

# Study of Transmission Characteristics of 2x2 MIMO System for OFDM Multiplexing and Bpsk Modulation with ZF Equalizer and MMSE Receivers

Bhagya. R, A. G. Ananth

**Abstract:** A Detailed Analysis Of 2x2 MIMO (Multiple Input Multiple Output) Combined With OFDM (Orthogonal Frequency Division Multiplexing) Transmission System And BPSK Modulation Has Been Carried Out. The BER Performance Of The System Has Been Determined For Additive White Gaussian Noise (AWGN) Presuming Flat Fading Rayleigh Channel. On The Receiver Side Linear Equalization Techniques Such As Zero Force Equalizer (ZF) And Minimum Mean Square Error (MMSE) Detectors Were Employed For Studying The BER Performance. The Simulation Results Show That For BER Of  $\sim 10^{-4}$ , The SNR Required Are  $\sim 34$  Db For ZF Equalizer And  $\sim 31$ db For MMSE Equalizer. The Simulation Results Indicate That The MMSE Equalizer Shows Better Performance  $\sim 3$  Db Over The ZF Equalizer. Further Comparison Of The 2X2 MIMO Performance Of OFDM With STBC Multiplexing Indicates Comparable BER Performances. The Simulation Results Are Presented And Discussed In The Paper.

**Key Words:** Multiple Input Multiple Output (MIMO), Orthogonal Frequency Division Multiplexing (OFDM), Space Time Block Coding (STBC), Binary Phase Shift Keying (BPSK), Bit Error Rate (BER), Signal To Noise Ratio (SNR), Zero Force Equalizer (ZFE) And Minimum Mean Square Error (MMSE).

## I. INTRODUCTION

The prime requirement of wireless communication systems is to provide high-data-rate wireless access and high quality of service (QoS), under the constraints of limited spectrum resource and hostile propagation conditions. This demands increase spectral efficiency and improvement in link reliability. OFDM is one of the important physical layer technologies for high data rate wireless communications due to its robustness to frequency selective fading, high spectral efficiency, and low computational complexity. This technique divides the frequency available into many closely spaced carriers which are individually modulated by low rate data streams. MIMO wireless systems use multiple antenna elements at transmit and receive to improve capacity over single antenna topologies in multipath channel characteristics play key role in determining communication performance. OFDM can be used in conjunction with a MIMO transceiver to increase the diversity gain and/or the

system capacity by exploiting spatial domain. Because the OFDM system effectively provides numerous parallel narrowband channels, MIMO-OFDM is considered a key technology in emerging high-data rate systems [2].

The combination MIMO-OFDM is beneficial since OFDM enables support of more antennas and larger bandwidths since it simplifies equalization dramatically in MIMO systems. By adopting MIMO and OFDM technologies, indoor wireless systems could reach data rates up to several hundreds of Mbits/s and achieve spectral efficiencies of several tens of bits/Hz/s, which are unattainable for conventional single-input single-output systems. The enhancements of data rate and spectral efficiency come from the fact that MIMO and OFDM schemes are indeed parallel transmission technologies in the space and frequency domains, respectively. When generated OFDM signal is transmitted through a number of antennas in order to achieve diversity or to gain higher transmission rate then it is known as MIMO-OFDM [6,8].

The present study involves a number of procedures namely simulation of the 2X2 MIMO transmission system, OFDM multiplexing, BPSK Digital modulation and Computation and comparison of BER for different SNR. Digital communication using MIMO has recently emerged as one of the most significant technical breakthroughs in modern communications and also known as volume to volume wireless link. The effect of fading and interference always causes an issue for signal recovery in wireless communication. This can be combated with application of an equalizer. Equalization compensates for Intersymbol Interference (ISI) created by multipath signal prorogation within time dispersive channels. The project will investigate the bit error rate performance characteristics of two types of equalizers namely, ZF and MMSE equalizers for MIMO-OFDM wireless receiver.

