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Parameter Estimation in Wireless Sensor Networks Based on Decisions Transmitted over Rayleigh Fading Channels

Zhenxing Luo

Abstract— In this paper, we present a distributed estimation method in wireless sensor networks (WSNs) based on decisions transmitted over Rayleigh fading channels. The fusion centre can uses either coherent receiver or non-coherent receiver to acquire decisions transmitted over Rayleigh fading channels. The estimation method using coherent receiver and the estimation method using non-coherent receiver are presented and the Cramer-Rao lower bounds (CRLBs) are derived. Simulation results showed that in ideal situations, the RMS errors given by the distributed estimation method were close to the CRLB. Moreover, simulation results highlighted the importance of the number of sensors, channel SNR, and accurate channel SNR information known to the fusion centre on estimation performance.

Keywords— Wireless sensor networks, maximum likelihood estimation, distributed estimation, Cramer-Rao lower bound, Rayleigh fading channel.

I. INTRODUCTION

Wireless sensor network (WSN) has become a popular research topic recently [1-12]. Usually, a WSN includes a large number of sensors to collect information and a fusion centre to process information [13]. The fusion centre has much more computational power and capacity than sensors. Therefore, the fusion centre, after collecting information from sensors, can perform many tasks, such as estimation, detection, and tracking [13].

In this paper, we only consider distributed estimation problem in WSNs. An energy-based target localization method was presented in [13] for a nonlinear model, the energy decay model. In this method, sensors send quantized data to the fusion centre, and the fusion centre uses a maximum likelihood estimation (MLE) method to estimate the target power and target position. However, this scheme suffers from several problems, such as sensor fault and communication channel errors. The imperfect communication channels were discussed and particularly, three different communication channel models were presented in [14]. These three communication channel models are binary symmetric channel (BSC), Rayleigh fading channel with coherent receiver, and Rayleigh fading channel with non-coherent receiver were discussed in [14]. However, research in [14] focused on the nonlinear estimation model. For a linear estimation model, the same problem exists. For a popular linear estimation model presented in [15]-[18], the communication channel problem also exists when sensors send decisions to the fusion centre. In [18], BSC channel was included into the MLE framework for linear estimation problems. However, to the best of knowledge, the Rayleigh fading channel has not been included into the MLE scheme for the linear estimation model presented in [15]-[18]. In many applications, Rayleigh fading channel is an important channel, particularly when multi-path transmission exists.

The main contribution of this paper is the inclusion of Rayleigh fading channel into the MLE scheme for a linear estimation model. Particularly, we considered Rayleigh fading channel with non-coherent receiver and Rayleigh fading channel with coherent receiver. Moreover, the Cramer-Rao lower bounds (CRLBs) corresponding to both receivers are presented. Furthermore, simulation results are presented to show the performance of the distributed estimation method in the presence of Rayleigh fading channels.

Section II presents the distributed estimation method based on decisions transmitted over Rayleigh fading channels with coherent receiver, and Section III presents the distributed estimation method based on decisions transmitted over Rayleigh fading channel with non-coherent receiver. The simulation setup are discussed in Section IV and Section V presents results and analysis. Conclusions are presented in Section VI.

