Signal Flow Graph Analysis of Linearized Takagi-Sugeno Fuzzy PI Controller

B. Amarendra Reddy, Praveen Adimulam, M. Sujatha

ABSTRACT: A systematic procedure for developing the signal flow graph model of linearized Takagi-Sugeno (TS) fuzzy PI controller is presented in this paper. This proposed method provides ease of model formulation and avoids the mathematical complexity involved in obtaining the linearized model from a non-linear model. As a first step in constructing the signal flow graph, the analytical structures of TS-fuzzy PI controller is needed. Triangular/trapezoidal membership functions are considered for input variables, Zadeh fuzzy logic AND operation and centroid defuzzifier, structural analysis of TS-fuzzy pi controller are considered. A TS-fuzzy PI controller is represented as a non-linear TS-fuzzy PI controller which is linearized around an operating point using perturbation method. For the linearized fuzzy TS-fuzzy PI controller signal flow graphs are developed.

Key words:- TS-fuzzy, PI controller.

I. INTRODUCTION

Since the first successful application of the idea of the fuzzy sets [1] to the control of dynamic plant by Mamdani and Assilian [2] there has been considerable worldwide interest in the subject of "Fuzzy Control System Engineering". It has been known that it is possible to control many complex systems effectively by human operators who have no knowledge of their underlying dynamics, while it is difficult to achieve the same with conventional controllers. It is this fact which has ultimately led to the prospective development of fuzzy control in a variety of applications [3] most of these applications have been based on the intuitive implementation of domain experts' experience."Analytical structure" we mean the mathematical expression of a fuzzy controller that represents precisely the fuzzy controller without any approximation. Note that this is never an issue for conventional control because the analytical structure of a conventional controller, linear or nonlinear, is always readily available for analysis and design. Thus the design goal is to design the controller structure and Parameters on the basis of the given system model so that resulting control System performance will meet user's performance specifications. For fuzzy control, in addition to this usual requirement, there exist few more major difficulties pertinent only to fuzzy control and irrelevant to conventional control. One of them is that the input-output structure of a fuzzy controller is usually mathematically unavailable after the controller is constructed; most fuzzy controllers are constructed via so called intelligent system approaches as opposed to the mathematical approaches exclusively used in conventional controller.The fuzzy controller have been treated and used as black-box controllers without the analytical structure information, precise and effective mathematical analysis and design are very difficult to achieve. Hence the foremost issue is revealing the analytical structure is sensible in the context of conventional control theory. This is to say that merely deriving the structure is not useful enough and the structure must be represented in a form clearly understandable from control theory point of view. Once the structure is well understood, analytical issues can be explored using the well-developed conventional control theory. Theoretical analysis coupled with signal flow graph depiction involving various system models demonstrates the effectiveness and superior performance of the simplest TS-fuzzy controller in comparison with linear PI controller.This paper organized as follows, section II describes the configuration of non-linear TS-fuzzy PI controller, Section III describes fuzzification algorithm and fuzzy control rules, section IV describes Fuzzy logic for the evaluation of fuzzy control rules. Section V describes Defuzzification Module and Structural analysis of the TS-fuzzy PI controller. Section VI describes Linearization using Perturbation Theory section VII describes the SFG analysis of the TS-fuzzy PI controller.

II. CONFIGURATION OF THE TS- FUZZY PI CONTROLLERS

The TS-fuzzy PI controller being developed is a nonlinear fuzzy PI controller. PI or Fuzzy PI controllers are the most used controllers in the industry, because the proportional (P) with the Integral actions in the proportional- integral (PI) controller eliminates the steady state error.

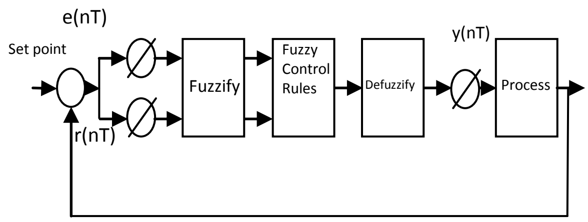

Fig 1:Block Diagram of a typical TS-fuzzy controller
e(nT) = SP(nT)-y(nT),
r(nT)=e(nT)-e(nT-T)

Where n is a positive integer, T is the sampling period and SP(nT) the set point. We denote e(nT) r(nT) , y(nT) as error, rate and process output.

