

Some New Operations of Intuitionistic Fuzzy Soft Sets

Manoj Bora, Tridiv Jyoti Neog, Dusmanta Kumar Sut

Abstract—In this paper, we have defined disjunctive sum and difference of two intuitionistic fuzzy soft sets and study their basic properties. The notions of (α, β) -cut soft set and (α, β) -cut strong soft set of an intuitionistic fuzzy soft set have been put forward in our work. Some related properties have been established with proof, examples and counter examples.

Index Terms— Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Soft Set, Disjunctive Sum, Difference, (α, β) -cut soft set, (α, β) -cut strong soft set..

I. INTRODUCTION

Most of the real life problems have various uncertainties. The Theory of Probability, Evidence Theory, Fuzzy Set Theory, Intuitionistic Fuzzy Set Theory, Rough Set Theory etc. are mathematical tools to deal with such problems. In 1999, Molodtsov [2] introduced the Theory of Soft Set and established the fundamental results related to this theory. In comparison, this theory can be seen free from the inadequacy of parameterization tool. It is a general mathematical tool for dealing with problems in the fields of social science, economics, medical sciences etc. In 2003, Maji, Biswas and Roy [5] studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In recent times, researchers have contributed a lot towards fuzzification of Soft Set Theory. Combining fuzzy sets with soft sets, Maji et al. [4] introduced the notion of fuzzy soft sets. They studied some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. These results were further revised and improved by Ahmad and Kharal [1]. They defined arbitrary fuzzy soft union and intersection and proved De Morgan Inclusions and De Morgan Laws in Fuzzy Soft Set Theory. Moreover Maji et al.[6] extended soft sets to intuitionistic fuzzy soft sets. Intuitionistic fuzzy soft set theory is a combination of soft sets and intuitionistic fuzzy sets initiated by Atanassov [3]. In [7] Neog and Sut have defined disjunctive sum and difference of two fuzzy soft sets. The notions of α -cut soft set and α -cut strong soft set of a fuzzy soft set have been put forward in their work. In this paper we have defined disjunctive sum and difference of two intuitionistic fuzzy soft sets. Further the notions of (α, β) -cut soft sets and (α, β) -cut strong soft set of an

intuitionistic fuzzy soft set have been put forward in our work. Some related properties have been established in our work with supporting proof, examples and counter examples.

II. PRELIMINARIES

A. Definition [2]

A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U .

In other words, the soft set is a parameterized family of subsets of the set U . Every set $F(\varepsilon), \varepsilon \in E$, from this family may be considered as the set of ε -elements of the soft set (F, E) , or as the set of ε -approximate elements of the soft set.

B. Definition [3]

An intuitionistic fuzzy set A over the universe U can be defined as follows –

$A = \{(x, \mu_A(x), \nu_A(x)) : x \in U\}$, where $\mu_A(x) : U \rightarrow [0,1], \nu_A(x) : U \rightarrow [0,1]$ with the property $0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad \forall x \in U$. The values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and non-membership of x to A respectively. $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$ is called the intuitionistic fuzzy index.

C. Definition [6]

Let U be an initial universe set and E be the set of parameters. Let IF^U denote the collection of all intuitionistic fuzzy subsets of U . Let $A \subseteq E$. A pair (F, A) is called an intuitionistic fuzzy soft set over U where F is a mapping given by $F : A \rightarrow IF^U$.

D. Definition [6]

A soft set (F, A) over U is said to be null intuitionistic fuzzy soft set denoted by ϕ if $\forall \varepsilon \in A, F(\varepsilon)$ is the null intuitionistic fuzzy set $\bar{0}$ of U where $\bar{0}(x) = 0 \quad \forall x \in U$.

We would use the notation (ϕ, A) to represent the intuitionistic fuzzy soft null set with respect to the set of parameters A .

E. Definition [6]

A soft set (F, A) over U is said to be absolute intuitionistic fuzzy soft set denoted by \tilde{A} if $\forall \varepsilon \in A, F(\varepsilon)$ is the absolute intuitionistic fuzzy set $\bar{1}$ of U where $\bar{1}(x) = 1 \quad \forall x \in U$.

We would use the notation $(\bar{1}, A)$ to represent the intuitionistic fuzzy soft absolute set with respect to the set of parameters A .

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Manoj Bora, Department of Mathematics, Jorhat Institute of Science and Technology, Jorhat, India.

Tridiv Jyoti Neog, Department of Mathematics, C.M.J. University, Shillong, India.

Dusmanta Kumar Sut, Department of Mathematics, N. N. Saikia College, Titabor, India.

F. Definition [6]

For two intuitionistic fuzzy soft sets (F, A) and (G, B) over (U, E) , we say that (F, A) is an intuitionistic fuzzy soft subset of (G, B) , if

- (i) $A \subseteq B$,
- (ii) For all $\varepsilon \in A$, $F(\varepsilon) \subseteq G(\varepsilon)$ and is written as $(F, A) \subseteq (G, B)$.

G. Definition [6]

Union of two intuitionistic fuzzy soft sets (F, A) and (G, B) over (U, E) is an intuitionistic fuzzy soft set (H, C) where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

and is written as $(F, A) \cup (G, B) = (H, C)$.

H. Definition [6]

Let (F, A) and (G, B) be two intuitionistic fuzzy soft sets over (U, E) . Then intersection (F, A) and (G, B) is an intuitionistic fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$.

