Arithmetic Operations on Symmetric Trapezoidal Intuitionistic Fuzzy Numbers

R. Parvathi , C. Malathi

Abstract - In this paper, Symmetric Trapezoidal Intuitionistic Fuzzy Numbers (STIFNs) have been introduced and their desirable properties are also studied. A new type of intuitionistic fuzzy arithmetic operations for STIFN have been proposed based on (α,β)-cuts. A numerical example is considered to elaborate the proposed arithmetic operations. These operations find applications in solving linear programming problems in intuitionistic fuzzy environment and also to find regression coefficient in intuitionistic fuzzy environment.

Index Terms - Intuitionistic Fuzzy Index, Intuitionistic Fuzzy Number, Intuitionistic Fuzzy Set, Symmetric Trapezoidal Intuitionistic Fuzzy Number, (α,β)-cuts.

I. INTRODUCTION

In real life, information available is sometimes vague, inexact or insufficient and so the parameters of the problem are usually defined by the decision maker in an uncertain way. Therefore, it is desirable to consider the knowledge of experts about the parameters as fuzzy data. Out of several higher order fuzzy sets, intuitionistic fuzy sets (IFS) [2], [3] have been found to be highly useful to deal with vagueness. There are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. But evaluation of nonmembership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives. Certainly, IFS theory is more suitable to deal with such problem. The intuitionistic fuzzy set has received more and more attention since its appearance. Gau and Buehrer [7] introduced the concept of vague set. But Bustince and Burillo [5] showed that vague sets are intuitionistic fuzzy sets. Shu, Cheng and Chang [10] gave the definition and operational laws of triangular intuitionistic fuzzy number and proposed an algorithm of the intuitionistic fuzzy set fault-tree analysis. Wang [11] gave the definition of trapezoidal intuitionistic fuzzy number which is an extension of triangular intuitionistic fuzzy number. Triangular intuitionistic fuzzy numbers and trapezoidal intuitionistic fuzzy numbers are the extensions of IF sets in another way, which extends discrete set to continuous set.

Manuscript received on April 26, 2012.

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Bellman and Zadeh (1970) [4] proposed the concept of decision making in fuzzy environments. Zimmermann [12], [13] proposed the first formulation of Fuzzy Linear Programming Problem. K.Ganesan and P.Veeramani [6] introduced a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and proposed a method for solving fuzzy linear programming problems.

In this paper, STIFNs, which are the extension of trapezoidal intuitionistic fuzzy numbers, have been introduced and arithmetic operations based on (α,β)-cuts have also been proposed. Also, the properties of STIFNs have been discussed.

The paper is organized as follows: Section 2 briefly describes the basic definitions and notations of IFS, IFN and TIFN. In Section 3, STIFNs have been introduced and their properties are discussed. Arithmetic operations of STIFN based on (α,β)-cuts are presented in Section 4. A numerical example is presented in Section 5. Section 6 concludes the paper. Scope and future work of the authors on the topic are also given.

II. PRELIMINARIES

Definition 2.1 [1] *An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the form $A = \{<x,\mu_A(x),\nu_A(x)>: x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of nonmembership of the element $x \in X$ respectively, and for every $x \in X$ in A, $0 \le \mu_A(x)+\nu_A(x) \le 1$ holds.*

Note

Throughout this paper, μ represents membership values and ν represents nonmembership values.

Definition 2.2 [1] *For every common fuzzy subset A on X, Intuitionistic Fuzzy Index of x in A is defined as $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in A.*

Obviously, for every $x \in X$, $0 \le \pi_A(x) \le 1$.