## II. MULTIPLE INPUT MULTIPLE OUTPUT (MIMO) WIRELESS COMMUNICATION SYSTEM

MIMO is an antenna technology for wireless communications in which multiple antennas are used at both the source (transmitter) and the destination (receiver). The antennas at each end of the communication circuit are

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combined to minimize errors and optimize data speed. MIMO is one of several forms of smart antenna technology. Wireless communication using MIMO systems enables increased spectral efficiency for a given total transmit power. Increased capacity is achieved by introducing additional spatial channels that are exploited by using space-time coding. The factors affecting MIMO system include channel complexity, external interference and channel estimation error. The ‘multichannel’ term indicates that the receiver incorporates multiple antennas by using space-time-frequency adaptive processing MIMO systems are a natural extension of developments in antenna array communication. While the advantages of multiple receive antennas such as gain and spatial diversity have been known and exploited for some time. The use of transmit diversity has only been investigated recently.

The advantages of MIMO communication, which exploits the physical channel between many transmit and receive antennas are currently receiving significant attention. While the channel can be so non-stationary that it cannot be estimated in any useful sense, MIMO systems provide a number of advantages over single-antenna-to-single-antenna communication. Sensitivity to fading is reduced by the spatial diversity provided by multiple spatial paths. Under certain environmental conditions, the power requirements associated with high spectral-efficiency communication can be significantly reduced by avoiding the compressive region of the information-theoretic capacity bound. Here, spectral efficiency is defined as the total number of information bits per second per Hertz transmitted from one array to the other [9, 2].

### III. ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

OFDM has grown to be the most popular communication system in high speed communication in the last decade. In fact, it has been said that OFDM technology is the future of wireless communications. OFDM was first proposed as a way of dealing with multipath. One of the problems with single carrier modulations (SCM) is that in a given environment, the symbol interval becomes much shorter than the delay spread as the symbol rate is increased. To solve this problem with multi-carrier modulation formats the symbol rate is instead decreased and the number of carriers is increased. The basic idea is that if a signal is sent over multiple low-rate carriers instead of a single high-rate carrier then inter-symbol interference (ISI) is eliminated and multipath effects can be compensated with a much simpler equalizer. As a modulation format, OFDM is very flexible in that it can be easily scaled to meet the needs of a particular application. OFDM is a Multi-Carrier Modulation technique in which a single high rate data-stream is divided into multiple low rate data-streams and is modulated using sub-carriers which are orthogonal to each other and can be thought of as a large number of low bit rate carriers transmitting in parallel. All these carriers transmitted using synchronized time and frequency, forming a single block of spectrum, to ensure that

the orthogonal nature of the structure is maintained. The OFDM signal includes many carrier signals with their own frequencies which are then fed into a guard time insertion circuit to reduce ISI. Since the duration of each symbol is long, it can be affordable to insert a guard interval between the OFDM symbols and thus the inter-symbols interference [ISI] can be eliminated [1, 4].

### IV. ZERO FORCING (ZF) EQUALIZER:

The zero-forcing equalizer removes all ISI, and is ideal when the channel is noiseless. However, when the channel is noisy, the zero forcing equalizer will amplify the noise greatly at frequencies  $f$  where the channel response  $H(j\omega)$  has a small magnitude (i.e. near zeroes of the channel) in the attempt to invert the channel completely. Zero-forcing equalizers ignore the additive noise and may significantly amplify noise for channels with spectral nulls [1,5].

#### Zero Forcing (ZF) Algorithm

The math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the second receive antenna is, Where

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

$y_1, y_2$  are the received symbol on the first and second antenna respectively,

$h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$x_1, x_2$  are the transmitted symbols and

$n_1, n_2$  is the noise on 1<sup>st</sup> and 2<sup>nd</sup> receive antennas.

We assume that the receiver knows,  $h_{1,1}$ ,  $h_{1,2}$ ,  $h_{2,1}$  and  $h_{2,2}$ .

The receiver also knows  $y_1$  and  $y_2$ . The unknowns are  $x_1$  and  $x_2$ . Two equations and two unknowns

For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Equivalently  $y=Hx+n$

To solve for  $x$ , we know that we need to find a matrix  $W$  which satisfies  $WH=I$ . The

Zero Forcing (ZF) linear detector for meeting this constraint is given by,

$$W = (H^H H)^{-1} H^H$$

Where  $W$  is the pseudo MATRIX

This matrix is also known as the pseudo inverse for a general  $m \times n$  matrix.

