Robust Stability Analysis with Time Delay using PI Controller

Gursewak Singh

Abstract- In this paper, real time implementation of PI and PID controllers is Investigated for stabilizing an arbitrary-order plant or system. Simple closed loop feedback ,PI controller can overcome all the problems of uncertainties like time delay in LTI (Linear Time Invariant) system.This paper will give us an implortant alogorithm to be imlemented in MATLAB for achieving the robust stability in LTI plant in presence of any type of perturbations or external disturbances. In this paper, a single input and single output (LTI) linear time invariant plant under perturbed conditions with additive uncertainity weight sensitivity is considered. In this paper especially, I have found all set of PI gains Kp,Ki which will promise us for the stability in a given disturbed (SISO) (LTI) system by achieving the robust stability constraint $\left|\frac{W(s)}{1+Gp(s)K(s)}\right|$<=1 can be found by obtaining the PI controller gains Kp,Ki using frequency domain analysis .

Keywords- PI Controller, Time delay, Robust Stability, Uncertainity weight, frequency domain.

I. INTRODUCTION

In today technologies, Robust control is a very high standard technology that help us to bring the improvement in stability and performance of a perturbed system. It has capability to cover all sets of uncertainties (structured or unstructured type) and to accommodate fault tolerance of arbitrary order plant. Time delay in LTI plant have increased burden on scientists and engineers because any time delay in plant which try to decrease the speed and actual performance of the system.For achieving the necessary requirements and robustness in system, I will design here an algorithim for tuning of PI controller which will accommodates all types of delay problems and uncertainties.To implement the robust control of systems in presence of uncertainty, the absence of the appropriate and valid techniques or algorithims for Robustness in system remain always a major hurdle . This hurdle can not give us right and best platform for the best performance and stability for robust control which cause a big challenge that prevents robust control from being implemented in safe manner. The applications of frequency domain analysis in this tuning technique decreases the complexities of plant modeling.In this paper, PI controller gains calculations are done which will provide us robust stability. Mr.S.P.Bhattacharyya and their colleagues used a mathematical generalization of the Hermite-Biehler theorem to find all stabilizing PID controllers for systems with time-delay [1].

Recently, a large amount of research has focused on finding the set of all stabilizing arbitrary-order controllers using frequency domain analysis as a result of this practical motivation[2,4,7,8].

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For a given arbitrary-order single input and single output LTI plant with variations of time delay , I proposed an technique to achieve the complete stabilizing set of PI controllers (Kp,Ki) which can be judged as best way for PI controller design using frequency domain analysis with extension of author on basis of earlier explained calculations and results [9]. I proposed only frequency domain stability analysis with time delay. Based on the result, this paper provide a direct parametric design method for PI controllers. The controller design problem is first converted into simultaneous all set of simple quasi-polynomial equation and then the entire set of all PI gains are found that can guarantee the stability of LTI system and the closed-loop characteristic equation is found. Robust stability criterian $\left|\frac{W(s)}{1+Gp(s)K(s)}\right|<1$ and some mathematical calucluation are discussed to achieve PI controller gains Kp and Ki in different planes (Kp,Ki) in section-VI.

In Figure 1 , consider a linear time invariant (LTI) system single input and single output with additive uncertainty where

Nominal plant is *Gp(s)* , PI controller is *K(s).*

The uncertainity weight. is $W_A(s)$, The input signal is *R(s)* , The output signal is *Z(s)* respectively.

The perturbed plant is $G\Delta(s)$ which includes $\Delta_A(s)$, which is any stable transfer function such that $|\Delta_A(s)| \le 1$, Δ = "uncertainty", or "perturbation" (unknown). *Gp(s)* is nominal plant $G\Delta$ is perturbed or uncertain plant because of uncertainity $\Delta_A(s)$.$W_A(s)$ is just like filter or scaling factor used to cover entire uncertainity, *K(s)* is PI controller .