Definition 2.3 [8] *An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is*

(i) an intuitionistic fuzzy subset of the real line,

(ii) normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1,\ \nu_{\tilde{A}^I}(x_0) = 0$,

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(iii) convex for the membership function $\mu_{\tilde{A}^I}(x)$, *that is,* $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)),$
for every $x_1, x_2 \in R, \quad \lambda \in [0,1],$

(iv) concave for the nonmembership function $\nu_A(x)$, *that is,* $\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq max(\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2))$
for every $x_1, x_2 \in R, \quad \lambda \in [0,1].$

Definition 2.4 $[8]$ *A Trapezoidal Intuitionistic Fuzzy Number (TIFN)* $\tilde{A}^I$ *is an IFS in R with membership function and nonmembership function as follows:*

$$\mu_{A^I}(x) = \begin{cases} \dfrac{x-(a_1-\alpha)}{\alpha} & \text{for } x \in [a_1-\alpha, a_1] \\ 1 & \text{for } x \in [a_1, a_2] \\ \dfrac{a_2+\beta-x}{\beta} & \text{for } x \in [a_2, a_2+\beta] \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{A^I}(x) = \begin{cases} \dfrac{a_1-x}{\alpha'} & \text{for } x \in [a_1-\alpha', a_1] \\ 0 & \text{for } x \in [a_1, a_2] \\ \dfrac{x-a_2}{\beta'} & \text{for } x \in [a_2, a_2+\beta'] \\ 1 & \text{otherwise} \end{cases}$$

where $a_1 \leq a_2$, $\alpha, \beta \geq 0$ *such that* $\alpha \leq \alpha'$ *and* $\beta \leq \beta'$.
A TIFN is denoted by $A^I_{TIFN} = [a_1, a_2, \alpha, \beta; a_1, a_2, \alpha', \beta'].$

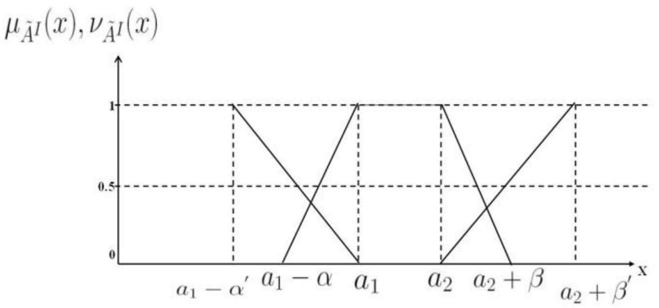

Fig. 1: Diagrammatic representation of a TIFN

Definition 2.5 $[8]$ *A set of* (α, β) *- cut, generated by an IFS* A^I, *where* $\alpha, \beta \in [0,1]$ *are fixed numbers such that* $\alpha + \beta \leq 1$ *is defined as* $A^I_{\alpha,\beta} =$

$$\left\{(x, \mu_{A^I}(x), \nu_{A^I}(x)) : x \in X, \mu_{A^I}(x) \geq \alpha, \nu_{A^I}(x) \leq \beta\right\}$$

(α, β) *-cut or* (α, β) *-level interval denoted by* $A^I_{\alpha,\beta}$ *is defined as the crisp set of elements x which belong to* A^I *atleast to the degree* α *and which belong to* A^I *atmost to the degree* β.

Definition 2.6 *The support of an IFS* A^I *on R is the crisp set of all* $x \in R$ *such that* $\mu_{A^I}(x) > 0, \nu_{A^I}(x) > 0$ *and* $\mu_{A^I}(x) + \nu_{A^I}(x) \leq 1.$

Definition 2.7 $[9]$ *An IFS A =* $\{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ *is called IF-normal if there exist atleast two points* $x_0, x_1 \in X$ *such that* $\mu_A(x_0) = 1, \nu_A(x_1) = 1.$

Therefore, a given intuitionistic fuzzy set A is IF-normal if there is atleast one point which belongs to A and atleast one point which does not belong to A.

III. SYMMETRIC TRAPEZOIDAL INTUITIONISTIC FUZZY NUMBERS

A TIFN is said to be STIFN if $\alpha = \beta$ (say h) and $\alpha' = \beta'$ (say h'). Hence the definition of STIFN is as follows:

An IFS $\tilde{A}^I$ in R is said to be a Symmetric Trapezoidal Intuitionistic Fuzzy Number (STIFN) if there exist real numbers a_1, a_2, h, h' where $a_1 \leq a_2$, $h \leq h'$ and $h, h' > 0$ such that the membership and nonmembership functions are as follows:

$$\mu_{A^I}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h} & \text{for } x \in [a_1-h, a_1] \\ 1 & \text{for } x \in [a_1, a_2] \\ \dfrac{a_2+h-x}{h} & \text{for } x \in [a_2, a_2+h] \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{A^I}(x) = \begin{cases} \dfrac{a_1-x}{h'} & \text{for } x \in [a_1-h', a_1] \\ 0 & \text{for } x \in [a_1, a_2] \\ \dfrac{x-a_2}{h'} & \text{for } x \in [a_2, a_2+h'] \\ 1 & \text{otherwise} \end{cases}$$

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where $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$.