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III. FUZZIFICATION ALORITHM

The following input membership functions are selected to transform inputs data of the FLC into two linguistic values, "P" and "N" for positive and negative input membership functions, as shown in fig.2 (a)

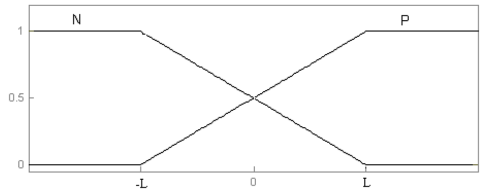

Fig.2 (a)the inputs membership functions of the FLC (error and rate)

The input membership functions of the FLC:

For error e(nT):

$$\mu_{\tilde{P}}(e) = \begin{cases} 0, & e(nT) < -L \\ \dfrac{e(nT)+L}{2L}, & -L \le e(nT) \le L \\ 1 & e(nT) > L \end{cases}$$

$$\mu_{\tilde{N}}(e) = \begin{cases} 0, & e(nT) < -L \\ \dfrac{-e(nT)+L}{2L}, & -L \le e(nT) \le L \\ 1 & e(nT) > L \end{cases}$$

Where $\mu_P(e)$ and $\mu_N(e)$ are the positive and negative membership functions of the error.

For rate r(nT):

$$\mu_{\tilde{P}}(r) = \begin{cases} 0, & r(nT) < -L \\ \dfrac{r(nT)+L}{2L}, & -L \le r(nT) \le L \\ 1 & r(nT) > L \end{cases}$$

$$\mu_{\tilde{N}}(r) = \begin{cases} 0, & r(nT) < -L \\ \dfrac{-r(nT)+L}{2L}, & -L \le r(nT) \le L \\ 1 & r(nT) > L \end{cases}$$

Where $\mu_P(r)$ and $\mu_N(r)$ are the positive and negative membership functions of the rate (change of error).

Fuzzy Rules base Module

The four fuzzy control rules are:

R1: if e(nT) is positive and r(nT)is positive then output is $a_1e(nT)+b_1r(nT)$

R2: if e(nT) is positive and r(nT) is negative then output is $a_2e(nT)+b_2r(nT)$

R3: if e(nT) is negative and r(nT) is positive then output is $a_3e(nT)+b_3r(nT)$

R4: if e(nT) is negative and r(nT) is negative then output is $a_4 e(nT)+b_4r(nT)$.

Here AND is Zadeh's logical "AND" defined by

$$\mu_A \text{ AND } \mu_B = \min\{\mu_A, \mu_B\}$$

for any membership value μ_A and μ_B on the fuzzy subsets A and B, respectively.

IV. FUZZY LOGIC FOR EVALUATION OF THE FUZZY CONTROL RULES

Zadeh fuzzy logic AND operator is used to realize the AND operations in antecedent part of the rules. Due to the use of Zadeh AND operator, the input space must be divided into number of regions in such a way that in each region a unique analytical inequality relationship can be obtained for each fuzzy rule between the two membership functions being AND.

Consider the first rule antecedent parts which contain two membership functions the boundary on which the membership value is same between two MFs is obtained by letting them equal.

Boundary division for rule1 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{P}}(r)$$

$$\frac{e(n)+L}{2L} = \frac{r(n)+L}{2L}$$

$$r(n) = e(n) \ldots [1]$$

Boundary division for rule2 is given by:

$$\mu_{\tilde{P}}(e) = \mu_{\tilde{N}}(r)$$

$$\frac{e(n)+L}{2L} = \frac{-r(n)+L}{2L}$$

$$r(n) = -e(n) \ldots [2]$$

Boundary division for rule3 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{P}}(r)$$

$$\frac{-e(n)+L}{2L} = \frac{r(n)+L}{2L}$$

$$r(n) = -e(n) \ldots [3]$$

Boundary division for rule3 is given by:

$$\mu_{\tilde{N}}(e) = \mu_{\tilde{N}}(r)$$

$$\frac{-e(n)+L}{2L} = \frac{-r(n)+L}{2L}$$

$$e(n) = r(n) \ldots [4]$$

Equations [1]-[4] are used to generate individually the space division (plane division) between error and rate.