II. DISTRIBUTED ESTIMATION METHOD BASED ON DECISIONS TRANSMITTED OVER RAYLEIGH FADING CHANNELS WITH COHERENT RECEIVER

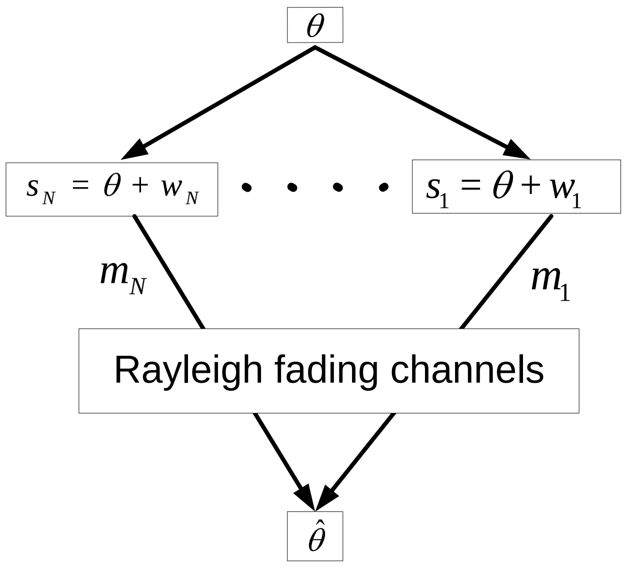

Figure 1: System Setup

The system setup was shown in Figure 1. A total number of N sensors are deployed in the field. Following the setup in [15][18], sensors measure an unknown but fixed variable θ. Because of the presence of noises, the measurement s_i will be corrupted by noise w_i. The measurement equation can be expressed as

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Zhenxing Luo, Department of Electrical and Computer Engineering, the University of Alabama at Birmingham, Birmingham, AL, USA.

$$s_i = \theta + w_i \,. \tag{1}$$

In (1), w_i is a Gaussian noise with zero mean and variance σ^2.

After conducting measurements, sensors transmit measurements to the fusion centre. To save energy and communication bandwidth, sensors will only transmit quantized data to the fusion centre [13]. We assume all sensors employ the same threshold γ. If the received signal s_i is greater than the threshold γ, the sensor will send decision 1 to the fusion centre. Otherwise, sensors will send decision 0 to the fusion centre. The quantization process can be denoted by:

$$m_i = \begin{cases} -1 & -\infty < s_i < \gamma \\ 1 & \gamma \le s_i < +\infty \end{cases} . \tag{2}$$

The probability that m_i assumes value m can be calculated by

$$p(m_i = m|\theta) = \begin{cases} 1 - Q(\gamma-\theta), & (m=-1) \\ Q(\gamma-\theta), & (m=1) \end{cases} . \tag{3}$$

In (3), $Q(x)$ is defined as

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt \,. \tag{4}$$

The transmitted decision vector by sensors to the fusion centre is $\mathbf{M} = [m_1 \ m_2 \ ... \ m_N]^T$. However, due to the presence of Rayleigh fading channel, the received decision vector $\tilde{\mathbf{M}} = [\tilde{m}_1 \ \tilde{m}_2 \ ... \ \tilde{m}_N]^T$ at the fusion centre will not be the same as the transmitted decision vector $\mathbf{M}$.

For Rayleigh fading channels with coherent receiver, the transition relations $p(\tilde{m}_i|m_i)$ between m_i and $\tilde{m}_i$ are defined as [14]

$$\begin{aligned} p(\tilde{m}_i|m_i) &= \frac{2\sigma_v}{\sqrt{2\pi}(1+2\sigma_v^2)} \\ &\times e^{\frac{-\tilde{m}_i^2}{2\sigma_v^2}} \left[1 + m_i\sqrt{2\pi}\alpha\tilde{m}_i e^{\frac{(\alpha\tilde{m}_i)^2}{2}} Q(-\alpha m_i \tilde{m}_i) \right] \end{aligned} \tag{5}$$

where

$$\alpha = 1 \big/ (\sigma_v \sqrt{1+2\sigma_v^2}) \,. \tag{6}$$

The fusion centre, based on $\tilde{\mathbf{M}}$, estimates θ by maximizing

$$\ln p(\tilde{\mathbf{M}}|\theta) = \prod_{i=1}^{N} \left[\sum_{m_i=o}^{L-1} p(\tilde{m}_i|m_i) p(m_i|\theta) \right] . \tag{7}$$

The maximum likelihood estimator tries to find the θ value to maximize

$$\hat{\theta} = \max_{\theta} \ln p\left(\tilde{\mathbf{M}}|\theta\right) . \tag{8}$$

For an unbiased estimate of θ, the CRLB can be derived by

$$E\{[\hat{\theta}(\tilde{\mathbf{M}}) - \theta][\hat{\theta}(\tilde{\mathbf{M}}) - \theta]^T\} \ge \mathbf{J}^{-1} \tag{9}$$

$$\mathbf{J} = -E\left[\nabla_\theta \nabla_\theta^T \ln p\left(\tilde{\mathbf{M}}|\theta\right) \right] . \tag{10}$$