We write $(F, A) \cap (G, B) = (H, C)$.

I. Definition [6]

Let (F, A) and (G, B) be two fuzzy soft sets in a soft class (U, E) with $A \cap B \neq \emptyset$. Then intersection of two fuzzy soft sets (F, A) and (G, B) in a soft class (U, E) is a fuzzy soft set (H, C) where $C = A \cap B$ and $\forall \varepsilon \in C$, $H(\varepsilon) = F(\varepsilon) \cap G(\varepsilon)$. We write $(F, A) \cap (G, B) = (H, C)$.

J. Definition [6]

The complement of an intuitionistic fuzzy soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c: A \rightarrow IF^U$ is a mapping given by $F^c(\varepsilon) = [F(\varepsilon)]^c$ for all $\varepsilon \in A$. Thus if $F(\varepsilon) = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}$, then $\forall \varepsilon \in A$, $F^c(\varepsilon) = (F(\varepsilon))^c = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) : x \in U\}$

K. Definition [6]

If (F, A) and (G, B) be two intuitionistic fuzzy soft sets, then “ (F, A) AND (G, B) ” is an intuitionistic fuzzy soft set denoted by $(F, A) \wedge (G, B)$ and is defined by $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where \cap is the operation intersection of two intuitionistic fuzzy sets.

L. Definition [6]

If (F, A) and (G, B) be two intuitionistic fuzzy soft sets, then “ (F, A) OR (G, B) ” is an intuitionistic fuzzy soft set denoted by $(F, A) \vee (G, B)$ and is defined by $(F, A) \vee (G, B) = (K, A \times B)$, where $K(\alpha, \beta) = F(\alpha) \cup G(\beta), \forall \alpha \in A$ and $\forall \beta \in B$, where \cup is the operation union of two intuitionistic fuzzy sets.

III. SOME NEW OPERATIONS OF INTUITIONISTIC FUZZY SOFT SETS

A. Definition (Disjunctive Sum of Intuitionistic Fuzzy Soft Sets)

Let (F, A) and (G, B) be two intuitionistic fuzzy soft sets over (U, E) . We define the disjunctive sum of (F, A) and (G, B) as the intuitionistic fuzzy soft set (H, C) over (U, E) , written as $(F, A) \oplus (G, B) = (H, C)$, where $C = A \cap B \neq \emptyset$ and $\forall \varepsilon \in C, x \in U$,

$$\begin{aligned} \mu_{H(\varepsilon)}(x) &= \max(\min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))) \\ \nu_{H(\varepsilon)}(x) &= \min(\max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))) \end{aligned}$$

B. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$

$$(F, A) = \{F(e_1) = \{(a, 0.5, 0.1), (b, 0.1, 0.8), (c, 0.2, 0.5)\},$$

$$F(e_2) = \{(a, 0.7, 0.1), (b, 0.0, 0.8), (c, 0.3, 0.5)\},$$

$$F(e_4) = \{(a, 0.6, 0.3), (b, 0.1, 0.7), (c, 0.9, 0.01)\}$$

$$(G, B) = \{G(e_1) = \{(a, 0.2, 0.6), (b, 0.7, 0.1), (c, 0.8, 0.1)\},$$

$$G(e_2) = \{(a, 0.4, 0.1), (b, 0.5, 0.3), (c, 0.4, 0.5)\},$$

$$G(e_3) = \{(a, 0.0, 0.6), (b, 0.0, 0.8), (c, 0.1, 0.5)\}$$

Then $(F, A) \oplus (G, B) = (H, C)$ where $C = A \cap B = \{e_1, e_2\}$ and

$$(H, C) = \{H(e_1) = \{(a, \max(\min(0.5, 0.6), \min(0.1, 0.2)),$$

$$\min(\max(0.1, 0.2), \max(0.5, 0.6)))$$

$$(b, \max(\min(0.1, 0.7), \min(0.8, 0.1)),$$

$$\min(\max(0.8, 0.7), \max(0.1, 0.1)))$$

$$(c, \max(\min(0.2, 0.1), \min(0.5, 0.8)),$$

$$\min(\max(0.5, 0.8), \max(0.2, 0.1)))\},$$

$$H(e_2) = \{(a, \max(\min(0.7, 0.1), \min(0.1, 0.4)),$$

$$\min(\max(0.1, 0.4), \max(0.7, 0.1)))$$

$$(b, \max(\min(0, 0.3), \min(0.8, 0.5)),$$

$$\min(\max(0.8, 0.5), \max(0, 0.3)))$$

$$(c, \max(\min(0.3, 0.5), \min(0.5, 0.4)),$$

$$\min(\max(0.5, 0.4), \max(0.3, 0.5)))\}$$

$$(H, C) = \{H(e_1) = \{(a, \max(0.5, 0.1), \min(0.2, 0.6)),$$

$$(b, \max(0.1, 0.1), \min(0.8, 0.1)),$$

$$(c, \max(0.1, 0.5), \min(0.8, 0.2))\},$$

$$H(e_2) = \{(a, \max(0.1, 0.1), \min(0.4, 0.7))$$

$$(b, \max(0, 0.5), \min(0.8, 0.3))$$

$$(c, \max(0.3, 0.4), \min(0.5, 0.5))\}$$

$$(H, C) = \{H(e_1) = \{(a, 0.5, 0.2), (b, 0.1, 0.1), (c, 0.5, 0.2)\},$$

$$H(e_2) = \{(a, 0.1, 0.4), (b, 0.5, 0.3), (c, 0.4, 0.5)\}$$

B. C. Proposition

Let (F, A) and (G, B) be two intuitionistic fuzzy soft sets over (U, E) . Then the following results hold.