The term,

$$H^H H = \begin{bmatrix} h_{1,1}^* & h_{1,2}^* \\ h_{2,1}^* & h_{2,2}^* \end{bmatrix} \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix}$$

Note that the off diagonal terms in the matrix  $H^H H$  are not zero. Because the off diagonal terms are not zero, the zero forcing equalizer tries to null out the interfering terms when performing the equalization, i.e when solving for  $x_1$  the interference from  $x_2$  is tried to be nulled and vice versa. While doing so, there can be amplification of noise. Hence Zero Forcing equalizer is not the best possible equalizer to do the job. However, it is simple and reasonably easy to implement. For BPSK modulation in Rayleigh fading channel, the bit error rate is derived as,

$$P_b = \frac{1}{2} \left( 1 - \sqrt{\frac{(E_b/N_0)}{(E_b/N_0)+1}} \right)$$

Where  $P_b$  represents the error rate

### V. MINIMUM MEAN SQUARE EQUALIZER: (MMSE)

The Minimum Mean-Square Error Linear Equalizer (MMSE-LE) balances a reduction in ISI with noise enhancement. The MMSE-LE always performs better than the ZFE and is of the same complexity of implementation. Nevertheless, it is slightly more complicated to describe and analyze than is the ZFE. The MMSE-LE uses a linear time-invariant filter  $w_k$  for  $R$ , but the choice of filter impulse response  $w_k$  is different than the ZFE. The MMSE-LE is a linear filter  $w_k$  that acts on  $y_k$  (output signal) to form an output sequence  $z_k$  that is the best MMSE estimate of  $x_k$  (input signal). That is, the filter  $w_k$  minimizes the Mean Square Error (MSE). The MSE criteria for filter design does not ignore noise enhancement because the optimization of this filter compromises between eliminating ISI and increasing noise power. Instead, the filter output is as close as possible, in the Minimum MSE sense, to the data symbol  $x_k$ .

MMSE equalization aims to minimize the average mean square error (MSE) between the transmitted symbol  $d_k$  and its estimate  $\hat{d}_{kat}$  the output of the equalizer. In other words, the  $\{w_i\}$ 's (the linear time variant filter output) are chosen to minimize  $E[d_k - \hat{d}_k]^2$ . Since the MMSE is a linear equalizer, its output  $\hat{d}_k$  is a linear combination of the input samples  $y[k]$ . In this way, for linear estimation, obtaining the optimal filter coefficients  $\{w_i\}$  becomes a standard problem. This is known as a standard Weiner filtering problem if the noise input to the equalizer is white. However, because of the matched filter  $g^*m(-t)$  at the receiver front end, the noise input to the equalizer is not white but coloured with power spectrum  $N_0/G^*m(1/z^*)/2$ . Against this backdrop, to apply known techniques for optimal linear estimation, the filter  $Heq(z)$  was expanded into two components, a noise whitening component  $1/G^*m(1/z_-)$  and an ISI removal component  $\hat{H}eq(z)$ . As it can be anticipated by the name itself, the noise whitening filter whitens the noise such that the noise component output from this filter has a constant power spectrum. Since the noise input to this receiver has power spectrum  $N_0/G^*m(1/z^*)/2$ , the appropriate noise whitening filter is  $1/G^*m(1/z^*)$ . The noise power spectrum at

the output of the noise whitening filter is then  $N_0/G^*m(1/z^*)/2/G^*m(1/z^*)/2 = N_0$ . Note that the filter  $1/G^*m(1/z^*)$  is not the only filter that will whiten the noise, and another noise whitening filter with more desirable properties (like stability) may be chosen. It might seem odd at first to introduce the matched filter  $g^*m(-t)$  at the receiver front end only to cancel its effect in the equalizer. However, the matched filter is meant to maximize the SNR prior to sampling. The math for extracting the two symbols which interfered with each other [3]. In the first timeslot, the received signal on the first receive antenna is,

$$y_1 = h_{1,1}x_1 + h_{1,2}x_2 + n_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$

