), it is possible to calculate the probability of all the test points including ADP and RDP.
Thus the acceptance probability **P_a (ω)** is obtained by summing of all ADP.

$$P_a (\omega) = \sum_{T_s} P_{ADP}(T_s, \omega) \quad (10)$$

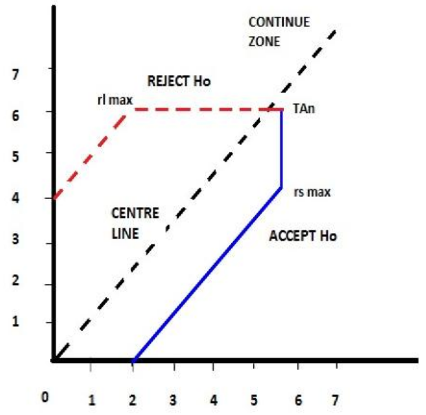

Figure 2: **Solution planes**

The real values of the error probabilities e1 and e2 of the truncated test is determined by

$$e1_{(real)} = 1- P_a (\omega_o) \quad (11)$$

$$e2_{(real)} = P_a (\omega_1) \quad (12)$$

The expected number of failures from the two tests is determined by

$$ASN (\omega) = \sum_{T_l} T_{PRDP} (T_\iota).\ P_{RDP} (T_\iota, \omega) + \sum_{T_l} T_{PADP} (T_s).\ P_{ADP} (T_\iota, \omega) \quad (13)$$

T_{PRDP}, T_{PADP} are the sum of failures in two tests on reaching RPD and ADP respectively:

$$T_{PRDP} (T_\iota)= T_\iota + R\,(T_\iota) \text{ and} \quad (14)$$

$$T_{PADP} (T_s) = T_s+A\,(T_s) \quad (15)$$

Average test duration is determined by,

$$ATD (\omega) = 2\,\theta_\iota.ASN (\omega) / (1+1/ \omega) \quad (16)$$

III. DISCRETIZING EFFECT OF TRUNCATION

From the equation [5], for the short term and long term test, the points with coordinates e1, e2 does not form a continuous plane which leads to the main problem in short tests.

Figure 2: shows that the solution planes for d=3.828A. d=6.657 respectively without the truncation.
For the construction of these planes $e1_{(real)}$, $e2_{(real)}$ were determined by using the geometric progression of the pair $e1^*/e2^*$ with 120 terms between the interval [0.02……….0.30], these steps were performed as trial calculation on considering all the points between the above interval.
The co-ordinate TA_N was taken for each pair depending on the values of ASN (ω_0) obtained for adjoining values of $e1^*/e2^*$.

TA_s=C. ASN (ω_0) **(17)**

From the above experiment, it is found that at C=12, the test is being stopped. On reaching the truncation lines, similarly the corresponding error of $e1_{(real)}$ and $e2_{(real)}$ is less than 0.01% of the actual value.
In the case of truncated test, value of C is low accordingly TA was close to the origin and probability of truncation lines being reached is high, and effect of truncation on the test characteristics is strong.
The co-ordinate TA_l was adjusted to TA_s with accuracy to nearest integer, so that truncation apex would fall on the center line.
From the solution planes, it is found that,

- For the short tests in question, the planes of possible pairs are characterized by extreme sparseness, which increases with decreasing ASN (i.e. increasing) and decreasing, and poses a substantial problem in planning such tests.
- Truncation affects the disposition of the points in the solution plane, especially in the drastic case; this provides extra possibilities in the search for a solution close to the given
- In Fig. 2, the zone of is demarcated by dashed lines. This extension of the exact relationship is dictated by the discreteness of the possible solutions, and by the fact that the narrower such a zone, the lower the number of possible solutions (points lying between the dashed lines). Considerations in choosing the zone width follows.

IV. EVALUATION OF TEST QUALITY

The test quality is evaluated by the following characteristics due to the reasons such as

*The test characteristics like OC, ASN and pmf is itself a function of multiple parameters and variables.

*The characteristics en bloc, in discrete fashion are affected by shifting of test boundaries and a rule that cannot be framed to construct two tests with different boundaries and the same operating characteristics or even with the same e1/e2 pair.

The criteria considered to fix the test quality are

- Closeness of the real OC.
- The degree of optimality of the test.

Closeness of OC:

The relative deviation R_d of $e1_{real}$ and $e2_{real}$ from the desired e1=e2 is given by

$$R_{d}=\sqrt{\left(\frac{e1real-e1}{e1}\right)2+\left(\frac{e2real-e2}{e2}\right)2} \quad \textbf{(18)}$$

The resultant is sought among the tests with e1real, $e2_{real}$ lying in solution plane within a circle with center e1=e2 and radius Rd.e1.