$$(i) (F, A) \oplus (G, B) = (G, B) \oplus (F, A)$$

$$(ii) (F, A) \oplus ((G, B) \oplus (H, C)) = ((F, A) \oplus (G, B)) \oplus (H, C)$$

Proof

$$(i) (F, A) = \{x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)\}, \forall x \in U, \forall \varepsilon \in A$$

$$(G, B) = \{ \langle x, \mu_{G(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in B \} = \mu_{F(\varepsilon)}(x)$$

Let $(F, A) \tilde{\ominus} (G, B) = (H, C)$ where $C = A \cap B$ and $\forall \varepsilon \in C, x \in U$,

$$\mu_{H(\varepsilon)}(x) = \max(\min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)))$$

$$\nu_{H(\varepsilon)}(x) = \min(\max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), \max(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)))$$

Let $(G, B) \tilde{\ominus} (F, A) = (I, D)$ where $D = A \cap B$ and $\forall \varepsilon \in D, x \in U$,

$$\mu_{I(\varepsilon)}(x) = \max(\min(\mu_{G(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)), \min(\nu_{G(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)))$$

$$\nu_{I(\varepsilon)}(x) = \min(\max(\nu_{G(\varepsilon)}(x), \mu_{F(\varepsilon)}(x)), \max(\mu_{G(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)))$$

It follows that $(H, C) = (I, D)$

Therefore $(F, A) \tilde{\ominus} (G, B) = (G, B) \tilde{\ominus} (F, A)$

Proof of (ii) can be done in a similar way.

C. D. Proposition

$$(i) (F, A) \tilde{\ominus} (\varphi, A) = (F, A)$$

$$(ii) (F, A) \tilde{\ominus} (U, A) = (F, A)^c$$

Proof

$$(i) \text{ Let } (F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(\varphi, A) = \{ \langle 0, 1 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let $(F, A) \tilde{\ominus} (\varphi, A) = (H, A)$, where $\forall \varepsilon \in A, x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \max(\min(\mu_{F(\varepsilon)}(x), \nu_{\varphi(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \mu_{\varphi(\varepsilon)}(x)))$$

$$= \max(\min(\mu_{F(\varepsilon)}(x), 1), \min(\nu_{F(\varepsilon)}(x), 0))$$

$$= \max(\mu_{F(\varepsilon)}(x), 0)$$

$$= \mu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \min(\max(\nu_{F(\varepsilon)}(x), \mu_{\varphi(\varepsilon)}(x)), \max(\mu_{F(\varepsilon)}(x), \nu_{\varphi(\varepsilon)}(x)))$$

$$= \min(\max(\nu_{F(\varepsilon)}(x), 0), \max(\mu_{F(\varepsilon)}(x), 1))$$

$$= \min(\nu_{F(\varepsilon)}(x), 1)$$

$$= \nu_{F(\varepsilon)}(x)$$

Therefore $(H, A) = \{ \langle \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall \varepsilon \in C = A, x \in U \}$

It follows that $(F, A) \tilde{\ominus} (\varphi, A) = (F, A)$

$$(ii) \text{ Let } (F, A) = \{ \langle x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x) \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

$$(U, A) = \{ \langle 1, 0 \rangle, \forall x \in U, \forall \varepsilon \in A \}$$

Let $(F, A) \tilde{\ominus} (U, A) = (H, A)$, where $\forall \varepsilon \in A, x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \max(\min(\mu_{F(\varepsilon)}(x), \nu_{U(\varepsilon)}(x)), \min(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x)))$$

$$= \max(\min(\mu_{F(\varepsilon)}(x), 0), \min(\nu_{F(\varepsilon)}(x), 1))$$

$$= \max(0, \nu_{F(\varepsilon)}(x))$$

$$= \nu_{F(\varepsilon)}(x)$$

$$\nu_{H(\varepsilon)}(x) = \min(\max(\nu_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x)), \max(\mu_{F(\varepsilon)}(x), \nu_{U(\varepsilon)}(x)))$$

$$= \min(\max(\nu_{F(\varepsilon)}(x), 1), \max(\mu_{F(\varepsilon)}(x), 0))$$

$$= \min(1, \mu_{F(\varepsilon)}(x))$$

It follows that

$$(H, A) = \{ \langle \nu_{F(\varepsilon)}(x), \mu_{F(\varepsilon)}(x) \rangle, \forall \varepsilon \in A, x \in U \}$$

$$= (F, A)^c$$

Consequently $(F, A) \tilde{\ominus} (U, A) = (F, A)^c$

D. Definition (Difference of Intuitionistic Fuzzy Soft Sets)

Let (F, A) and (G, B) be two intuitionistic fuzzy soft sets over (U, E) . We define the difference of (F, A) and (G, B) as the intuitionistic fuzzy soft set (H, C) over (U, E) , written as $(F, A) \tilde{\ominus} (G, B) = (H, C)$, where $C = A \cap B \neq \varphi$ and $\forall \varepsilon \in C, x \in U$,

$$\mu_{H(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$$

$$\nu_{H(\varepsilon)}(x) = \max(\nu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x))$$

E. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2, e_4\} \subseteq E$, $B = \{e_1, e_2, e_3\} \subseteq E$

$$(F, A) = \{ F(e_1) = \{ \langle a, 0.5, 0.1 \rangle, \langle b, 0.1, 0.8 \rangle, \langle c, 0.2, 0.5 \rangle \},$$

$$F(e_2) = \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0, 0.8 \rangle, \langle c, 0.3, 0.5 \rangle \},$$

$$F(e_4) = \{ \langle a, 0.6, 0.3 \rangle, \langle b, 0.1, 0.7 \rangle, \langle c, 0.9, 0.01 \rangle \}$$

$$(G, B) = \{ G(e_1) = \{ \langle a, 0.2, 0.6 \rangle, \langle b, 0.7, 0.1 \rangle, \langle c, 0.8, 0.1 \rangle \},$$

$$G(e_2) = \{ \langle a, 0.4, 0.1 \rangle, \langle b, 0.5, 0.3 \rangle, \langle c, 0.4, 0.5 \rangle \},$$

$$G(e_3) = \{ \langle a, 0.0, 0.6 \rangle, \langle b, 0, 0.8 \rangle, \langle c, 0.1, 0.5 \rangle \}$$

Let $(F, A) \tilde{\ominus} (G, B) = (H, C)$, where $C = A \cap B = \{e_1, e_2\}$. Then

$$(H, C) = \{ H(e_1) = \{ \langle a, \min(0.5, 0.6), \max(0.1, 0.2) \rangle, \langle b, \min(0.1, 0.1), \max(0.8, 0.7) \rangle, \langle c, \min(0.2, 0.1), \max(0.2, 0.8) \rangle \},$$

$$H(e_2) = \{ \langle a, \min(0.7, 0.4), \max(0.1, 0.4) \rangle, \langle b, \min(0, 0.3), \max(0.8, 0.5) \rangle, \langle c, \min(0.3, 0.5), \max(0.5, 0.4) \rangle \}$$

$$= \{ H(e_1) = \{ \langle a, 0.5, 0.2 \rangle, \langle b, 0.1, 0.8 \rangle, \langle c, 0.1, 0.8 \rangle \},$$

$$H(e_2) = \{ \langle a, 0.4, 0.4 \rangle, \langle b, 0, 0.8 \rangle, \langle c, 0.3, 0.5 \rangle \}$$

F. Proposition

$$(i) (F, A) \tilde{\ominus} (\varphi, A) = (F, A)$$

$$(ii) (F, A) \tilde{\ominus} (U, A) = (\varphi, A)$$

Proof

(i) Let $(F, A) \tilde{\ominus} (\varphi, A) = (H, A)$, where $\forall \varepsilon \in A, x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), \nu_{\varphi(\varepsilon)}(x))$$

$$= \min(\mu_{F(\varepsilon)}(x), \nu_{\varphi(\varepsilon)}(x))$$



$$= \min(\mu_{F(\varepsilon)}(x), 1)$$

$$= \mu_{F(\varepsilon)}(x)$$

$$v_{H(\varepsilon)}(x) = \max(v_{F(\varepsilon)}(x), \mu_{\varphi(\varepsilon)}(x))$$

$$= \max(v_{F(\varepsilon)}(x), 0)$$

$$= v_{F(\varepsilon)}(x)$$

Therefore $(H, A) = (\mu_{F(\varepsilon)}(x), v_{F(\varepsilon)}(x)), \forall \varepsilon \in A, x \in U$

It follows that $(F, A)\tilde{\Theta}(\varphi, A) = (F, A)$

(ii) Let $(F, A)\tilde{\Theta}(U, A) = (H, A)$, where $\forall \varepsilon \in A, x \in U$, we have

$$\mu_{H(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), v_{U(\varepsilon)}(x))$$

$$= \min(\mu_{F(\varepsilon)}(x), 0)$$

$$= 0$$

$$v_{H(\varepsilon)}(x) = \max(v_{F(\varepsilon)}(x), \mu_{U(\varepsilon)}(x))$$

$$= \max(v_{F(\varepsilon)}(x), 1)$$

$$= 1$$

Therefore $(H, A) = \{(0, 1), \forall \varepsilon \in A, x \in U\}$

It follows that $(F, A)\tilde{\Theta}(U, A) = (\varphi, A)$

G. Definition (α, β) - Cut Soft Set of an Intuitionistic Fuzzy Soft Set

Let (F, A) be an intuitionistic fuzzy soft set over (U, E) . We define the (α, β) - cut soft set of the intuitionistic fuzzy soft set (F, A) , denoted by $(F, A)_{(\alpha, \beta)}$ as the soft set $(F_{(\alpha, \beta)}, A)$, where $\forall \varepsilon \in A$,

$$F_{(\alpha, \beta)}(\varepsilon) = \{x : \mu_{F(\varepsilon)}(x) \geq \alpha, v_{F(\varepsilon)}(x) \leq \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$$

H. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_2, e_3, e_4\} \subseteq E$.

