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Parameter Estimation of Warranty Cost Model using Genetic Algorithm

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Abstract*— Parameter estimation can also be classified as an optimization where the objective is to find the values of some unknown parameters in which the objective function is minimized or maximized. Several methods can be used in estimating parameter including Genetic Algorithm (GA). GA has been widely used in many applications specifically in optimization problem. This paper propose GA as a method to estimate the parameter of warranty cost model with warranty claim data collected from Malaysian automotive industry. Various combinations of GA operators are carried out. Maximum Likelihood Estimation (MLE) method is employed in order to have a comparable solution for the model parameters of warranty cost. The performance of GA in warranty cost model is measured based on Residual Sum of Squares (RSS) and Mean Squared Error (MSE).

Index Terms*— Genetic Algorithm, parameter estimation, likelihood function, warranty.

I. INTRODUCTION

In mathematical model, parameter estimation is required in identifying the best value of unknown parameters. Parameter estimation for warranty cost model presented by Hairudin and Scarf (2011) was achieve through Maximum Likelihood Estimation (MLE). Sometimes, it is difficult to solve the optimal value analytically. In that case, we are motivated to propose an appropriate approach in parameter estimation for the same case study.

Based on previous findings, we found that many studies proposed GA as a method to estimate parameter. For example, Rashid *et. al* (2005) discussed a comparison of performance of GA and Gauss-Newton method in nonlinear kinetic parameter estimation. The performance shows that GA gives a better result in finding the global optimum solution. Parameter estimation has also been performed in different combination of probability of crossover and mutation. Halim *et. al* (2009) carried out 15 combination of crossover and mutation operator in GA for parameter estimation of STARMA model. The indicator of the best combination operator is measured based on the minimum value of RSS. In addition, Abutaleb (1997) also performed various crossover and mutation probability in estimating parameter of sinusoids in a noisy environment. The result of different combination operators gives effect on accuracy of the estimate values.

GA has been frequently used for parameter estimation in many studies. However, the application of GA for determining the parameters of the warranty cost model has less been explored. The purpose of this paper is to estimate the parameters of warranty cost model using GA and to compare the results obtained with MLE method. This paper is organized as follows. Section 2 describes a brief framework of this paper. Then, the discussion of likelihood function for parameter estimation is described in section 3. Section 4 presents the procedure of estimating parameter using GA followed by section 5 which presenting experimental results and some discussion. Section 6 provides the conclusions drawn from this paper.

II. FRAMEWORK

Figure below shows the framework of study. Framework is very useful to outline the stages of activities to be carried out in this paper.

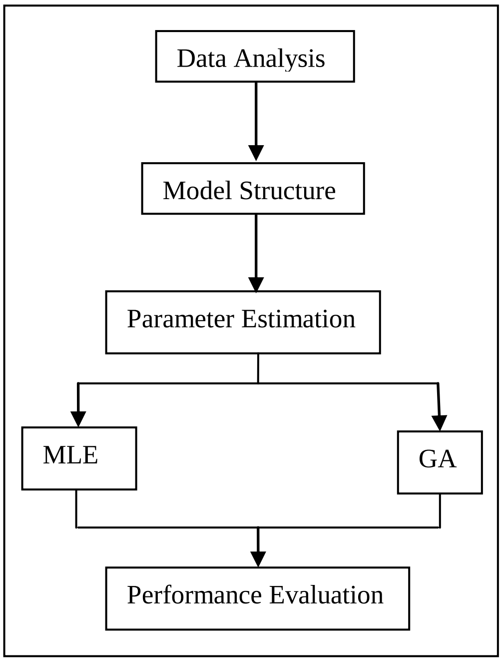

Figure 1: Framework of study.

Fig. 1 shows a brief framework of this paper. Firstly, we start with analyzing the warranty claim data. Then, a model structure is needed in order to estimate parameters. According to Meran (2006), the model structure indicates the goodness of solution. The parameters of the model structure are estimated using GA. The parameter values are compared with MLE. Lastly, the performance evaluation is measured based on the Residual Sum of Squares (RSS) and Mean Squared Error (MSE). A small value of RSS and MSE signifies a better result.