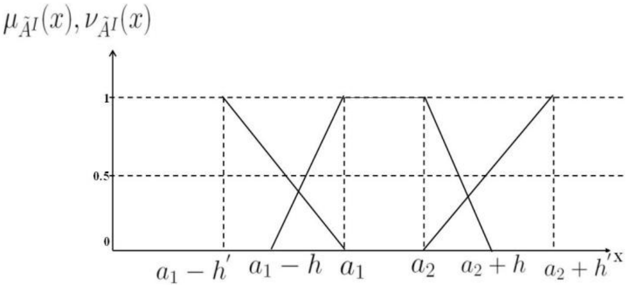

Fig. 2: Diagrammatic representation of a STIFN

Theorem 3.1 *If a STIFN is of the form* $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$ *then,* $h \le h'$.

Proof

From the definition of membership and nonmembership functions of a STIFN, for $x \in [a_1 - h, a_1]$, it can be written as

$$\mu_{\tilde{A}^I}(x) + \frac{a_1 - x}{h'} \le 1 \text{ [Since } \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1]$$

That is,

$$\frac{x-(a_1-h)}{h} + \frac{a_1 - x}{h'} \le 1 \Rightarrow \frac{x - a_1}{h} + \frac{a_1 - x}{h'} \le 0$$

[Since $x \le a_1 \Rightarrow a_1 - x \ge 0$]

$\Rightarrow h \le h'$.

Similarly, for $x \in [a_2, a_2 + h]$, it can be written as,

$$\frac{(a_2+h)-x}{h} + \frac{x-a_2}{h'} \le 1$$

$$\frac{x-a_2}{h'} - \frac{x-a_2}{h} \le 0$$

$$\frac{1}{h'} \le \frac{1}{h} \quad [x - a_2 \ge 0 \text{ as } x \ge a_2]$$

$$\Rightarrow h \le h'$$

Hence, in general, a STIFN can be represented as

$$\tilde{A}^I_{STIFN} = [a_1, a_2, h, h; a_1, a_2, h', h' : h \le h']$$

Theorem 3.2 *Transformation rule for the STIFN* $\tilde{A}^I_{STIFN} = [a_1, a_2, h, h; a_1, a_2, h', h']$ *to STFN* $\tilde{A}_{STFN} = [a_1, a_2, h, h]$ *is that* $h = h'$ *and* $\nu_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x)$, *for every* $x \in R$. *In this case, hesitancy degree becomes zero.*

Proof

By considering $\nu_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x)$ for every $x \in R$ and by taking $h = h'$, membership and nonmembership functions of STIFN are given as follows:

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{x-(a_1-h)}{h} & \text{for } x \in [a_1 - h, a_1] \\ 1 & \text{for } x \in [a_1, a_2] \\ \dfrac{(a_2+h)-x}{h} & \text{for } x \in [a_2, a_2 + h] \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

and

$$\nu_{\tilde{A}^I}(x) = \begin{cases} \dfrac{a_1 - x}{h} & \text{for } x \in [a_1 - h, a_1] \\ 0 & \text{for } x \in [a_1, a_2] \\ \dfrac{x - a_2}{h} & \text{for } x \in [a_2, a_2 + h] \\ 1 & \text{otherwise} \end{cases} \quad (2)$$

From (1) and (2), it is clear that $\nu_{\tilde{A}^I}(x)$ is the complement of $\mu_{\tilde{A}^I}(x)$ and hence the above transfomation rule transforms STIFN to STFN.

Theorem 3.3 *Transformation rule for the STIFN* $\tilde{A}^I_{STIFN} = [a_1, a_2, h, h; a_1, a_2, h', h']$ *to a real number 'a' is that* $h = h' = 0$ *and* $a_1 = a_2 = a$.

