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Simulation of Non-Equilibrium Transport of Suspended Sediment in Open Channels

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Abstract*— Present paper deals with development of a new mathematical model to simulate non-equilibrium transport of suspended sediment in open channels, with special emphasis on sediment overloading. Equilibrium transport of bed sediment is also included. Sources of the suspended sediment are from the catchment (such as construction sites, mining areas etc.). This sediment is brought into rivers and canals by flowing rainwater. Its overloading can result in large changes in bed and water levels of the rivers and also in heavy silting of the canal beds. A new mathematical model to simulate this effect is derived based on the control volume approach. The model is tested using data available in literature and results are found satisfactory. The developed model is then applied to a hypothetical flood and sediment routing problem in a river and analyses of the results are given.

***Index Terms*— Open channels, Suspended sediment, Non-equilibrium transport, Mathematical model, Simulation**

I. INTRODUCTION

The problems caused by suspended sediment overloading in open channels are of great importance to the hydraulic engineers. One of the important problems caused by the sediment overloading during flood seasons is the change in bed level and corresponding water level of rivers due to its deposition on the river bed. At the upstream reaches, the bed slope is high, velocity is high and hence the rivers carry large amount of sediment during flood seasons. Source of this sediment can be mining areas, construction sites, agricultural fields etc. and is brought into the rivers by flowing rain water. This type of sediment move as suspended load. When the rivers reach downstream flat plains, its velocity decreases, hence the excess sediment over the river transport capacity will act as sediment overload. So it will be deposited on the river bed, as a result the bed level rises, water level rises and hence more damages are caused during flood seasons. Another problem caused by the suspended sediment overloading is the heavy silting of the beds of irrigation canals off-taking from these rivers. De-silting of the canals to retain their original shape is highly expensive.

Transport and deposition of the suspended sediment, as described above, will be affected by hydraulics of the channel flow [1]-[3]. Present study focuses on development of a mathematical model to simulate the movement of water and associated sediment in an open channel, deposition of excess sediment, over the transport capacity, on the channel bed and consequent changes in the bed and water levels.

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II. LITERATURE REVIEW

The simulation codes, which are being used for flood and sediment routing, are of two types. The first type solves the continuity and momentum equation for water along with the sediment continuity equation. Examples for this type are FCM [4], MOBED [5], HEC-RAS [6]. This type cannot be used for the purpose of simulation of transport of sediment from catchment. This is because they are using sediment transport formulae to determine the sediment discharge, which give discharge of bed sediment only [7]. The second type can be used for the purpose of simulation of transport of bed sediment and also sediment from catchment. An example for this type is the SEDICOUP code [8]. Along with the equations used by the first type, this type used the advection-diffusion equation combined with a source term, to simulate the transport of suspended sediment from catchment. The source term is given by

$S = -S_d + S_e$

Where S_e and S_d are the source terms corresponding to entrainment and deposition of the sediment respectively. For the case of transport of suspended sediment from catchment, none of that type will be present in the bed initially so that $S_e = 0$. Thus there will always be deposition as the sediment moves forward, even if the existing sediment load is less than the river transport capacity (ref. test 2., [9]), which is physically incorrect.

From the above discussion, it is clear that the available simulation procedures are not sufficient to solve the present problem, which points to the necessity of a new model.

III. DEVELOPMENT OF A NEW MODEL

The new mathematical model developed is described below. The unknowns to be computed are Depth of flow (h), Water discharge (q), Bed level (Z) and Concentration (C_w) of sediment derived from sources other than river bed (that is, from catchment).

