Production Inventory Model with Price Dependent Demand and Deterioration

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Abstract:- In this paper many inventory models demand rate are either constant or time dependent but independent of the stock level. However for certain types of commodities particularly consumer goods, the demand rate of may be depend on the on hand inventory. For this type of commodity the sale would increase as the amount of inventory increase Most of the researchers have assumed that as soon as the items arrive in stock, they begin to deteriorate at once, but for many items this is not true. In practice when most of the items arrive in stock they are fresh and new and they begin to decay after a fixed time interval called life-period of items.

Key words:- Particularly Consumer Goods.

I. INTRODUCTION

Food items, pharmaceuticals and radioactive substances are example of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Effect of deterioration is very important in inventory system. In the present paper we consider random deterioration. Since deterioration of some items depends upon the fluctuation of humidity, temperature etc. Therefore it is more reasonable and realistic if we assume the deterioration function □to depend upon one more parameter □ in addition to time which ranges over a space in which some probability density function is defined. In this paper, a production inventory model for deteriorating items with lifetime is developed for a fixed and finite time horizon. . **Mandal and Phaujdar (1989)** presented an inventory model for deteriorating items and stock dependent consumption rate. An ordering policy for decaying inventory was developed by **Aggarwal and Jaggi (1989)**. **Su et al (1996)** developed an inventory model under inflation for stock dependent consumption rate and exponential decay. A deterministic production inventory model for deteriorating items and exponential declining demand was represented by **Lin et al (1999). Naresh Kumar and A. K. Sharma (2000)** formulated deterministic production inventory model for deteriorating items with an exponential declining demand. **Dutta** and **Pal (1990)** formulated inventory model for deteriorating items with demand rate to be a linear function of an hand inventory. **Goyal** and **Giri (2001)** presented an EOQ model for deteriorating items with time varing demand and partial backlogging. **Perumal (2002)** presented inventory model with two rates of production and backorders. **Tengel al (2003)** developed inventory model for deteriorating items with time varying demand and partial backlogging.

II. ASSUMPTIONS AND NOTATIONS

A Production inventory model for decaying items with lifetime and price dependent demand for finite time horizon is developed under following assumptions and notations:

(i) A variable fraction $\theta = \theta_0(\alpha, t)$ of the on hand inventory deteriorates per unit time only after the expiry of the life period μ of the item.

The deterioration function θ is taken as follows:-

$$\theta(t) = \theta_0(\alpha, t)H(t-\mu) \quad , t, \mu > 0$$

Where H(t - μ) is Heaviside unit function defined as follows :-

$$H(t-\mu) = 1 \quad , t > \mu$$

$$= 0 \quad , t \geq \mu$$, and

$$\theta_0(\alpha, t) = \theta_0(\alpha)t \quad , 0 < \theta_0 \quad (\alpha) << 1 \quad , t > 0$$

is a special form of two parameter Weibull function .The function is some function of the random variable α which ranges over a space Γ and in which a probability density function p(α) is defined such that—

$$\int_{\Gamma} p(\alpha) \; d\alpha = 1$$

(ii) D (p) is the demand rate when the selling price is p.

(iii) I (t) is the inventory level at any time t, and I'(t) is the derivative with respect to t.

(iv) Production rate is the linear combination of on hand inventory and demand rate. i.e.

$$P(t, p) = I(t) + bD(p) \; , \; 0 \leq b < 1 \; , \; t \geq 0$$

(v) A single item is considered over the prescribed period T unit of time, which is subject to a linear deterioration rate.

(vi) No replacement or repair of deteriorated items is made during a given cycle.

(vii) Shortages are allowed and they are fully backlogged.

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(viii) C_2 is the shortage cost per unit time.

(ix) C_3 is the setup cost for each new cycle.

(x) C_d is the cost of deteriorated items.

(xi) $T(=t_1+t_2+t_3+t_4)$ is the cycle time.

(xii) Q_1 is the maximum inventory level.

(xiii) Q_2 is the unfilled order backlog.

(xiv) Q be the inventory level at time μ , $0 \le \mu \le t_1$

(xv) μ is the life time of items.

(xvi) C_1 is the inventory carrying cost per unit time.

(xvii) K is the total average cost of the system.

III. MATHEMATICAL FORMULATION AND ANALYSIS OF THE MODEL

In the proposed system, a production/inventory cycle can be divided into five periods . The inventory starts and ends with zero stocks. Initially the production starts at time with the concept of non instantaneous deterioration and after life time μ when inventory level become Q , deterioration take place, and after t1 units of time it reaches to maximum inventory level Q1. After this production stopped and at time, t2 the inventory level becomes zero. At this time shortage starts developing at time t_3, it reaches to maximum shortage level Q2. At this stage fresh production starts to clear the backlogged by the time $t=t_4$. We want to find out mathematical expression for optimum values of t_1, t_2, t_3, t_4, Q_1 and Q_2 which minimize the total average cost K.