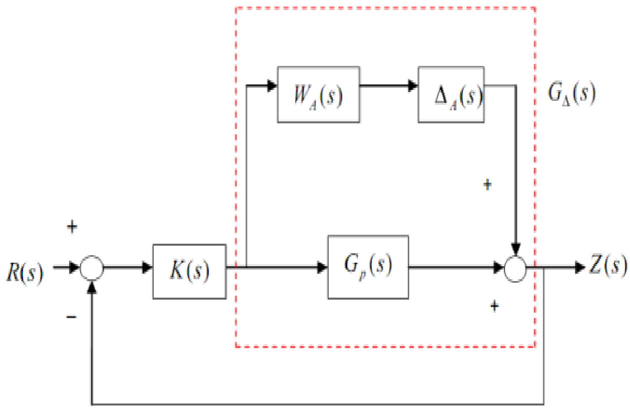

Figure 1: Block diagram of system with unstructured uncertainty weight [4]

So for maintaining Δ_A uncertainity , we have to multiply $W_A(s)$ with $\Delta_A(S)$, Now $G\Delta(s) = Gp + \Delta_A(s) W_A(s)$ s).In figure 1,The SISO LTI plant with (τ) time delay can be described as

$$Gp(s) = \frac{N(s)}{D(s)} e^{-j\tau s}$$
(1)

whereas $N(s)$ and $D(s)$ are coprime polynomials in s, defined as

$N(s)=V_m s^m + V_{m-1} s^{m-1} + \dots\dots\dots + V_1 s + V_0$
$D(s)= s^n + U_{n-1} s^{n-1} + \dots\dots\dots + U_1 + U_0$
(2)

here V_0, V_1, V_m and U_0, U_1, U_{n-1} are real numbers, and $n>m$,

$K(s)$ is PI controller with form

$K(s)= Kp+\frac{Ki}{s}$ here $s \rightarrow j\omega$,

Write PI controller as given below.

$K(j\omega)= Kp+\frac{Ki}{j\omega}$
(3)

II. SENSITIVITY CONCEPT [5]

1. Sensitivity:

Uncertainties in LTI plant give parameter variations that affect the desired performance of a control system. The utilization of closed loop feedback in a control system decreases the effect of parameter variations. The term sensitivity linked to a control systems gives us an information of the system performance as affected due to parameter variations. All these uncertainties in open-loop system will result in low performance and in-accuracy in output. However, a feedback closed-loop system can destroy this disadvantage. From figure-2 is sensitivity function written below for closed loop [5]

$S(j\omega) = 1/(1+Gp(jw)\ K(jw))$
(1)

2. Robustness:

The stability of LTI (SISO) linear time invariant (Single Input and Single Output) in presence of perturbed conditions conditions is called robustness .A primary advantage of a closed-loop feedback for control system is its capability to reduce the system 's sensitivity to nonlinearities ,disturbances and perturbed conditions(see figure-2) [5].

Sensitivity analysis → Robust control

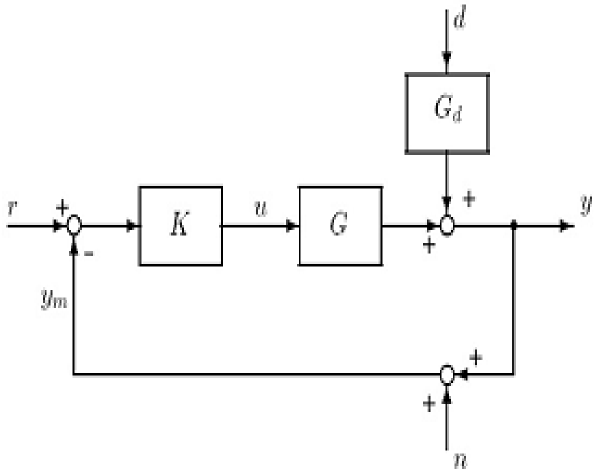

Figure 2 System G under effect of disturbance or perturbed conditions [6]

In Figure 2, *r* is desired force, *d*, *n* is disturbance and noise, *G* is nominal plant , *K* is PID controller ,*y* is actual force

III. DEFINATIONS FOR CLOSED LOOP

Robust stability (RS): system stable for "all" perturbations Δ

Robust performance (RP): system satisifies performance requirements for "all" perturbations.