The received signal on the second receive antenna is,

$$y_2 = h_{2,1}x_1 + h_{2,2}x_2 + n_2 = \begin{bmatrix} h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_2$$

Where

$y_1, y_2$  are the received symbol on the first and second antenna respectively,

$h_{1,1}$  is the channel from 1<sup>st</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{1,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 1<sup>st</sup> receive antenna,

$h_{2,1}$  is the channel from 1<sup>st</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$h_{2,2}$  is the channel from 2<sup>nd</sup> transmit antenna to 2<sup>nd</sup> receive antenna,

$x_1, x_2$  are the transmitted symbols and

$n_1, n_2$  is the noise on 1<sup>st</sup> and 2<sup>nd</sup> receive antennas.

We assume that the receiver knows  $h_{1,1}$ ,  $h_{1,2}$ ,  $h_{2,1}$ , and  $h_{2,2}$ . The receiver also knows  $y_1$  and  $y_2$ . For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

Equivalently,

$$y = Hx + n$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient  $W$  which minimizes the criterion,

$$E\{[Wy - x][Wy - x]^H\}$$

Solving,

$$W = [H^H H + N_0 I]^{-1} H^H$$

When comparing to the equation in Zero Forcing equalizer, apart from the  $N_0 I$  term both the equations are comparable. In fact, when the noise term is zero, the MMSE equalizer reduces to Zero Forcing equalizer.

### V. RESULTS AND DISCUSSION

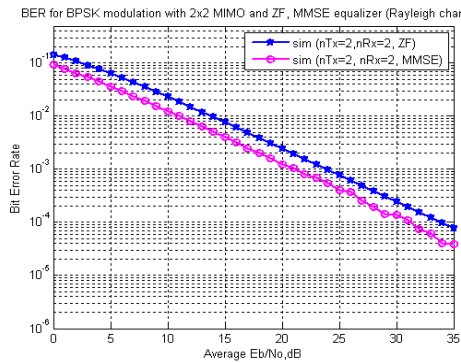
The simulation results for the performance for 2X2 MIMO-OFDM using BPSK digital modulation scheme has





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been determined with Zero Forcing (ZF) Equalizer and MMSE equalizer for AWGN channels using MATLAB. The BER values as a function of varying SNR has been determined for the MIMO-OFDM system for each of the two detector receiver configurations. The Figure 1 shows the BER performance as a function of SNR for the two detector configurations. The BER values of both the detectors are shown in the same figure for comparing their relative performances.



**Figure 1: BER plot for 2x2 MIMO with ZF receiver (Top) and MMSE (Below) for BPSK modulation in Rayleigh channel.**

The simulation results shown on Figure 1 demonstrate that for BER values sharply decreases as the SNR increases. For the BER  $\sim 10^{-4}$  the ZF equalizer shows an SNR  $\sim 34$  dB and for the same BER values the MMSE equalizer shows that SNR  $\sim 31$  dB. It is clearly seen that for the same BER values between the ZF and MMSE detectors, the 2x2 MIMO-OFDM transmission shows a significant improvement in SNR  $\sim 3$  dB. The simulation results indicate that the performance of MMSE receiver exhibits an improvement in SNR  $\sim 3$  dB compared to the ZF receiver. This is especially true at low spectral efficiencies, where the MMSE equalizer achieve full spatial diversity. A comparison of the performances of MIMO - OFDM system with that of MIMO-STBC system [7] with BPSK modulation shows that the BER performance for both the systems are exactly similar and the SNR values are same for the two types of receivers irrespective of the different multiplexing schemes employed for the MIMO transmission system.

## VI. CONCLUSIONS:

It can be concluded from the results presented,

1. For 2x2 MIMO system with OFDM multiplexing techniques and BPSK modulation, the MMSE detectors shows better BER performances for digital transmission.
2. For BER values of  $10^{-4}$ , the SNR for MMSE detectors  $\sim 31$  dB and ZF detector  $\sim 34$  dB suggesting that the MMSE receiver shows a better performance compared to ZF equalizer receivers.
3. The MMSE receiver system shows  $\sim 3$  dB improvement in the SNR compared to the ZF receiver system for achieving same BER values.

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