Proof

The proof is obvious.

IV. ARITHMETIC OPERATIONS ON STIFN BY (α, β)-CUTS

A Addition

If $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$ and $\tilde{B}^I = [b_1, b_2, k, k; b_1, b_2, k', k']$ are two STIFNs, then $\tilde{C}^I = \tilde{A}^I + \tilde{B}^I$ is also a STIFN and is given by

$$\tilde{C}^I = [a_1 + b_1, a_2 + b_2, h + k, h + k; a_1 + b_1, a_2 + b_2,$$

Proof

With the transformation $z = x + y$, the membership function and nonmembership function of IFS $\tilde{C}^I = \tilde{A}^I + \tilde{B}^I$ can be established by (α, β)-cut method.

α-cut for membership function of $\tilde{A}^I$ is $[(a_1 - h) + \alpha h, (a_2 + h) - \alpha h]$ for every $\alpha \in [0,1]$.

That is, $x \in [(a_1 - h) + \alpha h, (a_2 + h) - \alpha h]$.

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α -cut for membership function of $\tilde{B}^I$ is $[(b_1-k)+\alpha k,(b_2+k)-\alpha k]$ for every $\alpha\in[0,1]$. That is, $y\in[(b_1-k)+\alpha k,(b_2+k)-\alpha k]$.

So, $z\ (=x+y)\in[a_1+b_1-(h+k)+\alpha(h+k);$
$a_2+b_2+(h+k)-\alpha(h+k)]$.

The membership function of $\tilde{C}^I=\tilde{A}^I+\tilde{B}^I$ is given by $\mu_{C^I}(z)=$

$$\begin{cases} \dfrac{z-[(a_1+b_1)-(h+k)]}{h+k} & \text{for } z\in[a_1+b_1-(h+k),a_1+b] \\ 1 & \text{for } z\in[a_1+b_1,a_2+b_2] \\ \dfrac{[(a_2+b_2)+(h+k)]-z}{h+k} & \text{for } z\in[a_2+b_2,a_2+b_2+(h+k)] \\ 0 & \text{otherwise} \end{cases}$$

For nonmembership function, β -cut of $\tilde{A}^I$ is $[a_1-\beta h',a_2+\beta h']$ for every $\beta\in[0,1]$. That is,

$x\in[a_1-\beta h',a_2+\beta h']$. β -cut for non-memebrship function of $\tilde{B}^I$ is $[b_1-\beta k',b_2+\beta k']$. That is, $y\in[b_1-\beta k',b_2+\beta k']$. So, $z=x+y$
$\in[a_1+b_1-\beta(h'+k'),a_2+b_2+\beta(h'+k')]$.

Therefore, the nonmembership function of $\tilde{C}^I=\tilde{A}^I+\tilde{B}^I$ is given by $\nu_{C^I}(z)=$

$$\begin{cases} \dfrac{a_1+b_1-z}{h'+k'} & \text{for } z\in[a_1+b_1-(h+k),a_1+b_1] \\ 0 & \text{for } z\in[a_1+b_1,a_2+b_2] \\ \dfrac{z-(a_2+b_2)}{h'+k'} & \text{for } z\in[a_2+b_2,a_2+b_2+(h+k)] \\ 1 & \text{otherwise} \end{cases}$$

B Additive Image of $\tilde{A}^I$

If $\tilde{A}^I=[a_1,a_2,h,h;a_1,a_2,h',h']$ is a STIFN, then the additive image of $\tilde{A}^I$ is also a STIFN and is given by

$\tilde{B}^I=-\tilde{A}^I=[-a_2,-a_1,h,h;-a_2,-a_1,h',h']$.

Proof

With the transformation $z=-x$, the membership function and nonmembership function of IFS $\tilde{B}^I=-\tilde{A}^I$ can be found by (α,β) -cut method.

α -cut for membership function of $\tilde{A}^I$ is $[(a_1-h)+\alpha h,(a_2+h)-\alpha h]$ for every $\alpha\in[0,1]$.