The solution for long tests can always be found within such a circle even at small R_d. But for short tests, the points are sparsely distributed in solution and R_d has to be increased, R_d beyond 0.1 will cause difficulties in finding optimal solution and second criterion to be applied. Thus for tests with d>3, R_d should be 0.1

A. Estimation Of Degree Of Optimality

The ASN will not be helpful to find the degree of optimality when two tests with different boundaries and same $e1_{real}/e2_{real}$ pair are used. So the degree of optimality is measured using R_{ASN}, the relative excess of the truncated tests over its non-truncated counterpart.

$$R_{ASN}=\left[\sum_{i=1}^{5}ASN(\omega_i)-\sum_{i=1}^{5}ASN_{nTr}(\omega_i)\right]/\sum_{i=1}^{5}ASN_{nTr}(\omega_i) \quad \textbf{(19)}$$

Where ω_i is values of ω forming the geometric progression.

$$\omega_o/(\sqrt{d})^3,\ \omega_o/(\sqrt{d})^2=\omega_1,\ \omega_o/\sqrt{d},\ \omega_o,\ \omega_o\sqrt{d} \quad \textbf{(20)}$$

ASN is calculated both recursive method and non-truncated binomial SPRT for same $e1_{real}/e2_{real}$ pair and represented as ASN () and ASN() respectively.

$$ASN(h)=\frac{(1+\omega(h)).(Pa(h).InB+(1-Pa(h)).InA)}{(1+\omega(h)).Inq+Ind} \quad \textbf{(21)}$$

Where

$\omega(h)=((d.q)^h-1)/(1-q^h)$

$P_a(h)=(A^h-1)/(A^h-B^h)$

$q=(1+\omega_o)/(d+\omega_o)$

$A=(1-e2_{real})/e1_{real}$

$B=e2_{real}(1-e1_{real})$

R_{ASN} is selected under several conditions.

(i) For, the non-truncated SPRT is known to be optimal. It is expressed in another way as among all possible test with the given e1/e2 pair. The test results with smallest ASN() and ASN ()

(ii) The ASN is determined by an analytical formula (16) which overcomes the difficulty due to the discreteness of the possible pairs. It is eminently suitable for purposes of comparison of the latter with the ideal one.

(iii) ASN () increases when a shift of the TA to the left of the centerline at e1≈e2. When $\omega > \omega_1$, the increase is larger .

The increases negligible. In spite of being shifted to the right of the centerline increases the The suitable values of R_{ASN} are 5% and 10%.When $R_{ASN\ <5\%}$the truncation boundaries lie too far from the origin so that the truncation benefits are lost. And when R_{ASN}>10%, the benefits are lost because of the steep increase of the ASN.

V. SEARCH LIMITS FOR E1*AND E2*

With the help of initial boundary lines (e1*, e2*) and (TA_l,TA_s) the test boundaries are defined. Narrow limits are always preferred since searching those limits with the help of a test is time consuming. This section determines the search limits for e1* and e2* .

$$d=1+ (\sqrt{2})^I \quad (22)$$

d should be always chosen as greater than 3 for short term tests.

e1*/ e2* values are compiled for $e1_{real}$ and $e2_{real}$. In the above figure the parameters d and R_{ASN} are found to have a narrow domain which in turn reduces the volume of analyzed variants. When the initial boundary lines (e1 and e2) value decreases, the e1*/ e2* value comes close to the bisector. When α/ β value is very low, the e1*/ e2* value crosses the bisector. The next figure is drawn by taking d=5, which are narrower than the above.

$$e1^*_U = [1-R_{ASN}-0.15.\ln (2. (d-1))].s.x, \quad (23)$$

$$e1^*_L=x-0.3.R_{ASN}, \quad (24)$$

$$e2^*_U=\min \{[1- R_{ASN} -0.05.\ln (2.(d-1))].s.x, \text{ and } x+0.1 \quad (25)$$

$$e2^*_L=x-0.2.\ R_{ASN} \quad (26)$$

here s is the slope and x=e1=e2.The above equations represent the upper and lower search limits which is close to the bisector when d decreases and R_{ASN} increases.

These formulas refer to d=1.5...9 and R_{ASN} = 5=10% .The search steps for d=3 …9 is given by,

$$\Delta e1^*= [e1^*_U- e1^*_L]/20 \quad (27)$$

$$\Delta e2^*= [e2^*_U- e2^*_L]/20 \quad (28)$$

VI. SEARCH LIMITS FOR TRUNCATION APEX COORDINATES

The requirements for the TA location are as follows,

- e1/e2 pair must not have a deviation exceeding R_d.
- The ASN must not exceed given R_{ASN} because of the truncation.

The highest truncation level must be in terms of the sample number and TA must be as close to as possible to the origin.

From equation(1) at e1=e2, the truncation apex falls on the centerline and TA_N coordinate with high accuracy determined for d$\leq$ 3.In the present paper, dependences were found for short tests by means of test searching in the intervals d=3.8,……9, and e1=e2=0.05,….0.25 with step 0.05.The search zones by e1=e2 should slightly overlap, so that the possibility of an "empty" zone is avoided by sitting R_D value as 0.1.The test with smallest $r_n$max was chosen in empty zone and the test boundary parameters for the search algorithm are explained is not section.

In terms of R_{ASN} three group of tests were established and requirements are

R_{ASN} <=5%,
R_{ASN} >=10%,
R_{ASN} >10%
TA_n values are plotted in figure.