A. Governing Equations:

Considering unit width of a wide rectangular channel, the equations used are as follows:

1) Continuity equation for water [10]

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

2) Momentum equation for water [10]

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$$\frac{\partial q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q^2}{h}+\frac{gh^2}{2}\right)+gh\frac{\partial Z}{\partial x}+ghS_f=0 \qquad (2)$$

3) Bed sediment conservation equation

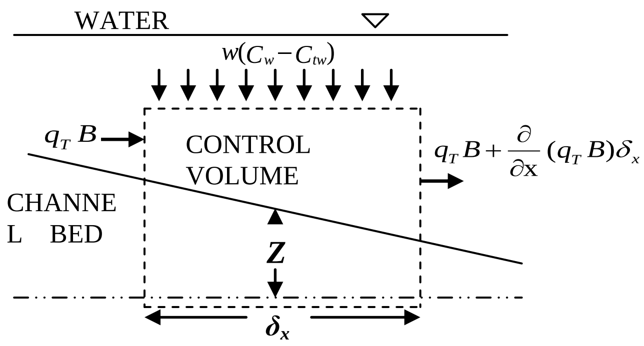

Figure1. Definition sketch for bed sediment continuity

With reference to Fig.1

$$w(C_w-C_{tw})B\delta_x+q_T B-\left[q_T B+\frac{\partial}{\partial x}(q_T B)\delta x\right]$$

$$=\frac{\partial}{\partial t}\left[ZB\delta_x(1-\lambda)\right]$$

where C_{tw} = channel transport capacity concentration of the suspended sediment; w = fall velocity of the suspended sediment; B = width of the channel ; q_T = total discharge of bed sediment per unit width; λ = porosity of channel bed, which is taken as zero. The resulting equation is

$$\frac{\partial Z}{\partial t}+\frac{\partial q_T}{\partial x}-w(C_w-C_{tw})=0 \qquad (3)$$

4) Suspended Sediment Conservation Equation

With reference to Fig. 2, neglecting the diffusive transport

$$q_w B-\left[q_w B+\frac{\partial}{\partial x}(q_w B)\delta_x\right]-w(C_w-C_{tw})B\delta_x=\frac{\partial}{\partial t}\left[C_w Bh\delta_x\right]$$

where q_w = discharge of suspended sediment, derived from sources other than river bed, per unit width = $C_w q$.

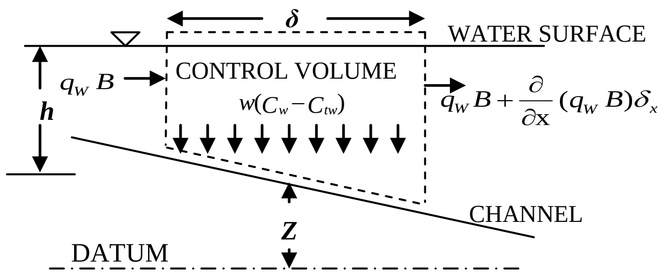

Figure 2. Definition sketch for suspended sediment continuity

The resulting equation is

$$\frac{\partial (C_w h)}{\partial t}+\frac{\partial (C_w q)}{\partial x}+w(C_w-C_{tw})=0 \qquad (4)$$

5) Channel transport capacity formula corresponding to suspended sediment derived from sources other than river bed

The model requires a formula to calculate value of C_{tw}. In the present model the equation proposed in [11] is used which is given by

$$C_{tw}=\alpha\frac{\tau_w}{(\rho_w-\rho)gh}\frac{U_m}{w} \qquad (5)$$

where $\tau_w = \rho g h S_f$; ρ_w = density of the suspended sediment; ρ = density of water; U_m = mean velocity of flow; S_f is the friction slope ; α is an empirical coefficient whose value depends on k_s / h where k_s is the equivalent sand - roughness height.

In the above equations, (3) and (4) take into account the transport and deposition of the suspended sediment. These formulae were derived based on the concept that as the sediment move forward the excess sediment will get deposited at the rate $w(C_w\text{-}C_{tw})$.

B. Initial and Boundary Conditions:

The values of q, h, C_w and Z corresponding to the beginning of first time step will have to be specified at all the nodes in the x-direction (along the canal) as initial conditions. Under subcritical flow condition, the model requires four boundary conditions as follows:

i) One upstream boundary condition and one downstream boundary condition for water.
ii) The bed wave travels in the downstream direction under subcritical flow condition [12]. Hence a condition for change in Z has to be imposed at the upstream boundary.
iii) The inflow suspended sediment hydrograph at the upstream boundary.

