An Implementation of a Forward Error Correction Technique using Convolution Encoding with Viterbi Decoding

Himmat Lal Kumawat, Sandhya Sharma

Abstract— This paper, as the name suggests, shows the working of a forward error correction (FEC) coding technique using convolutional encoding with Viterbi decoding. It can be used by anyone interested in designing or understanding wireless digital communications Technique systems. This paper initially explains the working of a convolutional encoder[1]. The encoded bit stream, is then passed through an additive white Gaussian noise (AWGN) channel, quantized and received at the decoder. Finally, the original data stream is recovered by either a hard decision Viterbi decoder or a soft decision Viterbi decoder. This entire FEC technique is demonstrated, both practically, using Matlab, and theoretically.

Also shown are simulation plots, characterizing the performance factors affecting the FEC coding technique. These factors include primarily the noise level as well as the encoder memory size. Convolutional encoding is widely used in modern communication systems. A software tool for simulating the processes of convolutional encoding and decoding is developed and presented in the paper. The convolutional encoders are investigated in respect of the parameter bit error rate. The decoding of convolutional codes is based on Viterbi algorithm[3]. The software tool is developed in MATLAB and Communication Toolbox.

In mathematics, computer science, telecommunication, and information theory, error detection and correction has great practical importance in maintaining information (data) integrity across noisy channels and less-than-reliable storage media. A channel code is a broadly used term mostly referring to the forward error correction (FEC) code and bit interleaving in communication and storage where the communication media or storage media is viewed as a channel[6]. The FEC code is used to protect data sent over the channel for storage or retrieval even in the presence of noise (errors) . There exist two main forms of channel codes – convolutional codes and block codes. Convolutional codes are often used to improve the performance of digital radio, mobile phones, satellite links, and Bluetooth implementations. Unlike block encoders, convolutional encoders are not memoryless devices.

A convolutional encoder accepts a fixed number of message symbols and produces a fixed number of code symbols, but its computations depend on the current set of input symbols and on some of the previous input symbols[13].

Keywords– Channel Coding, Convolutional Codes, Non-Systematic Convolutional Codes, Viterbi Algorithm, MATLAB, AWGN and BPSK.

I. INTRODUCTION

In wireless, satellite, and space communication systems, reducing error is critical. Wireless medium is quite different from the counterpart using wires and provides several advantages, for example; mobility, better productivity, low cost, easy installation facility and scalability. On the other hand, there are some restrictions and disadvantages of various transmission channels in wireless medium between receiver and transmitter where transmitted signals arrive at receiver with different power and time delay due to the reflection, diffraction and scattering effects[3]. Besides the BER (Bit Error Rate) value of the wireless medium is relatively high. These drawbacks sometimes introduce destructive effects on the wireless data transmission performance[5].

As a result, error control is necessary in these applications. During digital data transmission and storage operations, performance criterion is commonly determined by BER which is simply: Number of error bits / Number of total bits. Noise in transmission medium disturbs the signal and causes data corruptions[12]. Relation between signal and noise is described with SNR (signal-to-noise ratio). Generally, SNR is explained with signal power / noise power and is inversely proportional with BER. It means, the less the BER result is the higher the SNR and the better communication quality [1].

There are two different types of FEC techniques, namely block codes and convolutional codes [2]. The Viterbi algorithm is a method for decoding convolutional codes proposed in 1967 by A. J. Viterbi. It has been counted as one of good decoding scheme up to date. This algorithm, however, is vulnerable to burst error which means a series of consecutive errors [3]. Since most physical channels make burst errors, it can be a serious problem. Furthermore, the complexity increases as the number of memories in the encoder increases, and the increase of the memory causes the increase of computation. To compensate these problems, a new solution can be applied to concatenation of a Reed-Solomon (RS) code and a convolutional code (CC) i.e., RS-CC or CC-RS concatenated codes.

II. ERROR CORRECTION TECHNIQUES

Channel coding and modulation provide the means of mapping information into waveforms such that the receiver (with an appropriate demodulator and decoder) can recover the information in a reliable manner. A block diagram derived from Figure 1, which shows this part of the digital communication link, is shown in Figure 1.