Super imposing the four input space divisions from the expression [1]-[4] to form a total of 20 input combinations. They are labeled from ICI to IC20, as shown in *Fig 4*

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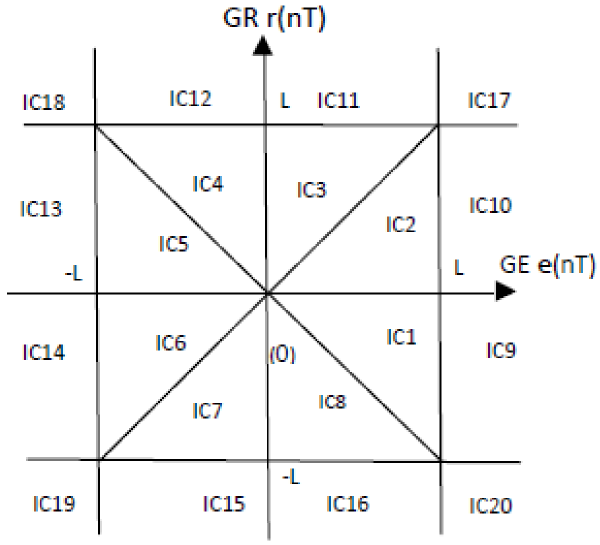

Fig.3 Possible input combinations (IC) of scaled error and rate of change of error, which must be considered when the fuzzy controller rules are evaluated as shown in Table. 1.

R1: if error is positive and rate is positive then output.

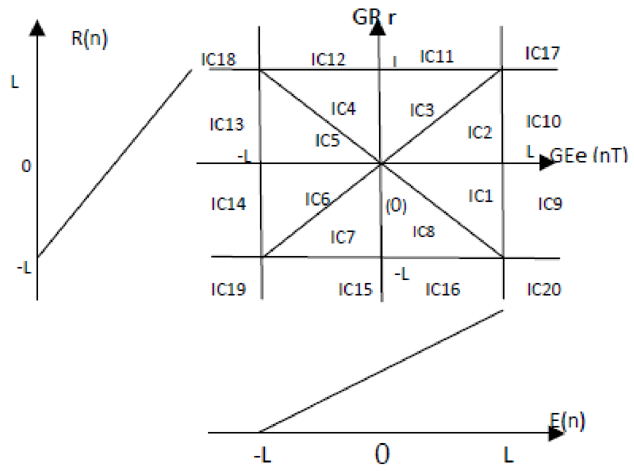

Fig. 4 division of input space for evaluation of Zadeh AND operation for fuzzy rule r1

The rule1 is associated with error positive and rate positive membership functions. Consider any point in the IC1 region, i.e., from 0 to L [0, L] error positive membership function will take the values from 0.5 to 1.0 and rate positive will take the values from 0 to 0.5. Applying Zadeh AND operation i.e., min(e.p, r.p), minimum of error positive and rate positive is rate positive. Similarly if we apply the minimum operation (Zadeh AND) to all regions and all rules, the results are tabulated as shown in Table 1.

These regions are necessary because they will result in each region of the 20 ICs, a unique inequality between error and rate when each of the four fuzzy rules evaluated by a Zadeh logic AND. After applying defuzzification algorithm to each region with resulting memberships, we obtain the expressions for different IC's which are in table2 in the next section.

Table-1 evaluation results for the four fuzzy control rules R1-R4 for all combinations of inputs using Zadeh AND fuzzy logic when scaled error and rate of process output are within the interval [-L, L] of the fuzzification algorithm. The input combinations of scaled error and rate are shown graphically in fig 4

IC No.	r1	r2	r3	r4
1,2	RP	RN	EN	EN
3,4	EP	RN	EN	RN
5,6	EP	EP	RP	RN
7,8	RP	EP	RP	EN
9,10	RP	RN	0	0
11,12	EP	0	EN	0
13,14	0	0	RP	RN
15,16	0	EP	0	EN
17	1	0	0	0
18	0	0	1	0
19	0	0	0	1
20	0	1	0	0

V. DEFUZZIFICATION MODULE

The defuzzification means the fuzzy to crisp conversions. The fuzzy results generated cannot be used such as to the applications hence it is necessary to convert the fuzzy quantities into crisp quantities for further processing. This can be achieved by using defuzzification process. Defuzzification method can also be called as "rounding off" method. The defuzzification reduces the collection of membership function values into a single scalar quantity.