C. Method of Solution:

The set of equations described in the above section, has been solved as a fully coupled set, using finite difference implicit method. The Priessmann scheme [13] has been used for discretizing the equations. The way in which discretization is done using this scheme, at the i^{th} node during j^{th} time step, is as follows (Fig. 3):

$$f(x,t)=\theta\left[\phi f_{i+1}^{j+1}+(1-\phi)f_i^{j+1}\right]+(1-\theta)\left[\phi f_{i+1}^{j}+(1-\phi)f_i^{j}\right]$$

$$\frac{\partial f}{\partial x}=\frac{1}{\Delta x}\left[\theta\left(f_{i+1}^{j+1}-f_i^{j+1}\right)+(1-\theta)\left(f_{i+1}^{j}-f_i^{j}\right)\right]$$

$$\frac{\partial f}{\partial t}=\frac{1}{\Delta t}\left[\phi\left(f_{i+1}^{j+1}-f_{i+1}^{j}\right)+(1-\phi)\left(f_i^{j+1}-f_i^{j}\right)\right]$$

Values of both θ and ϕ used in the simulation were 0.5 for which the scheme is unconditionally stable [14].

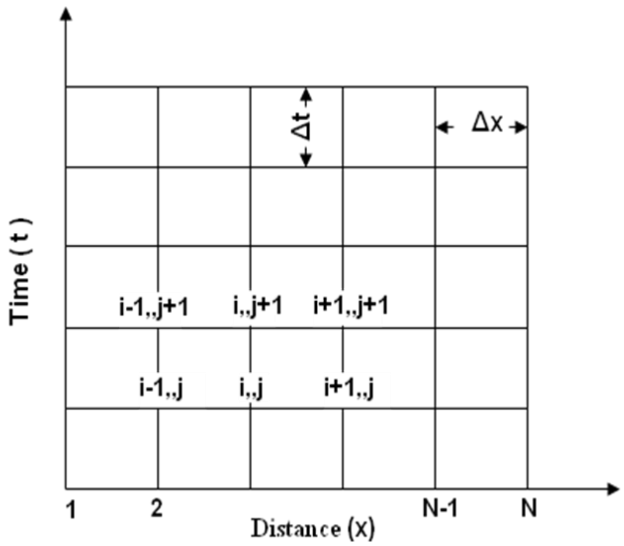

Figure 3. Computational domain for the mathematical model: N = number of nodes in the x – direction, i = node number in the x – direction, j = node number in the t – direction.

IV. MODEL TESTING

Two types of model applications were performed to test the proposed model:

TEST 1: Simulation of Bed Sediment Transport

For this, the reservoir sedimentation problem described in [15] was studied. Concentration of suspended sediment from catchment was zero in this case. A river reach 13.6 km upstream of a reservoir was considered for simulation. The median size of bed sediment was d = 0.4 mm with the geometric standard deviation of sediment size distribution σ_g =1.0. The friction factor and transport capacity concentration of river for bed sediment were predicted from Brownlie's formulae [16]. The initial river bed slope was 0.0005. Inflow water discharge was 5.0 m^3/s. Inflow bed sediment discharge was 5 kg/s, and this corresponds to transport capacity of the river, as a result of which no bed level change has taken place at upstream end. The downstream water level was kept constant equal to 5.0 m. A uniform spatial grid with Δx =340 m and Δt = 46,260 s has been used for computation. The bed growth profiles obtained after 257 hours of simulation are shown in Fig .4

Figure 4. Deposition upstream of a dam: H = critical depth, L = length of river reach

It can be seen that the values obtained from present model match well with the values from Lyn's computation proving that the proposed model can be used successfully as a fully coupled one to simulate the transport of bed sediment.