Himmat Lal Kumawat, Electronics and Communication Department, Suresh Gyan Vihar University, Jaipur, India.
Sandhya Sharma, Electronics and Communication Department, Suresh Gyan Vihar University, Jaipur, India.
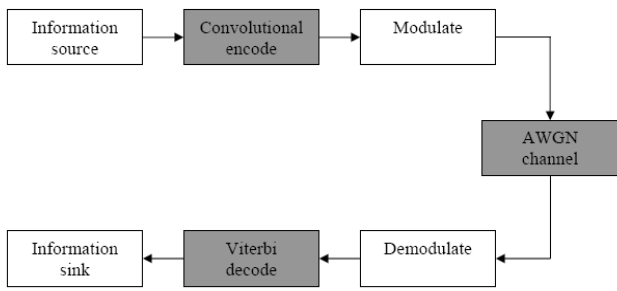
Figure 1: Encode/decode and modulate/demodulate portions of a digital communication link.

As seen in Figure 1, convolutional encoding is one way of performing channel coding. Another method uses block codes. In these methods, redundant bits are used to help determine the occurrence of an error due to noise present in the channel. In the receiver, Viterbi decoding is a way of performing channel decoding. Another method is turbo codes[11]. Turbo codes can be applied to the encoding process too. In these methods, errors can be "automatically" corrected (within specified limitations) to recover the original information. Error correction is a technique defined by the methods of encoding and decoding.One such technique, called the *automatic repeat request* (ARQ), simply recognizes the occurrence of an error and requests the sender retransmit the message. Another technique is known as the *forward error correction* (FEC) technique. This technique allows for "automatic" correction of errors[8].

III. CONVOLUTIONAL CODING

A convolutional encoder accepts a sequence of message symbols and produces a sequence of code symbols. Its computations depend not only on the current set of input symbols but, on some of the previous input symbols as well. In practice, convolutional codes operate on a block at a time and so like block codes, have intra-block memory, and possibly no inter-block memory[7].

*Convolutional Encoder***:** A convolutional encoder is made of a fixed number of shift registers. Each input bit enters a shift register and the output of the encoder is derived by combining the bits in the shift register. The number of output bits depends on the number of modulo 2-adders used with the shift registers.

*Encoder Parameters: -*Convolutional codes are commonly specified by the three parameters (n , k ,m)

where, $n =$ number of output bits

$k =$ number of input bits and,

$m =$ number of memory registers.

The quantity k / n called the code rate is a measure of the bandwidth efficiency of the code. Commonly k and n parameters range from 1 to 8, m from 2 to 10, and the code rate from 1/8 to 7/8 except for deep space applications where code rates as low as 1/100 or even longer can be employed. Convolutional codes can also be specified by the parameters (n,k,L) where, L is known as the constraint length of the code and is defined as the number of bits in the encoder memory that affects the generation of the n output bits. The convolutional codes discussed here will be referred to as (n,k,L) and not as (n,k,m) codes[2].

Encoder Structure:- The term *generator polynomial*(g) characterizes the encoder connections. The selection of which

bits (in the memory registers) are to be added (using modulo-q adders) to produce the output bits is called the generator polynomial for that output bit. There are many choices for polynomials for any m order code. Good polynomials are normally found by trial and error through computer simulations[8]. We used two sets of generator polynomials in this thesis which are listed in the Matlab. One set was that of the standard generator polynomials as listed in [1]. The other was another set, obtained by us, which closely matches the performance characteristics of the standard set of generator polynomials.

In order to understand the working of a convolutional encoder and further the forward error correction technique, the following assumptions have been made:
(a) A (2, 1, 3) convolutional encoder is used.
(b) A 3-bit input sequence is used specified by the bits [1 01].
(c) A 2 generator polynomials are used, specified by the bits [1 1 1] and [1 0 1].