IV. WEIGHT SELECTION

So, Sensitivity must start to decrease to external environment conditions. So We will have to increase the value of $W_A(s)$ uncertainity weight for decreasing sensitivity to unwanted disturbance. It means $W(s)$ become inversily proportional to sensitivity

$W_A(s) \propto 1/S(s)$

After eliminating Proportional sign

$W_A(s) = \gamma / S(s)$ or

$W_A(s)^* S(s) = \gamma$.

Condition 1 : consisder γ is as value 1 for robust stability condition.In this case, System will be controlled by PI controller as well as uncertainity Weight $W_A(s)$,

Now Criteria finally for Robust stability is magnitude of multiplication of W(s) and S(s) must be less than or equal to γ. $\|W_A(j\omega)^* \quad S(j\omega) \quad \| \quad <\gamma$
(2)

The goal of the paper is to determine the set of (Kp, Ki) for which satisfies the following H∞ performance index,From equation 1 and 2,Criterion $\left|\frac{W(s)}{1+Gp(s)K(s)}\right|<1$ for robust .

V. PRELIMINARY KNOWLEDGE OF THEOREMS

Lemma 1 *consider that*

$P(s)(\ U(s)\,e^{\tau s} + E(s))=Q(s)$ (3)

P(s) is stable where Q(s) , U(s) and E(s) are polynominals respectively with degree $[Q(s)] = \vartheta$,degree $[U(s)] =\mu$, degree$[E(s)]=\rho$, $\vartheta \le \mu$, $\vartheta \le \rho$, *c,d and j* are the highest order coefficient of Q(s), U(s), E(s) ,respectively.

The Inequality $\|P(j\omega)\|$ <1 holds if and only if

1. $\|c\|>\|j\|$ if $\mu > \rho$, $\|d\|>\|j\|$ if $\mu < \rho$ or $\|c+d\|> \|j\|$ if $\mu=\rho$
2. $[U(s)\,e^{\tau s} + E(s)]+e^{\tau s}Q(s)$ is stable for all θ in$[0,2\pi]$ lemma 1 proof is given in [1,8] that is omitted here delebrately.

Let us view the about Hermite –Biehler theorem [1,8]

Consider the quasipolynomial

$H(s) = \sum_{i=0}^{m}\sum_{j=0}^{n} b_{ij} \quad s^j \quad e^{s\theta_i}$
(4)

where $b_{mn}\neq0$ *m and n* is positive integers, b_{ij} is real or complex number ,and $\theta_0,\theta_1,\dots\dots\dots\theta_m$ are real numbers satisfying $0<\theta_0<\theta_1<\dots\dots\dots\theta_m$. The term $b_{mn}s^n e^{s\theta_m}$ is the principle term.For stability of quasiupolynomial H(s) in equation (4) ,the extended Hermite –Biehler theorem is explained as follows.

Theorem 1 [1,8].H(s) in equation (4) is stable if and only if

1. $H_p(j\omega)$ and $H_q(j\omega)$ have onl real zeros and these zeros interlace;
2. $H_q'(j\omega)\ H_p(j\omega)\ -H_q(j\omega)\ H_p'(j\omega) > 0$ for some$\omega\epsilon(-\infty,+\infty)$.

Here , $H_p(j\omega)$, $H_q(j\omega)$, $H_p'(j\omega)$, $H_q'(j\omega)$ denotes the real and imaginary parts of H(jω) and their first derivatives with respect to ω,respectively.