The inventory level I (t) at time t ($0 \le t \le T$) satisfies the differential equations:

$$I'(t) = P(t,p) - D(p),\quad 0 \le t \le \mu \qquad \text{... (1)}$$

$$I'(t) = -\theta(t)I(t) + P(t,p) - D(p),\quad \mu \le t \le t_1 \qquad \text{... (2)}$$

$$I'(t) = -\theta(t)I(t) - D(p),\quad 0 \le t \le t_2 \qquad \text{... (3)}$$

$$I'(t) = -D(p),\quad 0 \le t \le t_3 \qquad \text{... (4)}$$

$$I'(t) = P(t,p) - D(p),\quad 0 \le t \le t_4 \qquad \text{... (5)}$$

boundary conditions are given by

$$I(t) = 0 \text{ at } t = 0,\ t_1 + t_2 \text{ and } T \qquad \text{... (6)}$$

$$I(\mu) = q,\ I(t_1) = Q_1,\ I(t_1+t_2+t_3) = Q_2 \qquad \text{... (7)}$$

The solutions of equations (1), (2), (3), (4) and (5) are given by

$$I(t) = (b-1)D(p)(e^t - 1),\ 0 \le t \le \mu \qquad \text{... (8)}$$

$$I(t) = (1-b)D(p)[1+\theta_0(\alpha)(t+1)] + qe^{t-\mu}\left[1+\frac{\theta_0(\alpha)}{2}(\mu^2-t^2)\right] - (1-b)D(p)e^{t-\mu}\left[1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)}{2}(\mu^2-t^2)\right],$$

$$\mu \le t \le t_1 \qquad \text{... (9)}$$

$$I(t) = D(p)\left[(t_2-t)+\theta_0(\alpha)\left\{\frac{t_2^3}{6}-\frac{t^2t_2}{2}+\frac{t^3}{3}\right\}\right],\quad 0 \le t \le t_2 \qquad \text{...(10)}$$

$$I(t) = -D(p)t,\quad 0 \le t \le t_3 \qquad \text{...(11)}$$

and

$$I(t) = (b-1)D(p)(e^{t-t_4} - 1),\quad 0 \le t \le t_4 \qquad \text{...(12)}$$

Using boundary conditions (6) and (7).

$$Q_1 = D(p)\left(t_2 + \frac{\theta_0(\alpha)t_2^3}{6}\right) = (1-b)D(p)\left[1+\theta_0(\alpha)(t_1+1)\right] + qe^{t_1-\mu}\left[1+\frac{\theta_0(\alpha)}{2}(\mu^2-t_1^2)\right] - (1-b)D(p)e^{t_1-\mu}\left[1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)}{2}(\mu^2-t_1^2)\right] \qquad \text{... (13)}$$

From this equation we observe that variables t1 and t2 are not independent variables. Therefore one can write

$$t_2 = f_1(t_1) \qquad \text{... (14)}$$

Also

$$Q_2 = -D(p)t_3 = (b-1)D(p)(e^{-t_4} - 1) \qquad \text{... (15)}$$

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From this equation it can be observe that variables t3 and t4 are dependent. Therefore

$$t_3 = (1-b)(e^{-t_4}-1) = f_2(t_4)$$

... (16)

The deterioration cost for the period (0, T) is given by

$$C_d\left[\int_\mu^{t_1}\theta(t)I(t)dt + \int_0^{t_2}\theta(t)I(t)dt\right]$$

$$= C_d\left[\theta_0(\alpha)\left\{e^{t_1-\mu}(t_1-1)-(\mu-1)\right\}.\{q-(1-b)D(p)\}\right.$$

$$\left.+\theta_0(\alpha)D(p)\left\{(1-b)\left(\frac{t_1^2-\mu^2}{2}\right)+\frac{t_2^3}{6}\right\}\right] \quad \ldots (17)$$

Inventory carrying cost over the period (0, T) is given by

$$C_1\left[\int_0^{\mu}I(t)dt+\int_\mu^{t_1}I(t)dt+\int_0^{t_2}I(t)dt\right]$$

$$=C_1\left[(b-1)D(p)(e^{\mu}-\mu-1)+(1-b)D(p)\left\{(1+\theta_0(\alpha))(t_1-\mu)+\frac{\theta_0(\alpha)}{2}(t_1^2-\mu^2)\right\}\right.$$

$$+\left\{q\left(1+\frac{\theta_0(\alpha)\mu^2}{2}\right)-(1-b)D(p)\left(1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)}{2}\mu^2\right)\right\}\left\{e^{t_1-\mu}-1\right\}$$

$$-\frac{\theta_0(\alpha)}{2}\{q-(1-b)D(p)\}\left\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\right\}$$

$$\left.+D(p)\left(\frac{t_2^2}{2}+\frac{\theta_0(\alpha)t_2^4}{12}\right)\right]$$