It is easy to construct a convolutional encoder. We first draw m boxes representing the m memory registers. Then we draw n modulo-2 adders representing the n output bits. Finally, we connect the memory registers to the adders using the bits specifying the generator polynomials[2].

Shown in Figure 2 is a (2, 1, 3) convolutional encoder. This encoder is going to be used to encode the 3-bit input sequence [1 0 1] with the two generator polynomials specified by the bits [1 1 1] and [1 0 1].u_1 represents the input bit, and $v1$and $v2$ represent the output bits 1 and 2 respectively. $u0$ and u_{-1} represent the initial state of the memory registers which are initially set to zero[9].
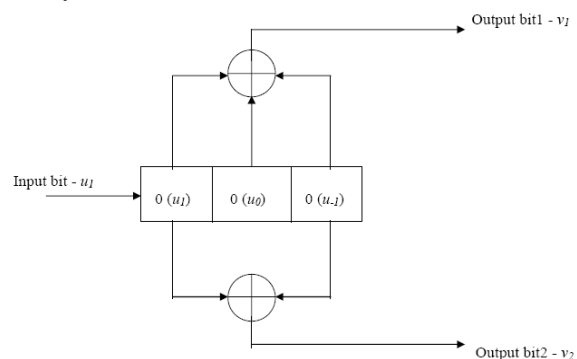

Figure 2: A (2, 1, 3) convolutional encoder.

IV. VITERBI DECODER

The Viterbi decoder algorithm proposed in 1967 is a decoding process for convolutional codes.Convolutional coding, as we all known, has been widely used in communication systems including deep space communications and wireless communications, such as IEEE 802.11a/g, WiMax, DAB/DVB, WCDMA and GSM.

Viterbi decoding algorithm is mostly applied to convolutional encoder and it uses maximum likelihood decoding technique [4]. Noisy channels cause bit errors at receiver. Viterbi algorithm estimates actual bit sequence using trellis diagram. Commonly, its decoding algorithm is used in two different forms. This difference results from the receiving form of the bits in the receiver[14].

Decoded information is received with hard decision or soft decision. Decoded information is explained with ±1 on hard decision operation while soft decision decoding uses multi bit quantization [4].
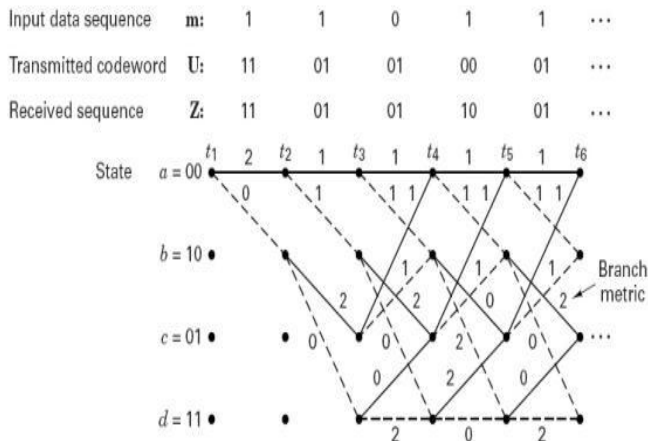
Figure 3: Trellis diagram of Viterbi decoder

Hard decision and soft decision viterbi decoding refer to the type of quantization used on the received bits. Hard decision decoding uses 1 bit quantization on the received channel values while soft decision decoding uses multi bit quantization on the received channel values[14]. For hard decision decoding, the symbols are quantized to one bit precision while for soft decision decoding, data bits are quantized to three or four bits of precision. The selection of quantization levels is an important design decision because of its significant effect on the performance of the link [10].
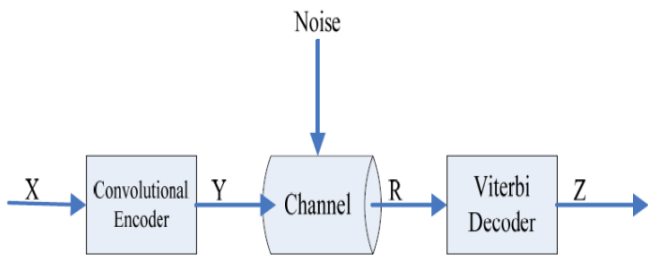
Figure 4: A simple Viterbi decoding system

In Viterbi decoding, unlike in sequential decoding, the decoding time is fixed and not variable thus making it more suitable to hardware implementation[14]. Here, we narrow the options systematically at each trellis stage. The principal used to reduce the choices are:

(i) The errors occur infrequently. The probability of error is small.