$\Rightarrow z(=-x)\in[-(a_2+h)+\alpha h,-(a_1-h)-\alpha h]$

So, the membership function of $\tilde{B}^I=-\tilde{A}^I$ is

$$\mu_{B^i}(z)=\begin{cases} \dfrac{z-(-a_2-h)}{h} & \text{for } z\in[-a_2-h,-a_2] \\ 1 & \text{for } z\in[-a_2,-a_1] \\ \dfrac{-a_1+h-z}{h} & \text{for } z\in[-a_1,-a_1+h] \\ 0 & \text{otherwise} \end{cases}$$

For nonmembership function, β -cut of $\tilde{A}^I$ is $[a_1-\beta h',a_2+\beta h']$ for every $\beta\in[0,1]$.

That is, $x\in[a_1-\beta h',a_2+\beta h']$

$\Rightarrow z(=-x)\in[-a_2-\beta h',-a_1+\beta h']$

So, the nonmembership function of $\tilde{B}^I=-\tilde{A}^I$ is

$$\nu_{B^I}(z)=\begin{cases} \dfrac{-a_2-z}{h'} & \text{for } z\in[-a_2-h',-a_2] \\ 0 & \text{for } z\in[-a_2,-a_1] \\ \dfrac{z-(-a_1)}{h'} & \text{for } z\in[-a_1,-a_1+h'] \\ 1 & \text{otherwise} \end{cases}$$

Diagrammatic Representation

Figure 3 describes the additive image of the STIFN given in Figure 2.

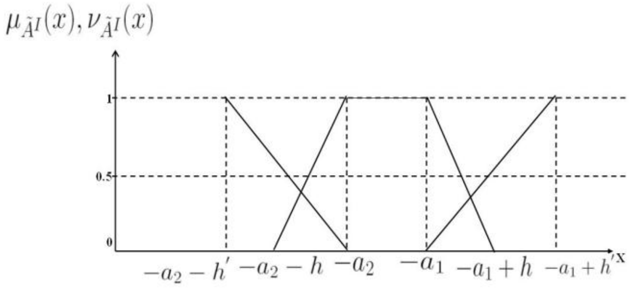

Fig. 3: Additive Image of a STIFN

C Subtraction

If $\tilde{A}^I=[a_1,a_2,h,h;a_1,a_2,h',h']$ and $\tilde{B}^I=[b_1,b_2,k,k;b_1,b_2,k',k']$ are two STIFNs, then $\tilde{D}^I=\tilde{A}^I-\tilde{B}^I$ is also a STIFN and is given by

$\tilde{D}^I=[a_1-b_2,a_2-b_1,h+k,h+k;a_1-b_2,a_2-b_1,$
$h'+k',h'+k']$.

Proof

With the transformation $z=x-y$, the membership function and nonmembership funciton of IFS $\tilde{D}^I=\tilde{A}^I-\tilde{B}^I$ can be found by using (α,β) -cut method.

α -cut for membership

function of $\tilde{A}^I$ is $[(a_1-h)+\alpha h,(a_2+h)-\alpha h]$ for every $\alpha\in[0,1]$. That is, $x\in[(a_1-h)+\alpha h,(a_2+h)-\alpha h]$. α-cut for the membership function of $\tilde{B}^I$ is $[(b_1-k+\alpha k),(b_2+k)-\alpha k]$

That is, $y\in[(b_1-k+\alpha k),(b_2+k)-\alpha k]$.

Therefore, α-cut for the membership function of additive image of $\tilde{B}^I$ is $[-(b_2+k)+\alpha k,-(b_1-k)-\alpha k]$. That is, $y\in[-(b_2+k)+\alpha k,-(b_1-k)-\alpha k]$. Therefore, $z(=x-y)\in[(a_1-h)+\alpha h-(b_2+k)+\alpha k,$ $(a_2+h)-\alpha h-(b_1-k)-\alpha k]$.

That is, $z\in[(a_1-b_2)-(h+k)+\alpha(h+k);$ $(a_2-b_1)+(h+k)-\alpha(h+k)]$.