Theorem 2 [1,8]. Imagine that all θ_i 's values in (4) are intergers and let $\jmath$ si constant so that the coefficients of the highest degrees term in , $H_p(j\omega)$, $H_q(j\omega)$ do not destroy at $\omega = \jmath$. The required and sufficient conditions under which $H_p(j\omega)=0$ or $H(j\omega)=0$ has only roots is that ,in the interval $-2g\pi+\jmath\le\omega\le2g\pi+\jmath$, , $H_p(j\omega)$ or $H_q(j\omega)$ has exactly $4g\theta_m+n$ real zeros starting with sufficiently large g ,respectively.

A quasipolynomial with general forms .

$$G(s) = D(s)e^{\theta s} + N(s) \qquad (5)$$

Where N(s) and D(s) are polynomials given by

$$N(s) = (\rho_n+j\sigma_n)\,s^n + (\rho_{n-1}+j\sigma_{n-1})\,s^{n-1} + \ldots\ldots(\rho_0+j\sigma_0)\,s^0$$

$$D(s) = s^m + D_{m-1}\,s^{m-1} + \ldots\ldots D_1\,s + D_0$$

Here ρ_0 , $\rho_1 \ldots\ldots\ldots \rho_n$, $\sigma_0, \sigma_1 \ldots\ldots \sigma_n$, D_0 , $D_1 \ldots\ldots\ldots D_{m-1}$ are the real numbers and m>n.

Theorem 3 [1 ,8] if m>n ,G(s) in equation (3) is stable if

(i) $G_g(z)$ has exactly 4g+m real zeros in $(-2g\pi-\varepsilon, 2g\pi-\varepsilon$;

(ii) All the zeros of $G_i(z)$ in $(-2g\pi-\ \varepsilon, 2g\pi-\varepsilon)$ interlace with those of $G_r(z)$

Here $z=\theta\omega$, g is a sufficient large integer , $G_r(z)$ and $G_i(z)$ are respectively ,the real and imaginary parts *of* $H(\frac{jz}{\theta})$ and ε is given by

$$\varepsilon = \begin{cases} -\frac{\pi}{2} \quad if\ m\ even\ and\ \rho_n\ \sigma_n = 0 \\ 0\ if\ m\ odd\ and\ \rho_n\ \sigma_n = 0 \\ \arctan\left(\frac{-\sigma_n}{\rho_n}\right) or\ \pi + \arctan\left(\frac{-\sigma_n}{\rho_n}\right)\ if\ m+n\ even \\ \arctan\left(\frac{\rho_n}{\sigma_n}\right) or\ \pi + \arctan\left(\frac{\rho_n}{\sigma_n}\right)\ if\ m+n\ odd \end{cases}$$

In interval [-π/2 ,π/2] in order to present the values if PI gains

(Kp,Ki).

VI. DESIGNING PROCEDURE

Now we will calculate the kp and ki gains with procedure given below. After calculating kp,ki gains , I will plot two regions: Robust region and Unstable region.

From equation (2), It can be re-written as follows,

$$\left|\frac{(W_A)}{1+G_p(s)K(s)}\right| << 1 \qquad (7)$$

Following is polynomial closed loop charactersitics equation

$$\delta\ (\theta,s,kp,ki)=0 \qquad (8)$$

Equation (7) can be re-written as follows

$$\frac{(W_A)}{1+G_p(s)K(s)} = \left|\frac{(W_A)}{1+G_p(s)K(s)}\right|\left(e^{j\theta}\right) \qquad (9)$$

$$\frac{W_A(e^{-j\theta})}{1+G_p K(s)} << 1 \qquad (10)$$

Where $\theta = \frac{(W_A)}{1+G_p(s)K(s)}$ in equation 10

Put equation (10) into (8)

$$\delta\ (\theta,s,kp,ki) = 1 + G_p(s)K(s) - W_A(e^{j\theta}) \qquad (11)$$

Closed loop characterstic equation can be found putting equation (1) and (3) in equation (11) as in following mannner.

$$\delta(\theta,s,kp,ki) = \{s+(s*kp+ki)G_p(s)\}\}-\{(sW_A\,e^{j\theta}\}=0 \qquad (12)$$

From equation (12) ,$\delta(\ \theta,s,kp,ki)=0$ must satisfy the robust stability constraint in equation (7) for all boundary of θϵ [0,2π] in kp and ki region.