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III. LIKELIHOOD FUNCTION FOR PARAMETER ESTIMATION

In this paper, the modeling of warranty cost is based on real data obtained from Malaysian Truck and Bus (MTB) Company. The data set cover five-year period that is from 1998 to 2002. We employ the model proposed by Hairudin and Scarf (2011) which consider a Non-Homogeneous Poisson Process (NHPP) model with asymptotically defect arrivals, $\lambda(u)$ and mixed exponential distribution, $f(h)$. The equation of asymptotically defect arrivals is as follows:

$$\lambda(u) = \frac{\lambda_0 + \lambda_1 u}{1 + u}$$

where

$\lambda_0 =$ Initial value of failure rate at time zero

$\lambda_1 =$ Steady state defect rate

Basically, there are two categories of delay time distribution which are exponential and mixed exponential distribution. Hairudin and Scarf (2011) introduce the mixed exponential distribution since zero delay time may occur in some defects. As a result, they found that the mixed exponential models is better than ordinary exponential. A likelihood function for NHPP model with asymptotically defect arrivals is as follows:

$$L = \sum_{i=1}^{N}\{-\log(n_i+1)! - \int_{s_i}^{t_i}\frac{\lambda_0+\lambda_1 u}{1+u}du\} + \sum_{i=1}^{N}\log\{p\lambda(t_i) + (1-p)\int_0^{t_i-s_i}(\frac{\lambda_0+\lambda_1(t_i-v)}{1+t_i-v})\beta e^{-\beta v}dv\}$$
$$+\sum_{i=1}^{N} n_i\log\{(1-p)\int_0^{t_i-s_i}(\frac{\lambda_0+\lambda_1(t_i-v)}{1+t_i-v})e^{-\beta v}dv\} + \sum_{j=1}^{M}\{-\log(m_i)!$$
$$-\int_{S_i}^{T_i}\frac{\lambda_0+\lambda_1 u}{1+u}du\} + \sum_{j=1}^{M} m_i\log\{(1-p)\int_0^{T_i-S_i}(\frac{\lambda_0+\lambda_1(t_i-v)}{1+t_i-v})e^{-\beta v}dv\}$$

where

λ_0 = Initial value of failure rate at time zero

λ_1 = Steady state of defect rate.

β = Mean of delay time

p = Probability of failure with zero delay time

Based on this model, there are four unknown parameters need to be estimated in order to obtain optimal value of warranty cost which are λ_0, λ_1, β and p. Therefore, an appropriate approach is proposed in order to obtain the optimal parameter values. The application of GA in solving nonlinear estimation problem performed by Ozturkler and Altan (2008) resulting close to the optimal solution. Since GA has been proved to be robust approach on estimating parameters on nonlinear problem, this paper proposes GA as an appropriate approach to solve the estimation problem. The fitness function for estimation is the likelihood function.

IV. PARAMETER ESTIMATION USING GENETIC ALGORITHM

In this paper, GA is used to obtain optimal values of parameters for the warranty cost model. This section discusses a general description and procedure of GA method in parameter estimation. The parameter estimation is performed using GA method since it is effectives in finding global optimum solution without being trapped at local minima. The application of GA has been widely use and the benefits of using GA technique are clear and well-known.

GA is one of the useful techniques in Artificial Intelligence which is based on the natural evolution in order to solve search and optimization problem. GA is very effective in finding a reasonable solution to a complex problem. GA can also solve nonlinear estimation problem since it is difficult to solve analytically. This is supported by Abutaleb (1996) which proposed GA method rather than maximum likelihood in solving nonlinear estimation problem. According to Lee *et. al* (2006), GA is robust to nonlinearity and complexity in the model and thus gives more flexible and direct way of solving in parameter estimation.