Now, the detailed steps to calculate the CRLB are presented. First, the (1, 1) element of $\mathbf{J}$ matrix is dereived:

$$\begin{aligned} &\frac{\partial^2 \ln p\left(\tilde{\mathbf{M}}|\theta\right)}{\partial P_0^2} = \\ &\sum_i \sum_l -\frac{\delta(\tilde{m}_i - l)}{p^2(\tilde{m}_i|\theta)} \left[\frac{\partial \ln p(\tilde{m}_i|\theta)}{\partial P_0} \right]^2 \\ &+ \frac{\delta(\tilde{m}_i - l)}{p(\tilde{m}_i|\theta)} \frac{\partial^2 p(\tilde{m}_i|\theta)}{\partial P_0^2} \end{aligned} . \tag{11}$$

Because the expectation of the second term of (11) is 0, the expectation of (11) is

$$\begin{aligned} &E\left[\frac{\partial^2 \ln p\left(\tilde{\mathbf{M}}|\theta\right)}{\partial P_0^2} \right] = \\ &\sum_i \sum_l -\frac{1}{p^2(\tilde{m}_i|\theta)} \left[\frac{\partial \ln p(\tilde{m}_i|\theta)}{\partial P_0} \right]^2 \end{aligned} . \tag{12}$$

In (12), $p(\tilde{m}_i|\theta)$ is defined in (3).

The derivative of $p(\tilde{m}_i|\theta)$ with respect to P_0 is

$$\frac{\partial p(\tilde{m}_i|\theta)}{\partial P_0} = \sum_{m_i=0}^{L-1} p(\tilde{m}_i|m_i) \frac{\partial p(m_i|\theta)}{\partial P_0} . \tag{13}$$

In (13), $\dfrac{\partial p(m_i|\theta)}{\partial P_0}$ can be obtained as

$$\begin{aligned} &\frac{\partial p(m_i=-1|\theta)}{\partial P_0} = \\ &\frac{\partial}{\partial P_0}\left[1 - Q(\gamma-\theta)\right] = -\frac{1}{\sqrt{2\pi}} e^{-\frac{(\gamma-\theta)^2}{2}} \end{aligned} \tag{14}$$

$$\begin{aligned} &\frac{\partial p(m_i=1|\theta)}{\partial P_0} \\ &= \frac{\partial}{\partial P_0}\left[1 - Q(\gamma-\theta)\right] = \frac{1}{\sqrt{2\pi}} e^{-\frac{(\gamma-\theta)^2}{2}} \end{aligned} \tag{15}$$

We can find other elements of $\mathbf{J}$ matrix in a similar way.

III. DISTRIBUTED ESTIMATION BASED ON DECISIONS TRANSMITTED OVER RAYLEIGH FADING CHANNEL WITH NON-COHERENT RECEIVER

If decisions sent by sensors are transmitted through Rayleigh fading channels with non-coherent receiver, the MLE method is the same as the MLE method used for Rayleigh fading channels with coherent receiver. The only difference is the transition relations in (5) and binary decisions used. For non-coherent receiver, 0s and 1s are used. For coherent receiver, -1s and 1s are used.

The transition relations corresponding to Rayleigh fading channel with non-coherent receiver are [15]

$$p(\tilde{m}_i|m_i=0) = \frac{1}{2\sigma_v^2} \times e^{\frac{-\tilde{m}_i}{2\sigma_v^2}} \tag{16}$$

$$p(\tilde{m}_i|m_i=1) = \frac{1}{1+2\sigma_v^2} \times e^{\frac{-\tilde{m}_i}{1+2\sigma_v^2}} \,. \tag{17}$$

The detailed steps to calculate the CRLB corresponding to the MLE method based on decisions transmitted through Rayleigh fading channels with non-coherent receiver are presented below.