Put $W_A=(e+jf)$ is uncertainity weight for unstrucured uncertainity and $G_p(s)$ plant $=\partial+j\sigma$, ∂,e is real part of plant and uncertainty weight and σ, f is imaginary part of plant and uncertainity weight , $e^{j\theta}=$ cosθ+jsinθ and kp,ki is proportional and integral controller ,$s\rightarrow j\omega$ in equation (12) and break equation (12) into real and imaginary part.

kp(r)+ki(s) = -1-e*cosθ+f*sinθ (13)

kp(u)+ki(v)= -e*cosθ+f*sinθ (14)

where

$$h=-1+e*\cos\theta-f*\sin\theta$$
$$i=-e*\sin\theta+f*\cos\theta$$
$$r=\partial$$
$$u=\sigma$$
$$s=\frac{\sigma}{\omega}$$
$$v=\frac{-\partial}{\omega}$$
(15)

Find kp and ki ,apply matrix theory in above equation (13) and (14) to find kp and ki.

$kp=\frac{i*s-h*v}{u*s-v*r}$,$ki=\frac{h*r-i*u}{v*r-u*s}$ put all equation of (15)

I have found kp,ki gains to find robust stability region

VII. STEPS FOR IMPLEMENTATION OF PID TUNING

Step1: Determine nominal system is stable, if yes move to step 2

Step 2: Select stable W_A (s) uncertainity weight for according to perturbed plant with time dealy variation and nomianl plant

See figure -1 $G\Delta(s) - Gp(s) = \Delta(s)\ W_A(s)$ (16)

Where GΔ is perturbed or uncertain plant with time delay under uncertain condition , Gp(s) is nominal plant without any uncertainity , Δ(s) is stable transfer function

$\Delta(s) <= 1$

(17)

From equation (16) and (17) ,$W_A(s) \geq G\Delta(s) - Gp(s)$ (18)

$W_A(s)$ must be greater than and equal to difference or mismatch between perturbed and nominal plant which will cover entire uncertainity of sytem gain variations.

Step 3 Select Condition for robust stability given below it must satisfy.

$$\frac{(W(j\omega)\, e^{\wedge}j\theta)}{(1+Gp(j\omega)K(j\omega))} < 1$$
(19)

Step 4 Select Angle for obtaining intersection of lines of magenta colour .$\theta_A \in [0, 2\pi]$ in figure-3.

Step 5 Check robust stability using frequency domain analysis.

VIII. RESULTS AND CALCULATIONS

If ,consider $G(s) = \frac{s+3.1}{(s^{\wedge}3+3.2*s^{\wedge}2+4.2*s+3)}$ (20)

$$Gp(s) = G(s) * e^{-0.9s}$$
(21)

Time delay *(A)* is 0.9s

Put equation (20) into (21) ,obtain final equation with time delay given below

Plant $Gp(s) = \frac{s+3.1}{(s^{\wedge}3+3.2*s^{\wedge}2+4.2*s+3)} e^{-0.9s}$ (22)

In this paper unkown time delay range is consisdered as

tau ϵ [0.1 , 0.17] (23)

From equation (18) and (23), we can select uncertainity weight as follows

$$|W_A(s)| \geq \left| \frac{s+3.1}{(s^{\wedge}3+3.2*s^{\wedge}2+4.2*s+3)} \right| |tau - 0.9|$$
(24)

Plot robust region (white) and unstable region (coloured) to meet robust stability criteria in following figure 3