(ii) The probability of two errors in a row is much smaller than a single error that is the errors are distributed randomly.

As it is showing in the figure 4, a data sequence x is encoded to generate a convolutional code word y. after y is transmitted through a noisy channel. The convolutional decoder takes the received vector r and generates an estimate z of the transmitted code word. The maximum likelihood (ML) decoder selects the estimate that maximizes the probability p (r|z), while the maximum a posteriori probability (MAP) decoder selects the estimate that maximizes p (z|r). If the distribution of the source bits x is uniform, the two decoders are identical. By Bayer's law, we could get:

$$P(r \mid z)\, p(z) = p(z \mid r)\ p(r)$$

The initial part first initializes the Viterbi decoder to the same FSM and the trellis diagram as the convolutional encoder as.

Then in each time clock, the decoder computes the four possible branches metric's Euclidean distance. For each state, ACS block compute the two possible paths Euclidean distance and select a small one. At the same time, ACS block will record the survival state metric.

A trellis structure is built along the way, where each state transition is noted with all the relevant information like path metric and the decision bit. When all the data bits have been received, and the trellis has been completed, the last stage of the trellis is used as the starting point for tracing back. The trace-back operation outputs the decoded data, as it traces back along the maximum likelihood path.

V. MAXIMUM HAMMING DISTANCE

Hamming distance between the received codeword and the allowable codeword is calculated by checking the corresponding bit positions of the two codewords[1]. For example the Hamming distance between the codewords 00 and 11 is 0 or the Hamming distance between the codewords 00 and 00 is 2. The Hamming distance metric is cumulative so that the path with the largest total metric is the final winner. Thus the hard decision Viterbi decoding makes use of the maximum Hamming distance in order to determine the output of the decoder[9].
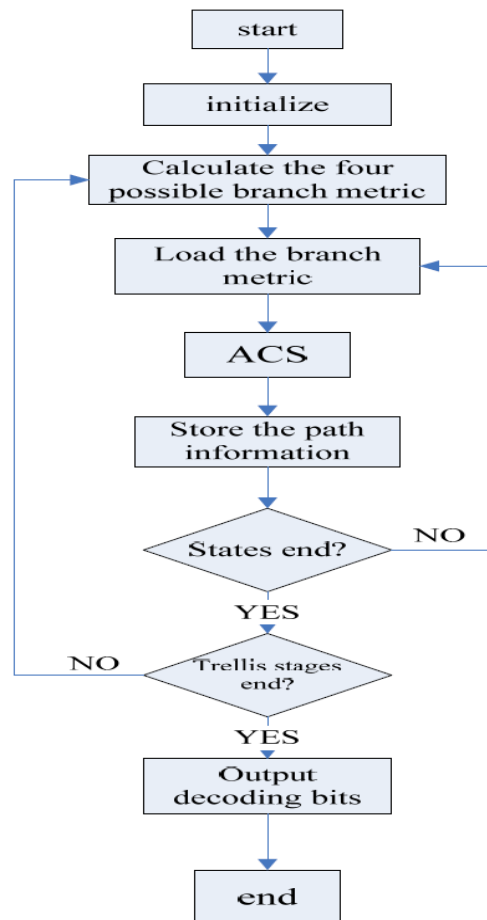

Figure 5: Show us the flow chat of the Viterbi decoding.

Note: Hamming distances as explained, can only be calculated when we use the binary bits 0 and 1. Thus, before the corrupted data signal is fed to the input of the hard decision Viterbi decoder, voltages that are less than and equal to zero are represented by the bit 0, and voltages that are greater than 0 are represented by the bit 1. This is also known as one-bit quantization[15].