First, the (1, 1) element of **J** matrix is determined:

$$\frac{\partial^2 \ln p\left(\tilde{\mathbf{M}}|\theta\right)}{\partial P_0^2} = \sum_i \sum_l -\frac{\delta(\tilde{m}_i - l)}{p^2(\tilde{m}_i|\theta)}\left[\frac{\partial \ln p(\tilde{m}_i|\theta)}{\partial P_0}\right]^2 + \frac{\delta(\tilde{m}_i - l)}{p(\tilde{m}_i|\theta)}\frac{\partial^2 p(\tilde{m}_i|\theta)}{\partial P_0^2}. \quad (18)$$

Because the expectation of the second term of (18) is 0, the expectation of (18) is

$$E\left[\frac{\partial^2 \ln p\left(\tilde{\mathbf{M}}|\theta\right)}{\partial P_0^2}\right] = \sum_i \sum_l -\frac{1}{p(\tilde{m}_i|\theta)}\left[\frac{\partial \ln p(\tilde{m}_i|\theta)}{\partial P_0}\right]^2. \quad (19)$$

In (19), $p(\tilde{m}_i|\theta)$ is defined in (16) and (17).

The derivative of $p(\tilde{m}_i|\theta)$ with respect to P_0 is

$$\frac{\partial p(\tilde{m}_i|\theta)}{\partial P_0} = \sum_{m_i=0}^{L-1} p(\tilde{m}_i|m_i)\frac{\partial p(m_i|\theta)}{\partial P_0}. \quad (20)$$

In (20), $\frac{\partial p(m_i|\theta)}{\partial P_0}$ can be obtained as

$$\frac{\partial p(m_i = -1|\theta)}{\partial P_0} = \frac{\partial}{\partial P_0}\left[1 - Q(\gamma - \theta)\right] = -\frac{1}{\sqrt{2\pi}}e^{-\frac{(\gamma-\theta)^2}{2}} \quad (21)$$

$$\frac{\partial p(m_i = 1|\theta)}{\partial P_0} = \frac{\partial}{\partial P_0}\left[1 - Q(\gamma - \theta)\right] = \frac{1}{\sqrt{2\pi}}e^{-\frac{(\gamma-\theta)^2}{2}}. \quad (22)$$

Other elements of **J** matrix can be determined in a similar way.

IV. SIMULATION SETUP

Because the simulations involving Rayleigh fading channels with coherent receiver are similar to simulations involving Rayleigh fading channels with non-coherent receiver, we only conduct simulations for Rayleigh fading channels with non-coherent receiver. To demonstrate the effect of the number of sensors on RMS estimation errors and the CRLB, simulations were run to calculate the RMS estimation errors given by the distributed estimation method based on decisions transmitted through Rayleigh fading channels with non-coherent receiver. We used $\theta = 5$, $\gamma = 5$, and channel SNR=6dB. Figure 2 shows the RMS estimation errors and the CRLB. To calculate the RMS estimation errors in Figure 2, 100 Monte Carlo simulations were used.

To demonstrate the effect of channel SNR on RMS errors and the CRLB, we used 10 sensors, set $\theta = 5$, $\gamma = 5$, and varied the channel SNR from 0dB to 25dB. Figure 3 shows the RMS estimation errors and the CRLB. To calculate the RMS estimation errors in Figure 3, 100 Monte Carlo simulations were used.

The distributed estimation method in this paper assumes that the fusion centre knows the exact channel SNR information. However, if the fusion centre does not know the exact channel SNR, the estimation performance will degrade. To demonstrate the situation when there is a mismatch between the actual channel SNR and the channel SNR assumed by the fusion centre, we set the actual channel SNR to 15dB and varied the assumed channel SNR by the fusion centre from 0dB to 25dB. In the simulation, we used 30 sensors, set $\theta = 5$ and set $\gamma = 5$. Figure 4 shows the RMS estimation errors and the CRLB. To calculate the RMS estimation errors in Figure 4, 100 Monte Carlo simulations were used.