TEST 2: Simulation of Transport of Suspended Sediment Derived From Sources Other Than River Bed

To test the capability of the proposed model to simulate this condition, the experimental data given in [17] has been used. The experiment was conducted in a re-circulating flume. Fine sediment was discharged continuously from a line source across the flume. The flume bed was roughened with wooden cleats whose height was such that (k_s /h) = 0.25. The sediment and geometric characteristics of the reach used were

a) Median grain size of sediment = 0.123 mm
b) Geometric standard deviation of sediment size distribution =1.0
c) Specific gravity of fine sediment =2.42
d) Median standard fall velocity of fine sediment = 1.09 cm /s
e) Depth of flow =0.407 m
f) Discharge = 0.24 m^3/s
g) Inflow suspended sediment concentration =168 ppm

The hydraulic conditions were such that uniform flow was established throughout the reach.

In the simulation using the proposed model a length of 40 m was considered. This length was discretized with 41 nodal points, so that Δx =1.0 m. The value of α in the suspended sediment transport capacity relation (5) will be affected by the value of k_s/h [11]. Taking this into account, the value of α used in the simulation was 0.001. No sediment contribution from the river bed was considered in the experiment and so the same was neglected in the computation. The model was run for such duration until steady conditions were established.

The sediment concentration profiles obtained from the experiment and from simulation using the proposed model are shown in Fig. 5. In the figure, '*x*' is the distance along the reach. From the figure it can be seen that the experimental data indicates equilibrium concentration has reached at (x/h) = 67.5. But the model results indicate that even at (x/h) = 100, the value of C_w/C_{tw} is slightly greater than 1.0. A similar result was obtained in [18] and there the conclusion was that equilibrium concentration might not have reached at (x/h) = 67.5. Another reason can be the difference between the individual fall velocity (which was taken in the calculation) and the group fall velocity with which the sediment will actually settle down. Further the nature of variation of the computed profile is similar to that of experimental data. Hence it can be concluded that the model can be used effectively for the simulation of suspended sediment overloading in open channels.

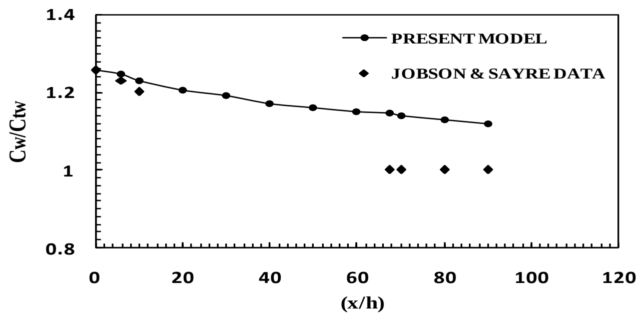

Figure 5. Transport of suspended sediment

V. APPLICATION OF THE MODEL TO THE PROBLEM OF A HYPOTHTICAL FLOOD IN RIVER

In order to study the effect of different percentages of suspended sediment overloading on the bed and water levels of a river, for a particular inflow discharge hydrograph, the model was used to simulate a hypothetical flood in a river. A river reach 80 km in length was chosen for simulation. The upstream boundary conditions used were:

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i). A water hydrograph, which is given by the relation

$$q(t) = q_b + q_b\left[1.0 - \cos\pi\frac{t}{t_p}\right]; \text{ for } 0<t<t_p$$

$$q(t) = q_b + q_b\left[1.0 - \cos\pi\frac{(t_b - t)}{(t_b - t_p)}\right]; \text{ for } t_p<t<t_b$$

where *q(t)* is value of discharge ordinate at time *t*, q_b is the base flow discharge, t_b is the time base (chosen as 259200 s) and t_p is the time to peak (chosen as 129600 s) of the discharge hydrograph.

ii). Sediment hydrograph: Corresponding to each water discharge obtained as described above, the corresponding ordinate of the sediment hydrograph was obtained by using (5) and applying the specified percentage overloading.

iii). The boundary condition for change in bed level at the upstream station was applied as

$$dz_1 = \Delta t\, w(C_w - C_{tw})$$

It was taken that inflow bed sediment concentration was equal to the river transport capacity so that no bed level change had taken place at the upstream station due to its effect.