VI.TRELLIS DIAGRAM

Trellis diagrams represent linear time sequencing events. They can also be used to encode messages. The trellis diagram of CC(2,1,3) encoder shown in figure 6 . It is built by using the Horizontal axis as a discrete time and all possible 2(L-1) states lined up on the vertical axis. One moves through the trellis every time step. New bits arrive every time step. Each state is connected to the next state by the allowable codewords for that state. There are only two possible choices at each state. These are determined by the arrival of input bits 0 or 1 which are shown between brackets in the diagram. It can be noticed that the arrows go upwards for a 0 input bit and they go downwards for a 1 input bit. The trellis diagram is unique to each code.

For instance to encode 1100 message ,one starts from the state 00 and goes downwards for each input bit 1 and goes upwards for each input bit 0 . After 4 time steps which correspond to the number of input bits, one gets the encoded word 11 01 01 11 (the reset bits are not considered here for simplicity). The trellis diagram of figure 6 will be used later to decode code words.
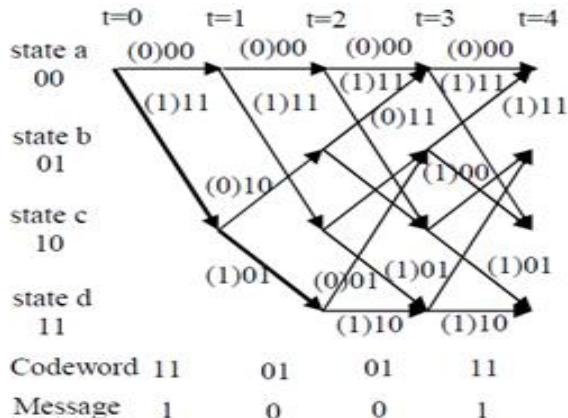
Figure 6: Trellis diagram for the code CC(2,1,3)

VII. SIGNAL TO NOISE RATIO (SNR)

The most direct factor to affect the performance of the FEC technique is the signal to noise ratio. For a fixed received signal power, noise level can be represented by the signal energy per bit to the noise power spectral density (*E/N*). As *E/N* increases, the noise level decreases. Thus, the performance of the FEC technique improves or the bit error rate (BER) decreases as *Eb/N0* increases.

To analyze how the noise level affects the performance of the FEC technique, a number of simulation tests were conducted using the Matlab codes. The number of input bits considered was 10000. These input bits were encoded using a rate 1/2 and a rate 1/3 convolutional encoder. The encoder was also tested for values of $m = 2$ and $m = 6$ for both the code rates.

VIII. FUTURE WORK

For years, convolutional encoding with Viterbi decoding has been the predominant FEC technique used in space applications, particularly in geostationary satellite communication networks, such as VSAT (very small aperture terminal) networks [3]. These networks most commonly use a rate 1/2 convolutional encoder with memory size of 6 and can transmit signals with at least 5 dB less power. This makes it useful in reducing transmitter and/or antenna cost or permitting increased data rates given the same transmitter power and antenna sizes.

If we further supplement the FEC technique with Reed-Solomon coding in the geostationary satellite communication arena, one can transmit signals with even lower power. The two coding techniques are implemented as serially concatenated block and convolutional coding, i.e., concatenated Reed - Solomon coding and convolutional coding with Viterbi decoding. Typically, the information to be transmitted is first encoded with the Reed - Solomon code, then with the convolutional code. On the receiving end, Viterbi decoding is performed first, followed by Reed – Solomon decoding. This is the technique that is used in most if not all of the direct broadcast satellite (DBS) systems, and in several of the newer VSAT products as well[9].

IX. SIMULATIONS

This section covers results/simulations done on MATLAB. Analysis of performance of the codes is done in terms of BER and E_b/N_o .Modulation scheme used in these simulations is BPSK. This section covers the BER plots for coded and un-coded data of bits. Then a comparison is made between when different code rates are used.