… (18)

Shortage cost is given by

$$C_2\left[-\int_0^{t_3}I(t)dt+\int_0^{t_4}I(t)dt\right] =$$

$$C_2\left[D(p)\frac{t_3^2}{2}+(b-1)D(p)(1-t_4-e^{-t_4})\right]$$

... (19)

Hence the total cost of the inventory system is, K = Set up cost + Deterioration cost + Inventory carrying cost + Shortage cost.

$$=\frac{C_3}{T}+\frac{C_d}{T}\left[\theta_0(\alpha)\left\{e^{t_1-\mu}(t_1-1)-(\mu-1)\right\}\{q-(1-b)D(p)\}+\theta_0(\alpha)D(p)\right.$$

$$\left.\left\{(1-b)\frac{t_1^2-\mu^2}{2}+\frac{t_2^3}{6}\right\}\right]+\frac{C_1}{T}\left[(b-1)D(p)(e^{\mu}-\mu-1)+(1-b)D(p)\right.$$

$$\left\{(1+\theta_0(\alpha))(t_1-\mu)+\frac{\theta_0(\alpha)}{2}(t_1^2-\mu^2)\right\}+\left\{q\left(1+\frac{\theta_0(\alpha)\mu^2}{2}\right)\right.$$

$$\left.-(1-b)D(p)\left[1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)\mu^2}{2}\right]\right\}\left(e^{t_1-\mu}-1\right)$$

$$-\frac{\theta_0(\alpha)}{2}\{q-(1-b)D(p)\}\left\{e^{t_1-\mu}(t_1^2-2t_1+2)\right.$$

$$\left.\left.-(\mu^2-2\mu+2)\right\}+D(p)\left\{\frac{t_2^2}{2}+\frac{\theta_0(\alpha)t_2^4}{12}\right\}\right]+\frac{C_2D(p)}{T}\left[\frac{t_3^2}{2}+(b-1)(1-t_4-e^{-t_4})\right]$$

(20)

IV. APPROXIMATE SOLUTION PROCEDURE

Equation (20) contains four variables t1, t2, t3 and t4. However these variables are not independent and are related by (14) and (16). Also we have K > 0.

For minimum K, we must have

$$\frac{\partial K}{\partial t_1}=0 \text{ and } \frac{\partial K}{\partial t_4}=0 \quad \ldots\ldots\ldots\ldots (21)$$

Provided these values of t_j satisfy the conditions

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$$\frac{\partial^2 K}{\partial t_1^2} > 0,\ \frac{\partial^2 K}{\partial t_4^2} > 0 \text{ and } \frac{\partial^2 K}{\partial t_1^2}\frac{\partial^2 K}{\partial t_4^2} - \left(\frac{\partial^2 K}{\partial t_1 \partial t_4}\right)^2 > 0$$

Now differentiating (20) with respect to t_1 and t_4, we get

$$\{1+f_1'(t_1)\}[C_3 + C_d\{\theta_0(\alpha)\left(e^{t_1-\mu}(t_1-1)-(\mu-1)\right)\left(q-(1-b)D(p)\right)$$

$$+\theta_0(\alpha)D(p)\left((1-b)\frac{t_1^2-\mu^2}{2}+\frac{f_1^3(t_1)}{6}\right)\} + C_1[(b-1)D(p)(e^{\mu}-\mu-1)$$

$$+(1-b)D(p)\left((1+\theta_0(\alpha))(t_1-\mu)+\frac{\theta_0(\alpha)}{2}(t_1^2-\mu^2)\right)$$

$$+\left\{q\left(1+\frac{\theta_0(\alpha)\mu^2}{2}\right)-(1-b)D(p)\left(1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)\mu^2}{2}\right)\right\}\left(e^{t_1-\mu}-1\right)$$

$$-\frac{\theta_0(\alpha)}{2}\{q-(1-b)D(p)\}\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\}$$

$$+D(p)\left\{\frac{f_1^2(t_1)}{2}+\frac{\theta_0(\alpha)f_1^4(t_1)}{12}\right\}] + C_2[D(p)\frac{f_2^2(t_4)}{2}$$

$$+(b-1)D(p)(1-t_4-e^{-t_4})]] - C_dT[\theta_0(\alpha)t_1e^{t_1-\mu}\{q-(1-b)D(p)\}$$

$$+\theta_0(\alpha)D(p)\left\{(1-b)t_1+\frac{f_1^2(t_1)f_1'(t_1)}{2}\right\}] - C_1T[(1-b)D(p)\{(1+\theta_0(\alpha))+\theta_0(\alpha)t_1\}$$

$$+\left\{q\left(1+\frac{\theta_0(\alpha)\mu^2}{2}\right)-(1-b)D(p)\left(1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)\mu^2}{2}\right)\right\}e^{t_1-\mu}$$

$$-\frac{\theta_0(\alpha)}{2}\left(q-(1-b)D(p)\right)t_1^2e^{t_1-\mu}+D(p)\left\{f_1(t_1)f_1'(t_1)+\frac{\theta_0(\alpha)f_1^3(t_1)f_1'(t_1)}{3}\right\}]=0$$