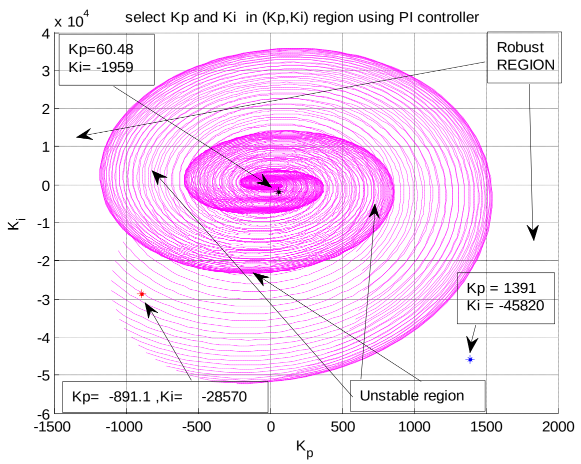

Figure-3 PI controller Tuning in (Kp,Ki) plane for value of Kdd=0

As per selected K1(s) and K2(s) and K3(s) PI controllers and WA(s), see robust stability criterion..For seleced K1(s) inside unstable region, the robust stability is not met here .But as per selected K2(s) on the robust region, Sytem will achieve robust stablity .But as per selected K3(s) on the unstable region, Sytem will not achieve robust stablity .

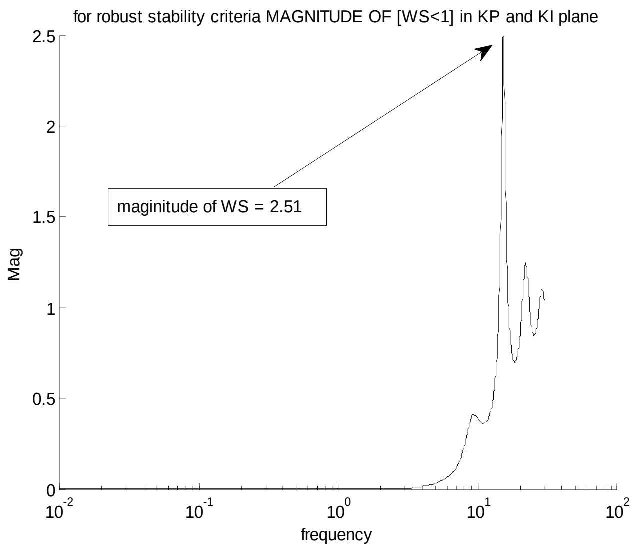

Figure-4:Magnitude of WS in (Kp,Ki)

Now we have selected PI K1(s) controller in figure-3(see black colour point). On selection of this PI controller we are not able to achieve robust stability criteria(see figure-4).Because, after putting the equations (22,24) and K1 PI controller into equation (7), Magnitude of equation (7) become more than one.

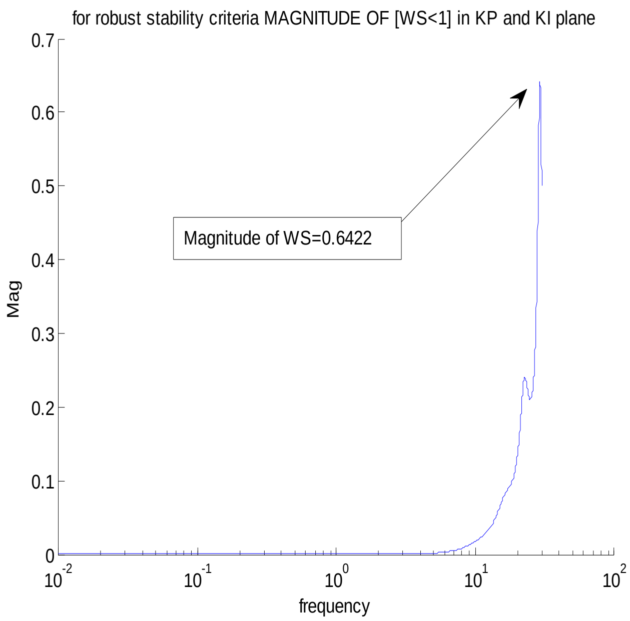

Figure-5:Magnitude of WS in (Kp,Ki)