The idea of GA was first introduced by John Holland in 1975. GA is an algorithm which makes it easy to search a large search space. In GA process, reproduction, crossover, and mutation are the three main operators that were used to find near-optimization of the problem. The fittest individual will survive more frequently and have high chances for reproduction. In this section, parameters of the models are estimated using the same data set. The information was extracted from a research made by Hairudin and Scarf (2011). The performance measure of the GA is minimum value of RSS and MSE. The simple steps of GA in parameter estimation of warranty cost model are explained as follows:

Step 1: At first generation (iteration) where t = 1, a population which contains 10 chromosomes is generated randomly. The random numbers ranged from 0 to 0.1. Each chromosome has 4 genes which represent the unknown parameters.

Step 2: Fitness function is evaluated for all chromosomes in the population. Fitness function is a function to be optimized. In this paper, the fitness function is a likelihood function which was maximized to give estimated values of parameters.

Step 3: Selection process is performed in which some chromosomes are chosen from the population to be parents. The selection operator is implemented in a number of ways. In this paper, we use Roulette-wheel selection method where a chromosome is selected from the population with a probability proportional to the fitness, F. The probability of the i^{th} selected string is

$$p_i = \frac{F_i}{\sum_{j=1}^{n} F_j}$$

where n is the population size.

Then, cumulative probability is required by adding the individual probabilities from the top of the list. In order to choose n chromosomes, n random numbers are created at random. Thus, the chromosomes that represent the random number lie between the ranges of cumulative probability are copied to the population. Thus, the chromosome with higher fitness value has a higher probability of being copied into the population.

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Step 4: After two parents are selected from the population, a crossover operation is performed. Here, we use one-point crossover method, which randomly select the cut point and exchanges the right parts of two parents to generate an offspring. Probability of crossover identifies number of pairs to be performed in crossover process. In this paper, we are using different combination of crossover probability. The probability of crossover is varied from 0.5 to 0.8.

Step 5: Mutation operator is performed in order to prevent the algorithm to be trapped in local minimum. Probability of mutation determines the number of mutations occurred in the population. We kept the probability of mutation at 0.1, 0.15 and 0.2. Then, old generation will be replaced by new generation.

Step 6: We consider maximum generation as the termination criteria. We set the maximum generation at 10. The old generation will be replaced by the new generation if the generation is more or equal to 10. Otherwise, breeding process is repeated until the termination criterion is fulfilled. When the termination criterion is fulfilled, the process has to be terminated and new generation is produced. Thus, the new generation gives optimal values of parameter.

Step 7: The generation index is increased by 1 where $t = t+1$. If the stopping criterion is not satisfied, go to step 3. Otherwise, stop the algorithm.

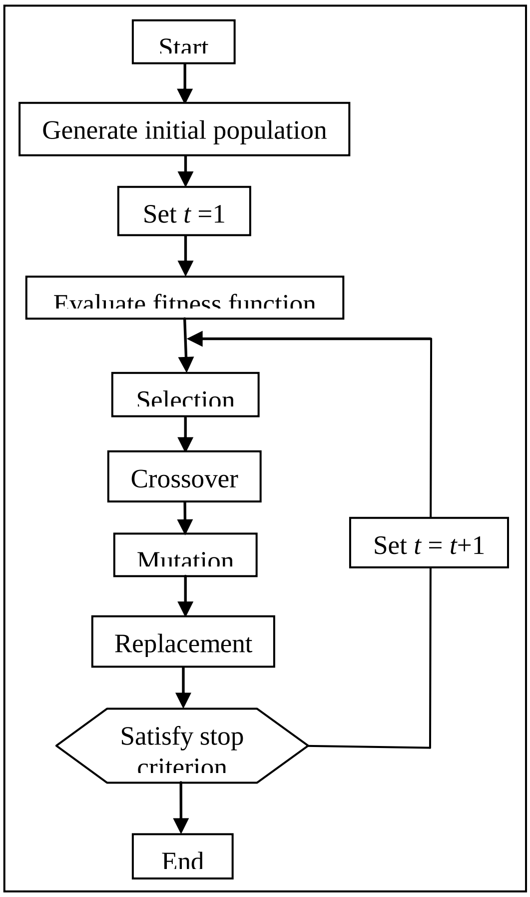

Figure 2: Flowchart of parameter estimation using GA.