(22)

nd

$$\{1+f_2'(t_4)\}[C_3 + C_d\{\theta_0(\alpha)\left(e^{t_1-\mu}(t_1-1)-(\mu-1)\right)\left(q-(1-b)D(p)\right)$$

$$+\theta_0(\alpha)D(p)\left((1-b)\frac{t_1^2-\mu^2}{2}+\frac{f_1^3(t_1)}{6}\right)\} + C_1[(b-1)D(p)(e^{\mu}-\mu-1)$$

$$+(1-b)D(p)\left((1+\theta_0(\alpha))(t_1-\mu)+\frac{\theta_0(\alpha)}{2}(t_1^2-\mu^2)\right)$$

$$+\left\{q\left(1+\frac{\theta_0(\alpha)\mu^2}{2}\right)-(1-b)D(p)\left(1+\theta_0(\alpha)(\mu+1)+\frac{\theta_0(\alpha)\mu^2}{2}\right)\right\}\left(e^{t_1-\mu}-1\right)$$

$$-\frac{\theta_0(\alpha)}{2}\{q-(1-b)D(p)\}\{e^{t_1-\mu}(t_1^2-2t_1+2)-(\mu^2-2\mu+2)\}$$

$$+D(p)\left\{\frac{f_1^2(t_1)}{2}+\frac{\theta_0(\alpha)f_1^4(t_1)}{12}\right\}] + C_2[D(p)\frac{f_2^2(t_4)}{2}$$

$$+(b-1)D(p)(1-t_4-e^{-t_4})]] - C_2T\{D(p)f_2(t_4)f_2'(t_4) + (b-1)D(p)(e^{-t_4}-1)\} = 0$$

... *(23)*

Where f1 (t1) and f2 (t4) are given by equations (14) and (16). From these two simultaneous non-linear equations, the optimum values of t1 and t4 can be found out using some suitable computational numerical method. The optimum values of t2, t3, Q1, Q2 and minimum average cost K can be obtained from (13), (15) and (20).

V. SPECIAL CASES

CASE I. If $\theta_0(\alpha) = 0$, b = 0 then the discussed model reduces to production inventory model without deterioration and in which production rate depends on inventory level. Then equation (20) reduces to

$$K = \frac{C_3}{T} + \frac{C_1}{T}\left[D(p)(1+\mu-e^{\mu})+D(p)(t_1-\mu) + \{q-D(p)\}(e^{t_1-\mu}-1)+D(p)\frac{t_2^2}{2}\right] + \frac{C_2D(p)}{T}\left[\frac{t_3^2}{2}-(1-t_4-e^{-t_4})\right]$$

CASE II. If b = 0, then the discussed model reduces to production inventory model in which production rate depends on inventory level.

VI. CONCLUSION

In this paper, a production inventory model for random deteriorating items with lifetime and price dependent demand is developed for a fixed and finite time horizon. Shortages and excess demand is backlogged.

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In the present model production rate is taken as linear combination of on-hand inventory and demand both. Special cases of model are also discussed.

REFERENCES

1. **B.N. Mandal and S. Phaujdar (1989):** An inventory model for deteriorating items and stock dependent consumption rate.
2. *J. Opl. Res. Soc. 40, 483-488.*
3. **S.P. Aggarwal and C.K. Jaggi (1989):** Ordering policy for decaying inventory.
4. *Int. J. System Sci. 20, 151-155.*
5. **Su., C.T., Tong, L.I. and H.C., Liao (1996):** An inventory model under inflation for stock dependent consumption rate and exponential decay. *Opsearch, 33, 71-82.*
6. **Su., C.T., Lin, C.W. and C.H., Tsai (1999):** A deterministic production inventory model for deteriorating items and exponential declining demand.
7. *Opsearch, 36, 95-106.*
8. **Naresh Kumar and Anil Kumar Sharma (2000):** On deterministic production inventory model for deteriorating items with an exponential declining demand.
9. Acta Ciencia Indica, Vol. XXVI M, 4, 305-310.
10. **Kun-Shan Wu, Liang-Yuh Ouyang and Chih-Te Yang (2005)** An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. International Journal of Production Economics.

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