Let us consider an intuitionistic fuzzy soft set (F, A) as

$$(F, A) = \{F(e_2) = \{(a, 0.3, 0.2), (b, 0.1, 0.8), (c, 0.4, 0.5)\},$$

$$F(e_3) = \{(a, 0.7, 0.2), (b, 0.4, 0.3), (c, 0.5, 0.1)\},$$

$$F(e_4) = \{(a, 0.6, 0.2), (b, 0.3, 0.5), (c, 0.3, 0.6)\}\}$$

Let $\alpha = 0.3, \beta = 0.5, \alpha, \beta \in [0, 1]$. Then

$$(F, A)_{(0.3, 0.5)} = (F_{(0.3, 0.5)}, A)$$

$$= \{F_{(0.3, 0.5)}(e_2) = \{a, c\}, F_{(0.3, 0.5)}(e_3) = \{a, b, c\},$$

$$F_{(0.3, 0.5)}(e_4) = \{a, b\}\}$$

I. Definition (α, β) - Cut Strong Soft Set of an Intuitionistic Fuzzy Soft Set

Let (F, A) be an intuitionistic fuzzy soft set over (U, E) . We define the (α, β) - cut strong soft set $(F_{(\alpha, \beta)^+}, A)$ of the intuitionistic fuzzy soft set (F, A) , denoted by $(F, A)_{(\alpha, \beta)^+}$ as the soft set, where $\forall \varepsilon \in A$,

$$F_{(\alpha, \beta)^+}(\varepsilon)$$

$$= \{x : \mu_{F(\varepsilon)}(x) > \alpha, v_{F(\varepsilon)}(x) < \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$$

J. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_2, e_3, e_4\} \subseteq E$. Let us consider an intuitionistic fuzzy soft set (F, A) as

$$(F, A) = \{F(e_2) = \{(a, 0.3, 0.2), (b, 0.1, 0.8), (c, 0.4, 0.5)\},$$

$$F(e_3) = \{(a, 0.7, 0.2), (b, 0.4, 0.3), (c, 0.5, 0.1)\},$$

$$F(e_4) = \{(a, 0.6, 0.2), (b, 0.3, 0.5), (c, 0.3, 0.6)\}\}$$

Let $\alpha = 0.3, \beta = 0.5, \alpha, \beta \in [0, 1]$. Then

$$(F, A)_{(0.3, 0.5)^+} = (F_{(0.3, 0.5)^+}, A)$$

$$= \{F_{(0.3, 0.5)^+}(e_2) = \{\},$$

$$F_{(0.3, 0.5)^+}(e_3) = \{a, b, c\}, F_{(0.3, 0.5)^+}(e_4) = \{a\}\}$$

K. Proposition

Let $(F, A), (G, B)$ be two intuitionistic fuzzy soft sets over (U, E) . Then the following results hold for all $\alpha, \beta \in [0, 1]$.

(i) $(F, A) \subseteq (G, B)$

$$\Rightarrow (F, A)_{(\alpha, \beta)} \subseteq (G, B)_{(\alpha, \beta)}, (F, A)_{(\alpha, \beta)^+} \subseteq (G, B)_{(\alpha, \beta)^+}$$

(ii) $(F, A) \tilde{\cap} (G, B)$

$$((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)} = (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)},$$

$$((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)^+} = (F, A)_{(\alpha, \beta)^+} \tilde{\cap} (G, B)_{(\alpha, \beta)^+}$$

(iii) $(F, A) \tilde{\cap} (G, B)$

$$((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)} = (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)},$$

$$((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)^+} = (F, A)_{(\alpha, \beta)^+} \tilde{\cap} (G, B)_{(\alpha, \beta)^+}$$

(iv) $(F, A)^c_{(\alpha, \beta)} = (F, A)^c_{(\beta, \alpha)^+}$

Proof

(i) Let $(F, A) \subseteq (G, B)$. Then $A \subseteq B$ and

$$\forall \varepsilon \in A, x \in U, \mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x), v_{F(\varepsilon)}(x) \geq v_{G(\varepsilon)}(x)$$

We assume that there are $\alpha_0, \beta_0 \in [0, 1]$ such that

$$(F, A)_{(\alpha_0, \beta_0)} \not\subseteq (G, B)_{(\alpha_0, \beta_0)}.$$

$$\text{Now } (F, A)_{(\alpha_0, \beta_0)} = (F_{(\alpha_0, \beta_0)}, A) = \{F_{(\alpha_0, \beta_0)}(\varepsilon) : \varepsilon \in A\}$$

Then there exists $x_0 \in F_{(\alpha_0, \beta_0)}(\varepsilon), x_0 \in U$ such that

$$x_0 \notin G_{(\alpha_0, \beta_0)}(\varepsilon) \text{ for at least one } \varepsilon \in A.$$

i.e. $\mu_{F(\varepsilon)}(x_0) \geq \alpha_0, v_{F(\varepsilon)}(x_0) \leq \beta_0$ and

$$\mu_{G(\varepsilon)}(x_0) < \alpha_0, v_{G(\varepsilon)}(x_0) > \beta_0. \text{ This is a contradiction,}$$

since $\forall \varepsilon \in A, x \in U, \mu_{F(\varepsilon)}(x) \leq \mu_{G(\varepsilon)}(x), v_{F(\varepsilon)}(x) \geq v_{G(\varepsilon)}(x)$

Thus for all $\alpha, \beta \in [0, 1]$ and $\forall \varepsilon \in A, F_{(\alpha, \beta)}(\varepsilon) \subseteq G_{(\alpha, \beta)}(\varepsilon)$.