Now we have selected PI K2(s) controller in figure-3 (see blue colour point). On selection of this PI controller, we are able to achieve robust stability criteria(see figure-5). Because, After putting the equations (22,24) and K2 PI controller into equation 7,Magnitude of equation (7) become less than one

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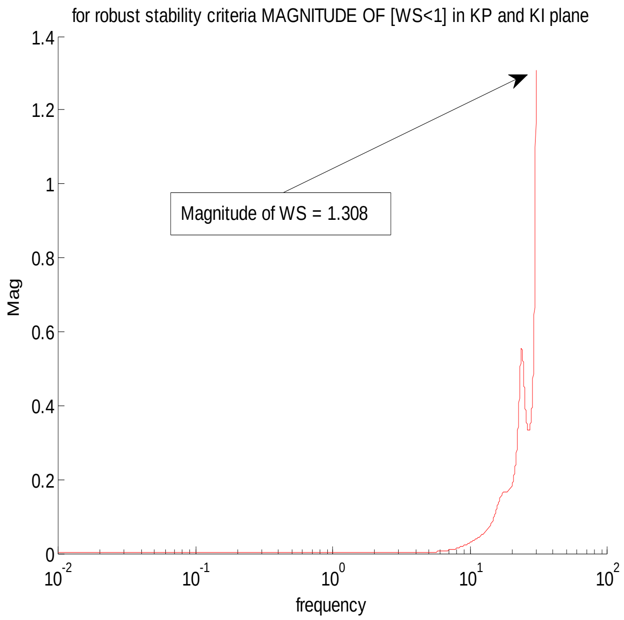

Figure-6:Magnitude of WS in (Kp,Ki)

Now we have selected PI , K3(s) controller in figure-3(see red colour point). On selection of this PI controller we are not able to achieve robust stability criteria(see figure-6). Because, after putting the equations (22,24) and K3 PI controller into equation (7), Magnitude of equation (7) become more than one.

Table 1. Robust performance for (Kp,Ki Plane)

Select from figure-3, K1(s) and K2(s) and K3(s) PI controllers (Kp,Ki) plane **Weight Selected** $WA(s) = \frac{s+0.3}{s+6}$	**Robust criterion is met or not ,must be** $\left\lvert \frac{WA(s)}{1+Gp(s)K(s)} \right\rvert$ **<1 and** **Plant with time delay =0.9sec and** $Gp(s) = \frac{s+3.1}{(s^3+3.2*s^2+4.2*s+3)} e^{-0.9s}$
K1(s)= 60.48- $\frac{19.59}{s}$ See figure-3	No (See figure- 4) and $\left\lvert \frac{WA(s)}{1+Gp(s)K1(s)} \right\rvert$ = 2.51 NO ROBUST STABILITY
K2(s)= 1391- $\frac{45820}{s}$ See figure-3	Yes(See figure- 5) and $\left\lvert \frac{WA(s)}{1+Gp(s)K2(s)} \right\rvert$ =0.642 YES ROBUST STABILITY
K3(s)= -891.1- $\frac{28570}{s}$ See figure-3	No (See figure- 6) and $\left\lvert \frac{WA(s)}{1+Gp(s)K3(s)} \right\rvert$ =1.308 NO ROBUST STABILITY

From Table 1 given above, observe robustness with selection of K1 and K2 and K3 PI controllers from figure-3. Robust criterion $\left\lvert \frac{W(s)}{1+Gp(s)K(s)} \right\rvert$ <1 can be achieved with selection of PI controller from constructed robust region in figure-3 ,look at figure-4,5,6.

IX. CONCLUSION

In this paper ,Graphical design method for obtaining all PI controllers that will satisfy a robust stability constraint for a LTI (Single input Single output system) with time delay variations have been obtained.Observing the results in Sections VIII for (Kp,Ki) plane , it can be concluded that the PI controllers selected from the robust stability regions in the planes satisfy the robust performance criterion under uncertain conditions.

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