The centroid defuzzification method is the mostly used method to convert the inference fuzzy control action to real number. The fuzzy logic controller output is obtained by,

$$\Delta u(nT) = \frac{\sum \Delta u_i \ \mu_{ri}}{\sum \mu_{ri}}$$

Where Δu_i is the value of the output member for i^{th} rule, μ_{ri} corresponding inferred input member for i^{th} rule.

Structural analysis of the fuzzy PI controller

The structure of the fuzzy PI controller from the defuzzification method is

$$\Delta u(nT) = \frac{\sum_{i=1}^{4} \Delta\mu_{ri}}{\sum_{i=1}^{4} \mu_{ri}} = \sum_{i=1}^{4} \frac{\mu_{ri}}{\sum_{i=1}^{4} \mu_{ri}} (a_i e(nT) + b_i r(nT))$$

$$= [K_I^i(e,r) r(nT) + K_P^i(e,r) e(nT)]$$

Where

$$K_p^i(e,r) = \frac{\mu_{ri} b_i}{\sum_i^4 \mu_{ri}}, \qquad K_I^i(e,r) = \frac{\mu_{ri} a_i}{\sum_i^4 \mu_{ri}}$$

$$K_P(e,r) = \sum_{i=1}^{4} K_p^i(e,r), \qquad K_I(e,r) = \sum_{i=1}^{4} K_I^i(e,r)$$

Where $K_P(e,r)$ and $K_I(e,r)$ are the dynamic proportional gain and integral gain respectively, they change with e(nT) and r(nT). This is to say the fuzzy controller is a nonlinear PI controller with variable proportional gain and integral gain. We have the structure related to PI controller. Since the only difference between them are the gains. We now derive the analytical expressions for $K_P(e,r)$ and $K_I(e,r)$.

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$$\Delta u(nT) = K_I(e,r)e(nT) + K_P(e,r)r(nT)$$

Table.2Mathematical input-output relations of the fuzzy controller for the overall input space division

IC NO.	$K_i = K_p$
IC1,IC2	$\frac{(L+r(nT))c_1+(L-r(nT))c_2+(L-e(nT))(c_3+c_4)}{2(2L-e(nT))}$
IC3,IC4	$\frac{(L+e(nT))c_1+(L-e(nT))c_3+(L-r(nT))(c_2+c_4)}{2(2L-r(nT))}$
IC5,IC6	$\frac{(L+e(nT))(c_1+c_2)+(L+r(nT))c_3+(L-r(nT))c_4}{2(2L-e(nT))}$
IC7,IC8	$\frac{(L-e(nT))c_4+(L+e(nT))c_2+(L+r(nT))(c_1+c_3)}{2(2L-r(nT))}$
IC9,IC10	$\frac{(L+r(nT))c_1+(L-r(nT))c_2}{2L}$
IC11,IC12	$\frac{(L+e(nT))c_1+(L-e(nT))c_3}{2L}$
IC13,IC14	$\frac{(L+r(nT))c_3+(L-r(nT))c_4}{2L}$
IC15,IC16	$\frac{(L+e(nT))c_2+(L-e(nT))c_4}{2L}$
IC17	c_1
IC18	c_3
IC19	c_4
IC20	c_2

Table 3 the coefficients C1C2C3and C4 in Table 2

K(e,r)	C_1	C_2	C_3	C_4
$K_P(e,r)$	b_1	b_2	b_3	b_4
$K_I(e,r)$	a_1	a_2	a_3	a_4

VI. LINEARIZATION USING PERTURBATION THEORY

Linearization is a method for assessing the local stability of a nonlinear system at an equilibrium point. Linearization makes it possible to use tools for studying linear systems to analyze the behavior of a nonlinear system near a given point. The linearization of a function is the first order term of its power series expansion around the point of interest. In this paper we linearize the nonlinear system using Perturbation theory. Perturbation theory leads to an expression for the desired solution in terms of a formal power series. The leading term in this power series is the solution of the exactly solvable non-linear problem.