It follows that $(F, A)_{(\alpha, \beta)} \subseteq (G, B)_{(\alpha, \beta)}$.

The reverse inclusion here is not valid which is clear from the following example –

L. Example

Let $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$, $A = \{e_1, e_2\} \subseteq E$,

$$B = \{e_1, e_2, e_4\} \subseteq E$$

$$(F, A) = \{F(e_1) = \{(a, 0.1, 0.8), (b, 0.2, 0.7), (c, 0.4, 0.5)\},$$

$$F(e_2) = \{(a, 0.6, 0.4), (b, 0.1, 0.5), (c, 0.5, 0.3)\}\}$$

$$(G, B) = \{G(e_1) = \{(a, 0.6, 0.3), (b, 0.7, 0.1), (c, 0.8, 0.2)\},$$

$$G(e_2) = \{(a, 0.3, 0.5), (b, 0.1, 0.7), (c, 0.4, 0.4)\},$$

$$G(e_4) = \{(a, 0, 0.4), (b, 0.2, 0.7), (c, 0.6, 0.2)\}\}$$

Here

$$\begin{aligned} (F, A)_{(0.3, 0.6)} &= (F_{(0.3, 0.6)}, A) \\ &= \{F_{(0.3, 0.6)}(e_1) = \{b, c\}, F_{(0.3, 0.6)}(e_2) = \{a, c\}\} \\ (G, B)_{(0.3, 0.6)} &= (G_{(0.3, 0.6)}, B) \\ &= \{G_{(0.3, 0.6)}(e_1) = \{a, b, c\}, \\ &\quad G_{(0.3, 0.6)}(e_2) = \{a, c\}, G_{(0.3, 0.6)}(e_4) = \{c\}\} \end{aligned}$$

It is clear that $(F, A)_{(0.3, 0.6)} \tilde{\subseteq} (G, B)_{(0.3, 0.6)}$ but $(F, A) \tilde{\not\subseteq} (G, B)$ as

$$\mu_{F(e_2)}(a) = 0.6 \text{ and } \mu_{G(e_2)}(a) = 0.3, \nu_{F(e_2)}(a) = 0.3 \text{ and } \nu_{G(e_2)}(a) = 0.4$$

Thus $\mu_{F(e_2)}(a) > \mu_{G(e_2)}(a), \nu_{F(e_2)}(a) < \nu_{G(e_2)}(a)$

$$\mu_{F(e_2)}(c) = 0.5 \text{ and } \mu_{G(e_2)}(c) = 0.4, \nu_{F(e_2)}(c) = 0.3 \text{ and } \nu_{G(e_2)}(c) = 0.4$$

$$\mu_{F(e_2)}(c) > \mu_{G(e_2)}(c) \nu_{F(e_2)}(c) < \nu_{G(e_2)}(c)$$

(ii) Let $(F, A) \tilde{\cap} (G, B) = (H, C)$. Then $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H(\varepsilon) = \begin{cases} F(\varepsilon), & \text{if } \varepsilon \in A - B \\ G(\varepsilon), & \text{if } \varepsilon \in B - A \\ F(\varepsilon) \cup G(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

i.e.

$$\mu_{H(\varepsilon)}(x) = \begin{cases} \mu_{F(\varepsilon)}(x), & \text{if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x), & \text{if } \varepsilon \in B - A \\ \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)), & \text{if } \varepsilon \in A \cap B \end{cases},$$

$$\nu_{H(\varepsilon)}(x) = \begin{cases} \nu_{F(\varepsilon)}(x), & \text{if } \varepsilon \in A - B \\ \nu_{G(\varepsilon)}(x), & \text{if } \varepsilon \in B - A \\ \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)), & \text{if } \varepsilon \in A \cap B \end{cases}$$

Now $((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)} = (H, C)_{(\alpha, \beta)} = (H_{(\alpha, \beta)}, C)$, where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$H_{(\alpha, \beta)}(\varepsilon)$$

$$= \begin{cases} \{x: x \in U, \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta\}, & \text{if } \varepsilon \in A - B \\ \{x: x \in U, \mu_{G(\varepsilon)}(x) \geq \alpha, \nu_{G(\varepsilon)}(x) \leq \beta\}, & \text{if } \varepsilon \in B - A \\ \{x: x \in U, \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha, \\ \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \leq \beta\}, & \text{if } \varepsilon \in A \cap B \end{cases}$$

Let $x \in H_{(\alpha, \beta)}(\varepsilon)$ for some $\varepsilon \in C$. Then

$$\begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha, \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in B - A \\ \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha, \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \leq \beta, & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha, \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in B - A \\ \mu_{F(\varepsilon)}(x) \geq \alpha \text{ or } \mu_{G(\varepsilon)}(x) \geq \alpha, \\ \nu_{F(\varepsilon)}(x) \leq \beta \text{ or } \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow x \in \begin{cases} F_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A - B \\ G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in B - A \\ F_{(\alpha, \beta)}(\varepsilon) \cup G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow x \in (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)}$$

Thus $(H, C)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)}$

For the converse part,

Let $(F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)} = (I, C)$ where $C = A \cup B$ and $\forall \varepsilon \in C$,

$$I(\varepsilon) = \begin{cases} F_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A - B \\ G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in B - A \\ F_{(\alpha, \beta)}(\varepsilon) \cup G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

Let $x \in I(\varepsilon)$ for some $\varepsilon \in C$.