So, the membership function of $\tilde{D}^I=\tilde{A}^I-\tilde{B}^I$ is given by $\mu_{D^I}(z)=$

$$\begin{cases}\dfrac{z-[(a_1-b_2)-(h+k)]}{h+k} & \text{for } z\in[a_1-b_2-(h+k),a_1-b_2]\\ 1 & \text{for } z\in[a_1-b_2,a_2-b_1]\\ \dfrac{(a_2-b_1)+(h+k)-z}{h+k} & \text{for } z\in[a_2-b_1,a_2-b_1+(h+k)]\\ 0 & \text{otherwise}\end{cases}$$

For nonmembership function, β-cut of $\tilde{A}^I$ is $[a_1-\beta h',a_2+\beta h']$ for every $\beta\in[0,1]$. That is, $x\in[a_1-\beta h',a_2+\beta h']$. β-cut for nonmembership function of $\tilde{B}^I$ is $[b_1-\beta k',b_2+\beta k']$. That is, $y\in[b_1-\beta k',b_2+\beta k']$.

Therefore,

$z(=x+(-y)=x-y)\in[a_1-b_2-\beta(h'+k'),$ $a_2-b_1+\beta(h'+k')]$.

So, the nonmembership function of $\tilde{D}^I=\tilde{A}^I-\tilde{B}^I$ is given by $\nu_{D^I}(z)=$

$$\begin{cases}\dfrac{a_1-b_2-z}{h'+k'} & \text{for } z\in[a_1-b_2-(h'+k'),a_1-b_2]\\ 0 & \text{for } z\in[a_1-b_2,a_2-b_1]\\ \dfrac{z-(a_2-b_1)}{h'+k'} & \text{for } z\in[a_2-b_1,a_2-b_1+(h'+k')]\\ 1 & \text{otherwise}\end{cases}$$

D Scalar Multiplication

Let $k\in R$. If $\tilde{A}^I=[a_1,a_2,h,h;a_1,a_2,h',h']$ is a STIFN, then $\tilde{E}^I=k\tilde{A}^I$ is also a STIFN and is given by

$$E^I=k\tilde{A}^I=\begin{cases}\left[ka_1,ka_2,kh,kh;ka_1,ka_2,kh',kh'\right] & \text{if k>0}\\ \left[ka_2,ka_1,-kh,-kh;ka_2,ka_1,-kh',-kh'\right] & \text{if k<0}\end{cases}$$

Proof

Case(i): Let $k>0$.

With the transformation $z=kx$, the membership and nonmembership functions of IFS $\tilde{E}^I=k\tilde{A}^I$ can be established by (α,β)-cut method. α-cut for membership function of $\tilde{A}^I$ is $[(a_1-h)+\alpha h,(a_2+h)-\alpha h]$ for every $\alpha\in[0,1]$. That is, $x\in[(a_1-h)+\alpha h,(a_2+h)-\alpha h]$.

So, z (= kx) $\in[k(a_1-h)+\alpha kh,k(a_2+h)-\alpha kh]$.

Thus, the membership function of $\tilde{E}^I=k\tilde{A}^I$ is given by

$$\mu_{E^I}(z)=\begin{cases}\dfrac{z-k(a_1-h)}{kh} & \text{for } z\in[k(a_1-h),ka_1]\\ 1 & \text{for } z\in[ka_1,ka_2]\\ \dfrac{k(a_2+h)-z}{kh} & \text{for } z\in[ka_2,k(a_2+h)]\\ 0 & \text{otherwise}\end{cases}$$

For nonmembership function, β-cut of $\tilde{A}^I$ is $[a_1-\beta h',a_2+\beta h']$. That is, x $\in[a_1-\beta h',a_2+\beta h']$.

So, z (= kx) $\in[ka_1-\beta kh',ka_2+\beta kh']$.

Thus, the nonmembership function of $\tilde{E}^I=k\tilde{A}^I$ is shown as

$$\nu_{E^I}(z)=\begin{cases}\dfrac{ka_1-z}{kh'} & \text{for } z\in[k(a_1-h'),ka_1]\\ 0 & \text{for } z\in[ka_1,ka_2]\\ \dfrac{z-ka_2}{kh'} & \text{for } z\in[ka_2,k(a_2+h')]\\ 1 & \text{otherwise}\end{cases}$$

Thus, scalar multiplication can be given as $\tilde{E}^I=k\tilde{A}^I=[ka_1,ka_2,kh,kh;ka_1,ka_2,kh',kh']$ which is also a STIFN when $k>0$.