We now consider incremental output of IC1 and apply perturbation theory to linearize it.

$$\Delta u = \frac{L(c_1+c_2+c_3+c_4)-e(nT)(c_3+c_4)+r(nt)(c_1-c_2)}{2(2L-e(nT))} \quad \dots (1)$$

$$u = u_0 + \Delta u \;\; ; \;\; e = e_0 + \Delta e \;\; ; \; r = r_0 + \Delta r \quad \dots (2)$$

Now put (2) in (1), we get

$$(u_0 + \Delta u) = \frac{L(c_1 + c_2 + c_3 + c_4) - (e_0 + \Delta e)(c_3 + c_4) + (r_0 + \Delta r)(c_1 - c_2)}{2(2L - e_0 - \Delta e)}$$

$$(u_0 + \Delta u)2(2L - e_0 - \Delta e) = L(c_1 + c_2 + c_3 + c_4) - (e_0 + \Delta e)(c_3 + c_4) + (r_0 + \Delta r)(c_1 - c_2)$$

$$\Delta u 2(2L - e_0) = L(c_1 + c_2 + c_3 + c_4) - \Delta e(c_3 + c_4 - 2u_0) + (r_0 + \Delta r)(c_1 - c_2)$$

$$\Delta u = \frac{L(c_1 + c_2 + c_3 + c_4) - \Delta e(c_3 + c_4 - 2u_0) + (r_0 + \Delta r)(c_1 - c_2)}{2(2L - e_0)}$$

Equation is now in linearized form of incremental output of IC1.

Table 4 shows all the linearized incremental outputs of IC1 – IC20.

IC NO.	$\Delta K_I = \Delta K_P$
1,2	$\frac{L(c_1 + c_2 + c_3 + c_4) + \Delta e(-c_3 - c_4 + 2u_0) + \Delta r(c_1 -}{2(2L - e_0)}$
3,4	$\frac{L(c_1 + c_2 + c_3 + c_4) + (c_2 + c_4 - 2u_0)\Delta r + (c_1 - c_3)\Delta e}{2(2L - r_0)}$
5,6	$\frac{L(c_1 + c_2 + c_3 + c_4) + (c_3 - c_4)\Delta r + (c_1 + c_2 + 2u_0)\Delta e}{2(2L - e_0)}$
7,8	$\frac{L(c_1 + c_2 + c_3 + c_4) + (c_2 - c_4)\Delta e + (c_1 + c_3 + 2u_0)\Delta r}{2(2L - r_0)}$
9,10	$\frac{(L + \Delta r)c_1 + (L - \Delta r)c_2}{2L}$
11,12	$\frac{(L + \Delta e)c_1 + (L - \Delta e(nT))c_3}{2L}$
13,14	$\frac{(L + \Delta r)c_3 + (L - \Delta r)c_4}{2L}$
15,16	$\frac{(L + \Delta e)c_2 + (L - \Delta e)c_4}{2L}$

For the linearized IC's we now draw the signal flow graphs which provides ease of model formulation and avoids the mathematical complexity involved in obtaining the linear fuzzy controller.

VII. SIGNAL FLOW GRAPH ANALYSIS

Signal-flow graph is a graphical representation of relationships between variables of a set of linear algebraic equations in a system. It is a directed graph consisting of nodes and branches. Its nodes are the variables of a set of linear algebraic relations. An SFG can only represent multiplications and additions. Multiplications are represented by the weights of the branches; additions are represented by multiple branches going into one node. A signal-flow graph has a one-to-one relationship with a system of linear equations. It can also be used to solve for ratios of these signals.

Key elements of a signal flow graph are:

1.The system must be linear,

2. Nodes represent the system variables,

3. Branches represent paths for signal flow.

4. Signals travel along branches only in the direction of the arrows.

Signal flow graphs (SFGs) can form an intuitive picture of the signal flow in a system. As an application, we will develop SFGs to all ICs from table 4.Signal flow graph as shown below