$$\text{Then } x \in \begin{cases} F_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A - B \\ G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in B - A \\ F_{(\alpha, \beta)}(\varepsilon) \cup G_{(\alpha, \beta)}(\varepsilon), & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha, \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in B - A \\ \mu_{F(\varepsilon)}(x) \geq \alpha \text{ or } \mu_{G(\varepsilon)}(x) \geq \alpha, \\ \nu_{F(\varepsilon)}(x) \leq \beta \text{ or } \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow \begin{cases} \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in A - B \\ \mu_{G(\varepsilon)}(x) \geq \alpha, \nu_{G(\varepsilon)}(x) \leq \beta, & \text{if } \varepsilon \in B - A \\ \max(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha, \min(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \leq \beta, & \text{if } \varepsilon \in A \cap B \end{cases}$$

$$\Rightarrow x \in H_{(\alpha, \beta)}(\varepsilon)$$

Thus $I(\varepsilon) \subseteq H_{(\alpha, \beta)}(\varepsilon) \forall \varepsilon \in C$

$\Rightarrow (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)} \tilde{\subseteq} (H, C)_{(\alpha, \beta)}$ and the result follows immediately.

The proof of second result is similar.

(iii) Let $(F, A) \tilde{\cap} (G, B) = (H, C)$.

Then $C = A \cap B$ and $\forall \varepsilon \in C$,

$$\mu_{H(\varepsilon)}(x) = \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)),$$

$$\nu_{H(\varepsilon)}(x) = \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x))$$

Now $((F, A) \tilde{\cap} (G, B))_{(\alpha, \beta)} = (H, C)_{(\alpha, \beta)} = (H_{(\alpha, \beta)}, C)$, where $C = A \cap B$ and $\forall \varepsilon \in C$,

$$H_{(\alpha, \beta)}(\varepsilon) = \{x: x \in U, \mu_{H(\varepsilon)}(x) \geq \alpha, \nu_{H(\varepsilon)}(x) \leq \beta\}$$

Let $x \in H_{(\alpha, \beta)}(\varepsilon)$ for some $\varepsilon \in C$. Then

$$\mu_{H(\varepsilon)}(x) \geq \alpha \Rightarrow \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha \Rightarrow \mu_{F(\varepsilon)}(x) \geq \alpha$$

and $\mu_{G(\varepsilon)}(x) \geq \alpha$,

$$\text{Also } \nu_{H(\varepsilon)}(x) \leq \beta \Rightarrow \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \leq \beta$$

$$\Rightarrow \nu_{F(\varepsilon)}(x) \leq \beta \text{ and } \nu_{G(\varepsilon)}(x) \leq \beta$$

$$\Rightarrow x \in F_{(\alpha, \beta)}(\varepsilon) \text{ and } x \in G_{(\alpha, \beta)}(\varepsilon)$$

$$\Rightarrow x \in (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)}$$

Thus $(H, C)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)}$

For the converse part,

let $(F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)} = (F_{(\alpha, \beta)}, A) \tilde{\cap} (G_{(\alpha, \beta)}, B) = (I, C)$

Where $C = A \cap B$ and $\forall \varepsilon \in C, I(\varepsilon) = F_{(\alpha, \beta)}(\varepsilon) \cap G_{(\alpha, \beta)}(\varepsilon)$.

Let $x \in I(\varepsilon)$ for some $\varepsilon \in C$.

$\Rightarrow x \in F_{(\alpha, \beta)}(\varepsilon)$ and $x \in G_{(\alpha, \beta)}(\varepsilon)$

$\Rightarrow \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta$ and $\mu_{G(\varepsilon)}(x) \geq \alpha,$

$\nu_{G(\varepsilon)}(x) \leq \beta$

$\Rightarrow \min(\mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x)) \geq \alpha, \max(\nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x)) \leq \beta$

$\Rightarrow x \in H_{(\alpha, \beta)}(\varepsilon)$

Thus $I(\varepsilon) \subseteq H_{(\alpha, \beta)}(\varepsilon) \forall \varepsilon \in C$

$\Rightarrow (F, A)_{(\alpha, \beta)} \tilde{\cap} (G, B)_{(\alpha, \beta)} \tilde{\subseteq} (H, C)_{(\alpha, \beta)}$ and the result follows.

The proof of second result similarly follows.

(iv) We have $(F, A)^c = (F^c, A)$, where $\forall \varepsilon \in A,$

$F^c(\varepsilon) = (F(\varepsilon))^c$ i.e.

$\forall \varepsilon \in A, x \in U \Rightarrow \mu_{F^c(\varepsilon)}(x) = \nu_{F(\varepsilon)}(x), \nu_{F^c(\varepsilon)}(x) = \mu_{F(\varepsilon)}(x)$

Now $(F, A)^c_{(\alpha, \beta)} = (F^c, A)_{(\alpha, \beta)} = (F^c_{(\alpha, \beta)}, A)$ where $\forall \varepsilon \in A,$

$F^c_{(\alpha, \beta)}(\varepsilon) = \{x: x \in U, \mu_{F^c(\varepsilon)}(x) \geq \alpha, \nu_{F^c(\varepsilon)}(x) \leq \beta\}$

Let $x \in F^c_{(\alpha, \beta)}(\varepsilon)$ for some $\varepsilon \in C$. Then

$\mu_{F^c(\varepsilon)}(x) \geq \alpha, \nu_{F^c(\varepsilon)}(x) \leq \beta \Rightarrow \nu_{F(\varepsilon)}(x) \geq \alpha, \mu_{F(\varepsilon)}(x) \leq \beta$

This means $x \in F_{(\beta, \alpha)}(\varepsilon)$ i.e. $x \in (F_{(\beta, \alpha)}(\varepsilon))^c$.