V. RESULTS AND ANALYSIS

In Figure 2, as the number of sensors increased, the RMS estimation errors given by the MLE method decreased. The results showed that the number of sensors has to be sufficient to provide satisfactory results. Moreover, the distributed estimation method is especially sensitive when the number of sensors is low. However, if the number of sensors passes a certain threshold, the number of sensors will not be so important. Increasing the number of sensors will not dramatically improve the estimation performance. Furthermore, the RMS estimation errors given by the distributed estimation method were close to the CRLB when the number of sensors was high. When the number of sensors was low, the RMS estimation errors were much higher than the CRLB.

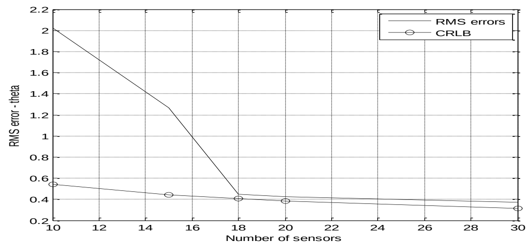

Figure 2: RMS estimation errors compared to the CRLB (Different sensor numbers)

Similarly, channel SNR will also affect the performance of the distributed estimation method. When channel SNR was high, which means better channel condition, the estimation performance was good, and the RMS estimation errors given by the MLE method were close to the CRLB. When channel SNR was low, which means channel condition was not good, the estimation performance was also not good, and the RMS estimation errors given by the distributed estimation method were much higher than the CRLB.

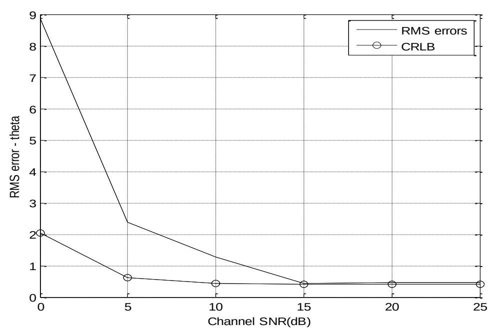

Figure 3: RMS estimation errors compared to the CRLB (Varying channel SNR value)

If the fusion centre does not have accurate channel SNR information, the estimation performance will degrade. In Figure 4, the actual channel SNR was 15 dB and the channel SNR assumed by the fusion centre varied from 0dB to 25dB. The RMS estimation errors given by the distributed estimation method increased when the channel SNR assumed by the fusion centre deviated from 15dB. The simulation results revealed the importance for the fusion centre to have the accurate channel SNR information. In practice, training methods can be used to acquire more accurate channel SNR information to improve the estimation performance.

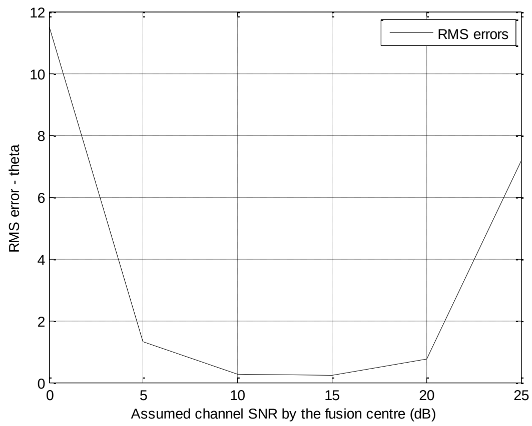

Figure 4: RMS estimation errors when the channel SNR assumed by the fusion centre is not the same as the actual channel SNR (Actual channel SNR=15dB and fusion center assumes different SNR values)

VI. CONCLUSION

In this paper, we presented a distributed estimation method based on decisions transmitted through Rayleigh fading channels with either coherent receiver or non-coherent receiver. Moreover, simulation results showed that the distributed estimation method based on decisions transmitted over Rayleigh fading channel with non-coherent receiver could provide satisfactory results when the channel SNR was high, the number of sensors was sufficient and the fusion centre knew the accurate channel SNR information. However, when the channel SNR was low, the number of sensors was not sufficient, or the fusion centre did not know the accurate channel SNR information, the estimation performance degraded. The research in this paper confirmed the importance of these factors in providing good estimation performance. Similar simulations can also be conducted for the distributed estimation method based on decisions transmitted over Rayleigh fading channel with coherent receiver.