For Encoder:-

A convolutional coding is done by combining the fixed number of inputs bits. The input bits are stored in the fixed length shift register and they are combined with the help of mod-2 adders. This operation is equivalent to binary convolution and hence it is called convolution coding. Convolutional coding and Viterbi decoding, along with binary phase-shift keyed modulation, is presented as an efficient system for reliable communication on power limited satellite and space channels. Performance results, obtained theoretically and through computer simulation, are given for optimum short constraint length codes for a range of code constraint lengths and code rates.[4]
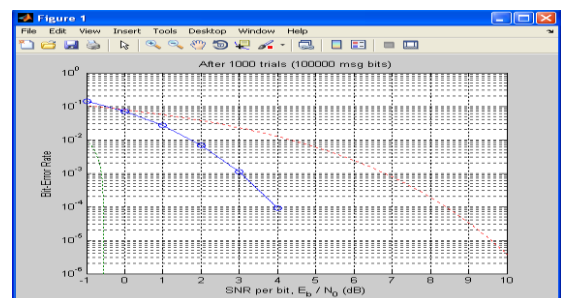
Figure 7: Plot of BER vs E_b/N_o for various combinations of the encoder

For Hard Decision Viterbi Decoding:-

In the hard-decision decoding, the path through the trellis is determined using the Hamming distance measure. Thus, the most optimal path through the trellis is the path

with the minimum Hamming distance. The Hamming distance can be defined as a number of bits that are different between the observed symbol at the decoder and the sent symbol from the encoder. Furthermore, the hard decision decoding applies one bit quantization on the received bits.
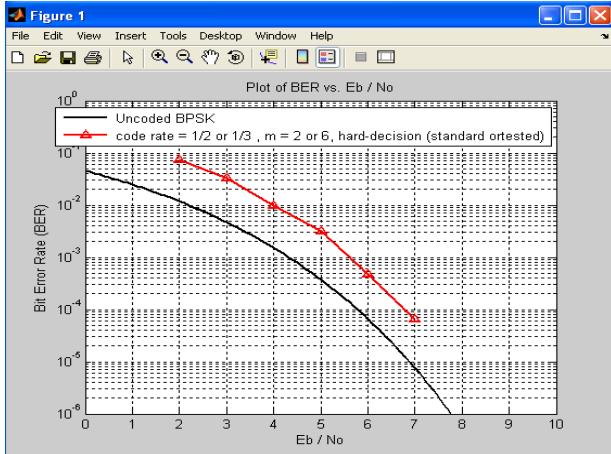

Figure 8: Plot of BER vs E_b/N_o for various combinations of the encoder, with hard decision decoder

For Soft Decision Viterbi Decoding:-

In the Viterbi Algorithm, there are two ways to calculate the distance to choose a most likelihood path. One is hamming distance which is related to the hard decision. The other one is Euclidean distance related with soft decision. Soft-decision decoding is applied for the maximum likelihood decoding, when the data is transmitted over the Gaussian channel. On the contrary to the hard decision decoding, the soft-decision decoding uses multi-bit quantization for the received bits, and Euclidean distance as a distance measure instead of the hamming distance. The demodulator input is now an analog waveform and is usually quantized into different levels in order to help the decoder decide more easily. A 3-bit quantization results in 8 output values.
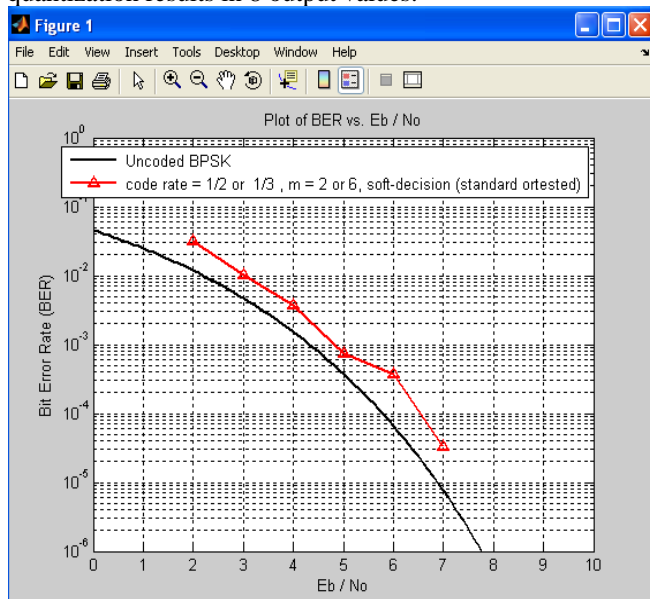