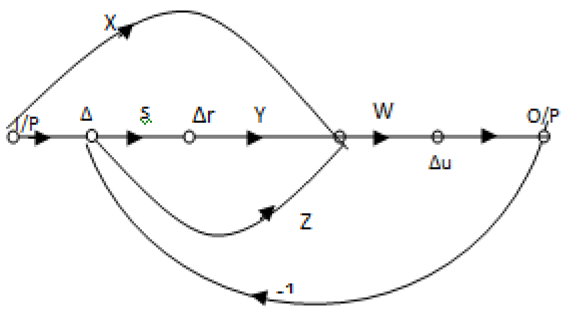


Fig-(a): signal flow graph model for 20 ICS

Table-5 Signal flow graph tabular form for X, Y, Z, W constants

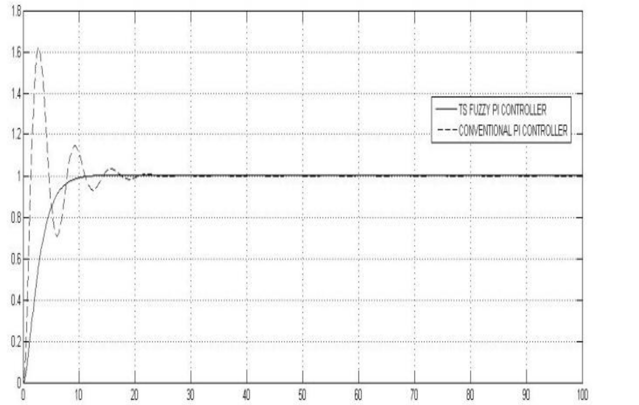
IC NO	X	Y	Z	W
IC1, IC2	$L(c_1 + c_2 + c_3 + c_4)$	$c_1 - c_2$	$-c_3 - c_4 + 2u_0$	$\frac{1}{2(2L - e_0)}$
IC3, IC4	$L(c_1 + c_2 + c_3 + c_4)$	$c_2 + c_4 - 2u_0$	$c_1 - c_3$	$\frac{1}{2(2L - r_0)}$
IC5, IC6	$L(c_1 + c_2 + c_3 + c_4)$	$c_3 - c_4$	$c_1 + c_2 + 2u_0$	$\frac{1}{2(2L - e_0)}$
IC7, IC8	$L(c_1 + c_2 + c_3 + c_4)$	$c_1 + c_3 + 2u_0$	$c_3 - c_4$	$\frac{1}{2(2L - r_0)}$
IC9,IC10	$L(c_1 + c_2)$	$c_1 - c_2$	0	$\frac{1}{2L}$
IC11,IC12	$L(c_1 + c_3)$	0	$c_1 - c_3$	$\frac{1}{2L}$
IC13,IC14	$L(c_3 + c_4)$	$c_3 - c_4$	0	$\frac{1}{2L}$
IC15,IC16	$L(c_2 + c_4)$	0	$c_3 - c_4$	$\frac{1}{2L}$
IC17	c_1	0	0	1
IC18	c_2	0	0	1
IC19	c_3	0	0	1
IC20	c_4	0	0	1

The Transfer functions from the signal flow graphs are develop using mason's gain formula and are tabulated in table-6.

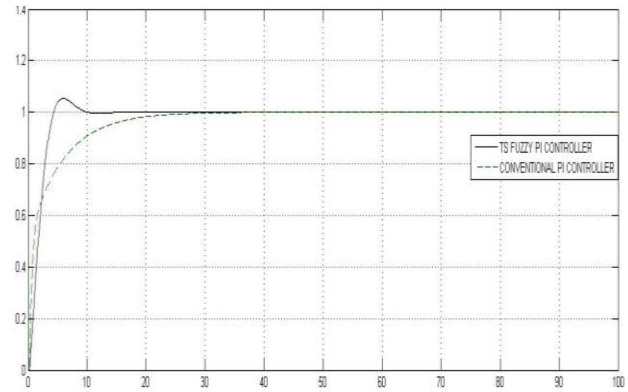
Table-6 transfer functions of the signal flow graphs

IC No.	$\Delta K_I = \Delta K_P$
1,2	$\frac{G[L(c_1 + c_2 + c_3 + c_4) - (c_3 + c_4 - 2u_0) + s(c_1 - c_2)]}{2(2L - e_0) + G[-(c_3 + c_4 - 2u_0) + s(c_1 - c_2)]}$
3,4	$\frac{G[L(c_1 + c_2 + c_3 + c_4) + (c_2 + c_4 - 2u_0)s + (c_1 - c_3)]}{2(2L - r_0) + G[(c_2 + c_4 - 2u_0)s + (c_1 - c_3)]}$
5,6	$\frac{G[L(c_1 + c_2 + c_3 + c_4) + (c_3 - c_4)s + (c_1 + c_2 + 2u_0)]}{2(2L - e_0) + G[(c_3 - c_4)s + (c_1 + c_2 + 2u_0)]}$
7,8	$\frac{G[L(c_1 + c_2 + c_3 + c_4) + (c_3 - c_4) + (c_1 + c_3 + 2u_0)s]}{2(2L - r_0) + G[(c_3 - c_4) + (c_1 + c_3 + 2u_0)s]}$
9,10	$\frac{G[(c_1 + c_2)L + s(c_1 - c_2)]}{2L + Gs(c_1 - c_2)}$
11,12	$\frac{G[L(c_1 + c_3) + (c_1 - c_3)]}{2L + G(c_1 - c_3)}$
13,14	$\frac{G[L(c_3 + c_4) + s(c_3 - c_4)]}{2L + Gs(c_3 - c_4)}$
15,16	$\frac{G[L(c_2 + c_4) + (c_2 + c_4)]}{2L + G(c_2 + c_4)}$