The downstream boundary condition applied was a relation between water discharge and water depth as given by Brownlie's formula [16].

Uniform flow conditions were existing initially with a base flow discharge (q_b) of 11.6 m^3/s and depth of 5.0 m. The initial bed slope was 0.00059. Initially the bed was covered with sand grains of size 0.4 mm with a median standard deviation of sediment size distribution of 1.0. Friction slope and concentration of bed sediment were predicted using Brownlie's formulae. The model was run for different percentages of overloading of inflow suspended sediment concentration, the source of which is different from the bed of the river reach taken for simulation. The size and fall velocity of the suspended sediment were 0.05 mm and 0.001 m/s respectively. The value of α used in (5), in order to determine the transport capacity of the river for sediment from sources other than the river bed and move in the form of suspended sediment, was 0.01.

The 80 km length of the river reach has been discretized with 81 nodal points by taking Δx as 1000 m. The time interval (Δt) used in the simulation was 7200 s. The model was run for a total time period of 259200 s (30 days).

The following two cases have been taken for study:

Case 1.When the river bed is considered fixed without any sediment transport (that is, with 0% overload)

Case 2.When the inflow suspended sediment concentration corresponds to an overloading of 2%

The results obtained from above simulation using the proposed model are shown in Figs. 6-11. In the figures, W. S. L and B. L indicate water surface level and initial bed level respectively and *L* indicates the length of reach. The variables W.S.L, *x* and *t* were normalized as specified below:

i) Normalized W.S.L = $\dfrac{\text{W.S.L}(x,t) - \text{W.S.L}(x,0)}{\text{W.S.L}(0,t_p) - \text{W.S.L}(0,0)}$

ii) Normalized distance = x / L

iii) Normalized time = t / t_p

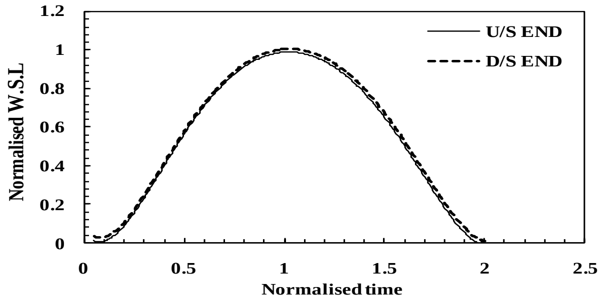

Figure 6. Satge hydrograph (0% overload)

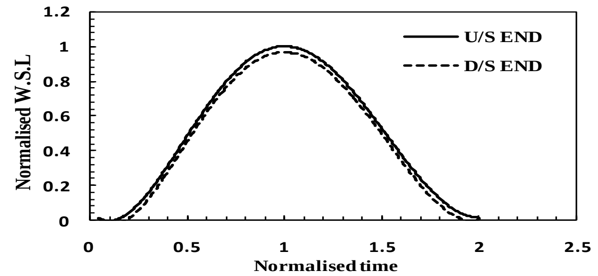

Figure 7. Satge hydrograph (2% overload)

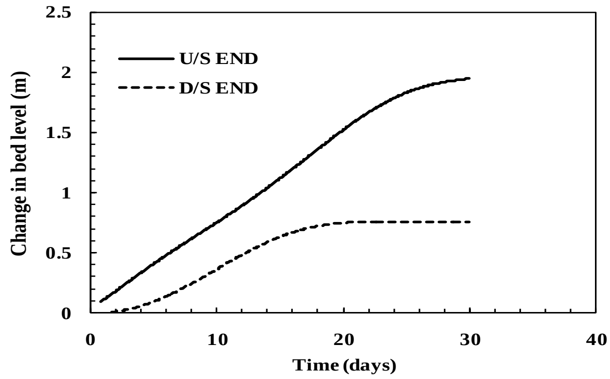

Figure 8. Bed level change with time (2% overload)