It follows that $(F, A)^c_{(\alpha, \beta)} \tilde{\subseteq} (F, A)^c_{(\beta, \alpha)}$.

It can also be verified that $(F, A)^c_{(\beta, \alpha)} \tilde{\subseteq} (F, A)^c_{(\alpha, \beta)}$ and the result follows immediately.

M. Proposition

Let (F, A) be an intuitionistic fuzzy soft set over (U, E) and $\alpha, \beta, \gamma, \delta \in [0, 1]$. Then the following results hold.

(i) $(F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\alpha, \beta)}$

(ii) $\alpha \leq \gamma, \beta \geq \delta \Rightarrow (F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\gamma, \delta)},$

$(F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\gamma, \beta)}$

Proof

(i) Let (F, A) be an intuitionistic fuzzy soft set over (U, E) .

Then $\forall \varepsilon \in A,$

$F_{(\alpha, \beta)}(\varepsilon)$

$= \{x: \mu_{F(\varepsilon)}(x) > \alpha, \nu_{F(\varepsilon)}(x) < \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$

$\subseteq \{x: \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$

$= F_{(\alpha, \beta)}(\varepsilon)$

Therefore $(F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\alpha, \beta)}$

(ii) Let (F, A) be an intuitionistic fuzzy soft set over (U, E)

and $\alpha \leq \gamma, \beta \geq \delta$

Then $\forall \varepsilon \in A,$

$F_{(\alpha, \beta)}(\varepsilon)$

$= \{x: \mu_{F(\varepsilon)}(x) \geq \alpha, \nu_{F(\varepsilon)}(x) \leq \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$

$\subseteq \{x: \mu_{F(\varepsilon)}(x) \geq \gamma, \nu_{F(\varepsilon)}(x) \leq \delta; x \in U, \gamma, \delta \in [0, 1], \gamma + \delta \leq 1\}$

$= F_{(\gamma, \delta)}(\varepsilon)$

Therefore $(F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\gamma, \delta)}$

Also $\forall \varepsilon \in A,$

$F_{(\alpha, \beta)}(\varepsilon)$

$= \{x: \mu_{F(\varepsilon)}(x) > \alpha, \nu_{F(\varepsilon)}(x) < \beta; x \in U, \alpha, \beta \in [0, 1], \alpha + \beta \leq 1\}$

$\subseteq \{x: \mu_{F(\varepsilon)}(x) > \gamma, \nu_{F(\varepsilon)}(x) < \delta; x \in U, \gamma, \delta \in [0, 1], \gamma + \delta \leq 1\}$

$= F_{(\gamma, \delta)}(\varepsilon)$

Therefore $(F, A)_{(\alpha, \beta)} \tilde{\subseteq} (F, A)_{(\gamma, \delta)}$

IV. CONCLUSION

In our work, we have put forward some new concepts such as disjunctive sum, difference, (α, β) - cut soft set and (α, β) - cut strong soft set of an intuitionistic fuzzy soft set. Some related properties have been established with examples and counter examples. It is hoped that our work will enhance this study in intuitionistic fuzzy soft sets.

REFERENCES

1. B. Ahmad and A. Kharal, "On Fuzzy Soft Sets", Advances in Fuzzy Systems, Volume 2009, pp. 1-6, 2009.
2. D. A. Molodtsov, "Soft Set Theory - First Result", Computers and Mathematics with Applications, Vol. 37, pp.19-31, 1999.
3. K. Atanassov, "Intuitionistic fuzzy sets", Fuzzy Sets and Systems 20 (1986), 87-96
4. P. K. Maji, R. Biswas and A. R. Roy, "Fuzzy Soft Sets", Journal of Fuzzy Mathematics, Vol 9, No. 3, pp. 589-602, 2001.
5. P. K. Maji and A. R. Roy, "Soft Set Theory", Computers and Mathematics with Applications 45 (2003) 555 - 562.
6. P. K. Maji, R. Biswas, A. R. Roy, "Intuitionistic fuzzy soft sets", The journal of fuzzy mathematics 9(3)(2001), 677-692
7. T. J. Neog, D. K. Sut, "Some New Operations of Fuzzy Soft Sets", Accepted for publication in Journal of Mathematical and Computational Sciences.

AUTHORS PROFILE

Manoj Bora received his M.Sc. degree in Mathematics from Dibrugarh University, India. He is an assistant professor in the department of Mathematics, Jorhat Institute of science and Technology, Jorhat, India.

Tridiv Jyoti Neog received his M.Sc. degree in Mathematics from Dibrugarh University, India. He is a research scholar in the department of Mathematics, Faculty of Science, CMJ University, Shillong, Meghalaya, India.

Dusmanta Kumar Sut received his M.Sc. and Ph.d in Mathematics from Dibrugarh University, India. He is an assistant professor in the department of Mathematics, N. N. Saikia College, Titabor, India. His research interests are in Fuzzy Mathematics, Fluid dynamics and Graph Theory.