Case(ii): Let $k < 0$.

Similarly, it can be proved that $\tilde{E}^I = k\tilde{A}^I$ if $z = kx$, $k < 0$ where

$$\mu_{E^I}(z) = \begin{cases} \dfrac{z-k(a_2+h)}{kh} & \text{for } z \in [k(a_2+h), ka_2] \\ 1 & \text{for } z \in [ka_2, ka_1] \\ \dfrac{k(a_1-h)-z}{kh} & \text{for } z \in [ka_1, k(a_1-h)] \\ 0 & \text{otherwise} \end{cases}$$

and

$$\nu_{E^I}(z) = \begin{cases} \dfrac{ka_2 - z}{kh'} & \text{for } z \in [k(a_2+h'), ka_2] \\ 0 & \text{for } z \in [ka_2, ka_1] \\ \dfrac{z-ka_1}{kh'} & \text{for } z \in [ka_1, k(a_1-h')] \\ 1 & \text{otherwise} \end{cases}$$

Thus, scalar multiplication can be given as $\tilde{E}^I = k\tilde{A}^I = [ka_2, ka_1, -kh, -kh; ka_2, ka_1, -kh', -kh']$ which is also a STIFN when $k < 0$.

A Numerical Example

An example is presented to depict the above mentioned arithmetic operations.

Consider two STIFNs $\tilde{A}^I = [7,10,1,1;7,10,3,3]$ and $\tilde{B}^I = [4,6,2,2;4,6,5,5]$

(i) Addition

$$\tilde{A}^I + \tilde{B}^I =$$
$$[7+4,\ 10+6,\ 1+2,\ 1+2;\ 7+4,\ 10+6,\ 3+5,\ 3+5]$$
$$= [11,\ 16,\ 3,\ 3;\ 11,\ 16,\ 8,\ 8]$$

(ii) Additive Images

Additive image of $\tilde{A}^I = -\tilde{A}^I$

$$= [-10,\ -7,\ 1,\ 1;\ -10,\ -7,\ 3,\ 3]$$

Additive image of $\tilde{B}^I = -\tilde{B}^I$

$$= [-6,\ -4,\ 2,\ 2;\ -6,\ -4,\ 5,\ 5]$$

(iii) Subtraction

$$\tilde{A}^I - \tilde{B}^I =$$
$$[7-6, 10-4,\ 1+2,\ 1+2;\ 7-6,\ 10-4,\ 3+5,\ 3+5]$$
$$= [1,\ 6,\ 3,\ 3;\ 1,\ 6,\ 8,\ 8]$$

(iv) Scalar Multiplication

For $k = 2$, $k\tilde{A}^I =$

$$[2\times7,\ 2\times10,\ 2\times1,\ 2\times1;\ 2\times7,\ 2\times10,\ 2\times3,\ 2\times3]$$
$$= [14,\ 20,\ 2,\ 2;\ 14,\ 20,\ 6,\ 6]$$

For $k = -2$, $k\tilde{A}^I = [(-2)10,\ (-2)7,\ -(-2)1,\ -(-2)1;\ (-2)10,\ (-2)7,\ -(-2)3,\ -(-2)3]$

$$= [-20,\ -14,\ 2,\ 2;\ -20,\ -14,\ 6,\ 6].$$

V. CONCLUSION

On the foundation of the theory of intuitionistic fuzzy sets, this paper extends the traditional research. A more general definition of STIFN has been defined. Operational laws of STIFNs have been proposed keeping central thought that the same has to be used to solve a class of intuitionistic fuzzy optimization problems in which the data parameters are STIFNs on which the authors are working. Also, this approach can easily be extended to more experiment and industrial applications concerning decision making in an uncertain environment with STIFNs as parameters.

ACKNOWLEDGMENT

The author R.Parvathi thanks the University Grants Commission, New Delhi, India for its financial support to UGC Research Award No. F. 30 - 1/ 2009 (SA - II) dated 2nd July 2009.

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