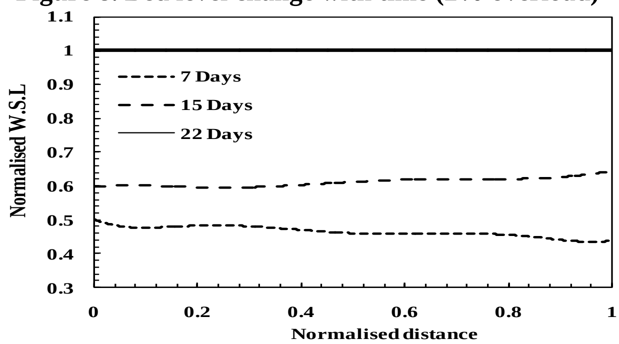

Figure 9. Water surface profile (0% overload)

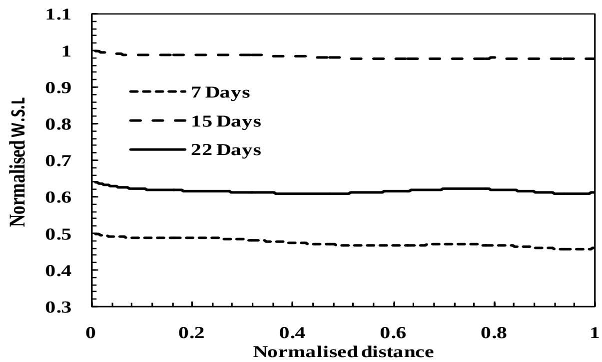

Figure 10. Water surface profile (2% overload)

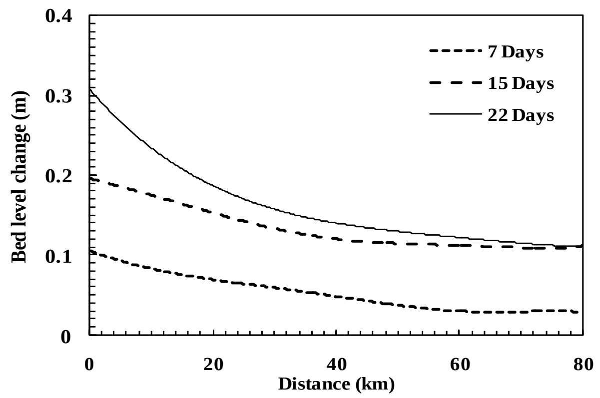

Figure 11. Bed profiles (2 % overload)

A. Analysis of the results

Effect of sediment overloading on water level: From figures (6) and (7), it can be realized that extent of sediment overloading directly affects the peak water level at a station, with overloading peak water level increases. It can be seen from these figures and from figures (9) and (10) that the sediment overloading has more effect on water level at upstream stations than those at downstream stations.

Effect of sediment overloading on bed level: From the plot of rise in bed level Vs time (Fig. 8), it can be concluded that percentage overloading directly affects the increase in bed level. Also it can be seen from the above figure and Fig. 11 that the maximum rise in bed level occurred at the upstream station and it become less while moving downstream. Again, it can be observed from these two figures that the bed level went on increasing as time increases. During both rising and falling of the flood, the rate of increase of bed level at upstream station was approximately the same, but the rate of increase of bed level at the downstream station was more during rising of the flood than during its falling. Thus the transport and deposition of excess sediment can take place in longer reaches during rising of the flood as compared to the distance in which they can occur during its falling.

VI. CONCLUSIONS

For simulation of non-equilibrium transport of suspended sediment derived from catchment along with bed sediment in open channels, a mathematical model has been developed. It has been verified using data available in literature and results were found satisfactory. The model was then used to simulate a hypothetical flood movement in a river corresponding to cases where the percentage overloading of sediment derived from catchment has been different for each one. The following conclusions are drawn from the analyses of the results obtained:

a) The peak water level at a station increases with increase in the percentage of the sediment overloading.
b) The transport and deposition of the sediment at the downstream reaches were more at the time of rising of the flood than when the flood falls.

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