The dependences are considerably less smooth than those for d<=3 It contributes to the distribution of the operative points which are highlighted as d increases. At the same time, curves for respective R_{ASN} at same d moves close together and differences not exceed 3 to 4 failures. Hence for any R_{ASN}, the lower search limit, can be set at min TA_N curves and upper search limit for R_{ASN} can be set at the curve lying four failures higher.

$$LTA_N=\min TA_N \quad (29)$$

$$UTA_N=LTA_n+4 \quad (30)$$

$$\text{Min } TA_N =g(d).f(x) \quad (31)$$

Where

$g(d)=\exp[0.5+0.06.(d-3)/d-1]-1$
$f(x) =16.(\ln1-x/x)^{1.6}-3.4$

A close approximation results from the root mean square error of ( ) for the data. Eqn ( ) is the product of two mutually s-in-dependent functions, one solely of d and the other solely of e1=e2, identical for all d in the examined interval d=1.5…9
i.e) for short and long tests, g(x) differs only slightly in counterpart for $d \leq 3$.

Optimal TA lies on the centerline, and so the upper and lower limits of TA_L (UTA_l and LTA_l respectively) are determined for each TA_s as follows.

$$LTA= floor(TA_s/S) \quad \textbf{(34)}$$

$$UTA=ceil(TA_s/s) \quad \textbf{(35)}$$

S indicates the test slope and floor and ceil indicates the rounding off functions to closest integer below and above respectively.

The dissimilarities prevailing in finding TA_s between the cases d>3 and d<=3 paves the way for the classification of tests into two groups as "short" and "long".

VII. ANALYSIS OF THE RELATIVE EFFICACY OF THE TEST

In case of parameters ω_0 and ω_1.the non- truncated SPRT is optimal. Because of the truncation test sets a limit on its maximal sample number. But this development is offset by the detrimental effect on the corresponding average sample number over ω. This can be defined by the index of R_{ASN}.

In this module we discuss about the efficiency of a truncated test between the average sample number line and a Fixed Sample Size Test (FSST).

For example the test includes the parameter like $\omega_0=1$, d=5, $e1_{real}=0.101$,$e2_{real}=0.099$,Tab=5,Tan=10 and $R_{ASN}=7.8\%$ are the limits and average sample number(ASN). Comparison for a non-truncated test with the same parameters like ω_0, d, $e1_{real}$, $e2_{real}$ as per in the example. This ASN are the average cumulative number of failures of short and long term systems.

A. Characteristics of FSST:

- Fixed number of failures should satisfy the relationship.

That is,

$$F_{\alpha,\ 2rn,2rb}=1/dF_{1-\beta,2rn,2rb} \quad \textbf{(36)}$$

Where, Fα, 2rn, 2rb are Inverse cumulative F-distribution function.

2rn is degrees of freedom in the numerator.2rb is degrees of freedom in the denominator.

- The minimal cumulative of SN_{FSST} is obtained when the $r_s=r_l$.

Substitute this in the equation (). we get $SN_{FSST}=10.92$ for achieving the given $e1_{real}$, $e2_{real}$ under the conditions,

The cumulative test duration for the two systems will be,

$$TD_{FSST}(\omega)=\theta.(1+\omega).SN_{FSST}/2 \quad \textbf{(37)}$$

- The SN of a real FSST should be an integer. But TD_{FSST} as to permit evaluation of relative efficiency of the test.

In figure the OC curve are practically coincident and the ω_0 and ω_1 are identical.
The ASN of truncated test exceeds only its non-truncated counterpart at $\omega\approx\sqrt{\ }\omega_0\omega_1$ (values are almost equal). Both of them are substantially lower than SN_{FSST}.

The superiority of SPRT is more obvious in term of ATD(Average Test Duration). The TD_{FSST} is much longer than those of the two SPRT. Regarding the relative quality of the latter and their ATD are practically coincident at ω<0.7, and close at ω>0.7.

Besides the ASN, of interest to a prospective performer of the SPRT is the Probability Mass Function (pmf),the respective probabilities PADP(r_s, ω), and PRDP(r_l, ω) of the test terminating at the given ADP/RDP. The pmf for a test with boundaries as per fig.2. and the relevant characteristics. corresponding to five ω– values arranged in a descending progression as per(15). The lower part of figure 2 shows the coordinates r_l of ADP versus r_s (curve 4), and r_s of RDP versus r_l (curve 3). These points demarcate the test boundaries, with the horizontal segments representing their truncated parts. The figure shows that in practice the truncated part can be reached only at $\omega_1 \leq \omega \leq \omega_0$, beyond which limits this probability is insignificant. The maximum SN in this case is 14(see fig 2), a number achievable with low probability only within the above limits.

Thus the document demonstrates that the proposed planning methodology for truncated SPRT is capable of yielding highly efficacious tests, not inferior in practice to their non-truncated counterparts.

VIII. CONCLUSIONS

In the previous system we had the disadvantage of having test with high average sample number which required high resolving power. This is overcome by the proposed system which has the advantage of economy in time requirement and cost .The proposed planning methodology is capable of yielding highly efficacious tests which increases reliability and feasibility of decisions.

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