Figure 9 : Plot of BER vs E_b/N_o for various combinations of the encoder, with a soft decision decoder

X. CONCLUSIONS

The forward error correction technique (FEC) technique is a technique, particularly suited for a white Gaussian noise (AWGN) channel, which has been implemented using convolutional encoding with Viterbi decoding. The encoding process was demonstrated using a (2, 1, 3) convolutional encoder. A 3-bit input stream was encoded as an example to show the working of this encoder. This process was shown and verified both theoretically[6].

To study how these performance factors affect the working of the FEC technique, the encoder for the FEC technique was considered for different values of code rates and the number of memory registers. The code rates considered were 1/2 and 1/3 and the number of memory registers considered were 2 and 6. These code rates were used with each of the two values of memory[3]. The decoder for the FEC technique was then tested with both a hard decision Viterbi decoder and a soft decision Viterbi decoder, for each one of the combinations considered at the encoder. These combinations included both a standard set of generator polynomials as listed in [1] and another set, which we found, by a trial and error method through computer simulations.

REFERENCES

1. B.Sklar, *Digital Communications Fundamentals and Applications*, 2nd edition ,Prentice Hall, Upper Saddle River, New Jersey, 2001.
2. A. J. Viterbi, "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm,"*IEEE Transactions on Information Theory*, vol.IT-13, pp. 260-269, April, 1967.
3. J.A.Heller and I. M. Jacobs, "Viterbi Decoding for Satellite and Space Communications,"*IEEE Transactions on Communication Technology*, vol. com-19, pp. 835-848, October 1971.
4. J.G.Proakis,*Digital Communications*, 3rd edition, WCB/McGraw-Hill, Boston,Massachusetts, 1995.
5. S.Haykin, *Communication Systems*, 3rd edition, John Wiley & Sons, New York,1994.
6. L. W. Couch, II,*Digital and Analog Communication Systems*, 4th edition, Macmillan Publishing Company, New York, 1993.
7. T . Mc Dermott, *Wireless Digital Communications: Design and Theory*, Tuscon Amateur Packet Radio Corporation, Tuscon, Arizona, 1996.
8. M.S. Roden, *Digital Communication Systems Design*, Prentice Hall, Englewood Cliffs, New Jersey, 1988.
9. S.Lin and D.J. Costello, *Error Control Coding*, Prentice Hall, Englewood Cliffs,New Jersey, 1982.
10. A.M. Michelson and A. H. Levesque, Error*Control Techniques for Digital Communication*, John Wiley & Sons, New York, 1985.
11. V.Pless, *Introduction to the Theory of Error-Correcting Codes*, 3rd edition, John Wiley & Sons, New York, 1998.
12. K.J. Larsen, "Short Convolutional Codes with Maximal Free Distance for Rates1/2, 1/3, and 1/4,"*IEEE Transactions on Information Theory*, vol. IT-19, pp. 371-372, May 1973.
13. G.C.Clark, Jr., and J. B. Cain,*Error-Correction Coding for Digital Communications*, Plenum Press, New York, 1981.
14. John.M.Geist, "Viterbi Decoder Performance in Gaussian Noise and PeriodicErasure Bursts,"*IEEE Transactions on Communications*, vol. com-28, no. 8, pp.1417-1422, August 1980.
15. J.K.Omura, "On the Viterbi Decoding Algorithm," IEEE*Transactions onInformation Theory*, vol. IT-15, pp. 177-179, January 1969.