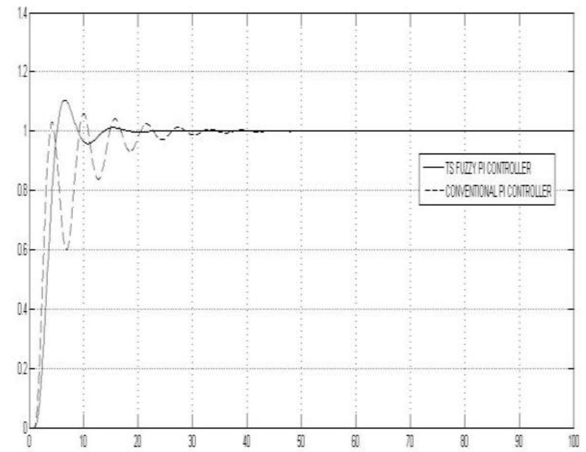
8. Simulation results:



Example 1: Consider a 1st Order time delay system, G(s) = $\frac{1}{s+1}e^{-0.2s}$. The control the performance of a system subjected to a unit step input is compared by considering conventional PI controller with TS-Fuzzy PI controller.



Example 2: Consider a 1st Order system with time delay, G(s) = $\frac{e^{-s}}{s^2+s+12}$. The control the performance of a system subjected to a unit step input is compared by considering conventional PI controller with TS-Fuzzy PI controller.



Example 3: Consider a 3rd Order system with time delay, G(s) = $\frac{1}{s^3+3s^2+3s+1}e^{-2s}$. The control the performance of a system subjected to a unit step input is compared by considering conventional PI controller with TS-Fuzzy PI controller.

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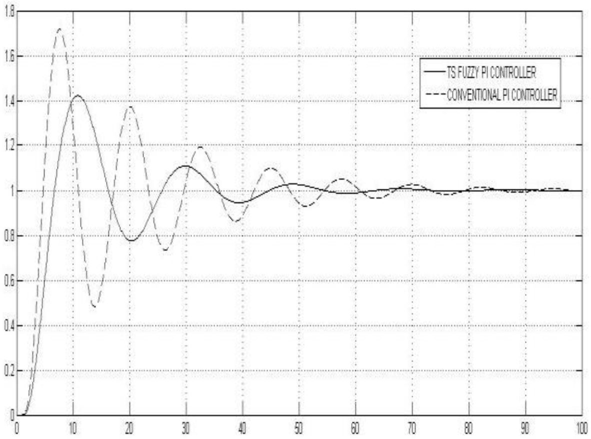

Example 4: Consider a non linear system, $\dot{y}(t) = -y(t) + \sqrt{|y(t)|} + u(t)$. The control the performance of a system subjected to a unit step input is compared by considering conventional PI controller with TS-Fuzzy PI controller.

VIII. CONCLUSION

In this paper we have derived the analytical input and output relationships for the linearized Takagi-Sugeno fuzzy PI controller having two input variables and output variable. Which is a function of input variables. There are two triangular/trapezoidal membership functions in each input variable. Zadeh AND operator is used to evaluate the antecedent part of the each of the rule. Since Zadeh AND is used, the input space is divided into 20 regions. Then the non-linear TS-fuzzy PI controller is linearized around an operating point using perturbation method. For the linearized TS-fuzzy PI controller signal flow graphs are developed for each IC and Transfer function from SFG for linearized fuzzy PI controllers was developed. Simulations show that the TS-Fuzzy PI controller is giving the good performance over the conventional PI controller.

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