Fig. 2 illustrates the flowchart of GA in estimating parameter of warranty cost model. The performance of the GA in estimating parameter is measured in the sense of the Mean Squared Error (MSE). The MSE is formulated as follows:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

where

n = number of parameters

y = actual parameter value

$\hat{y}$ = estimated parameter value

V. EXPERIMENTAL RESULTS

This paper attempts to estimate the parameters using GA and to compare the results with those calculated using MLE method. Different combination of GA operator was performed in estimating parameter values. We use 12 combination of GA operator for parameter estimation of warranty cost model. For each combination, GA process was carried out by using Matlab software to obtain minimum value of MSE. Each combination is evaluated through the minimum MSE. The results for combination of GA operator are shown in Table 1 below.

Table 1: Results of combination of GA operator.

Probability of crossover, *Pc*	Probability of mutation, *Pm*	MSE
0.5	0.1	0.00380
0.5	0.15	0.00208
0.5	0.2	0.00430
0.6	0.1	0.00254
0.6	0.15	0.00258
0.6	0.2	0.00272
0.7	0.1	0.00231
0.7	0.15	0.00352
0.7	0.2	0.00400
0.8	0.1	0.00283
0.8	0.15	0.00216
0.8	0.2	0.00201

Based on Table 1, it is clear that the combination of probability of crossover 0.8 and probability of mutation 0.2 gives the lowest value of MSE. This indicates that the combination of both probabilities is appropriate to be used in this case study to obtain a better output. The MSE of likelihood function is plotted against probability of crossover, *Pc* as illustrated in Fig. 3. It can be seen that there is an optimum combination of the two probabilities.

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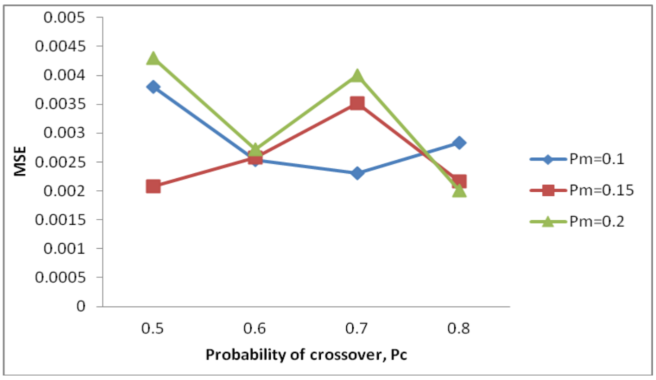

Figure 3: Plot of different combination of crossover and mutation probability.

In this paper, we use 10 generations with population size of 10 in GA process. The population size does not affect the minimum RSS as proved by Halim (2009). The results of estimated parameter values using GA and MLE is shown in Table 2 below.

Table 2: Parameter estimation results using GA and MLE

Parameter	Result			
	MLE		GA	
	Estimated value	Error	Estimated value	Error
λ_0	0.9339	0.9175	0.0360	0.0197
λ_1	0.0164	0.0093	0.0113	0.0042
β	0.0066	0.0023	0.0605	0.0314
p	0.1805	0.1514	0.0905	0.0816
RSS	0.86482		0.1368	

Table 2 shows the comparison on parameter values obtained by GA and MLE method. These results were the product of 10 generations with population size of 10, and a probability of crossover and mutation of 0.8 and 0.2 respectively. The outputs revealed that the estimated values of GA is more accurate compared to MLE since the RSS for GA is significantly smaller than MLE. Thus, the performance of GA is better than MLE method. However, the MSE values of both methods do not differ significantly.

VI. CONCLUSION

This paper provides a parameter estimation of warranty cost model using GA. Our findings show that the probability of crossover 0.8 and the probability of mutation 0.2 have been sufficient to achieve good results. The performance is evaluated based on minimum value of RSS and MSE. Also, we see that GA can perform better than MLE method since parameter values obtained by GA are more accurate compared to values obtained by MLE. Small error indicates a better result. As a conclusion, GA can be considered as an appropriate approach in solving parameter estimation of warranty cost model.

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