REFERENCES

1. D. Li, K. D. Wong, Y.H.Hu, and A. N. Sayeed, "Detection, Classification, and Tracking of Targets", IEEE Signal Processing Magazine, vol.19, no. 3, pp. 17-29, Mar. 2002.
2. Z. X. Luo and T. C. Jannett, "Optimal Threshold for Locating Targets within a Surveillance Region Using a Binary Sensor Network", in Proceedings of the International Joint Conferences on Computer, Information, and Systems Sciences, and Engineering (CISSE 09), Dec. 2009.
3. Z. X. Luo and T. C. Jannett, "A Multi-Objective Method to Balance Energy Consumption and Performance for Energy-Based Target Localization in Wireless Sensor Networks", in Proc. of the 2012 IEEE Southeastcon, Orlando, FL, Mar. 2012.
4. X. Sheng and Y. H. Hu, "Maximum Likelihood Multiple-Source Localization Using Acoustic Energy Measurements with Wireless Sensor Networks", IEEE Transactions on Signal Processing, vol.53, no.1, pp. 44-53, Jan. 2005.
5. Z. X. Luo and T. C. Jannett, "Performance Comparison between Maximum Likelihood and Heuristic Weighted Average Estimation Methods for Energy-Based Target Localization in Wireless Sensor Networks", in Proceedings of the 2012 IEEE Southeastcon, Orlando, FL, Mar. 2012.
6. Z. X. Luo and T. C. Jannett, "Modelling Sensor Position Uncertainty for Robust Target Localization in Wireless Sensor Networks", in Proceedings of the 2012 IEEE Radio and Wireless Symposium, Santa Clara, CA, Jan. 2012.
7. Z. X. Luo and T. C. Jannett, "Energy-Based Target Localization in Multi-Hop Wireless Sensor Networks", in Proceedings of the 2012 IEEE Radio and Wireless Symposium, Santa Clara, CA, Jan. 2012.
8. Z. X. Luo, "A censoring and quantization scheme for energy-based target localization in wireless sensor networks," Journal of Engineering and Technology 2012, no 2, pp. 69-74.
9. Z. X. Luo, "Anti-attack and channel aware target localization in wireless sensor networks", International Journal of Engineering and Advanced Technology, vol. 1, no.6, Aug. 2012.
10. Z. X. Luo, "Robust energy-based target localization in wireless sensor networks in the presence of Byzantine attacks", International Journal of Innovative Technology and exploring Engineering, vol. 1, no.3, Aug. 2012.
11. Z. X. Luo, "A coding and decoding scheme for energy-based target localization in wireless sensor networks", International Journal of Soft Computing and Engineering, vol. 2, no. 4, Sept. 2012.
12. Z. X. Luo, "Distributed Estimation in Wireless Sensor Networks with Heterogeneous Sensors", International Journal of Innovative Technology and Exploring Engineering, vol. 1, no. 4, Sept. 2012.
13. R. X. Niu and P. K. Varshney, "Target Location Estimation in Sensor Networks with Quantized Data", IEEE Transactions on Signal Processing, vol. 54, pp. 4519-4528, Dec. 2006.
14. O. Ozdemir, R. X. Niu, and P. K. Varshney, "Channel Aware Target Localization with Quantized Data in Wireless Sensor Networks," IEEE Trans. Signal Process., vol. 57, pp. 1190-1202, 2009.
15. A. Ribeiro, and G. B. Giannakis, "Bandwidth-constrained Distributed Estimation for Wireless Sensor Networks-part I: Gaussian case," IEEE Trans. Signal Process., vol. 54, no. 3, pp.1131-43, March 2006.
16. A. Ribeiro, and G. B. Giannakis, "Bandwidth-constrained Distributed Estimation for Wireless Sensor Networks-part II: Unknown Probability Density Function," IEEE Transactions on Signal Process., vol. 54, no. 7, pp. 2784-96, July 2006.
17. C. Hao and P. K. Varshney, "Nonparametric One-Bit Quantizers for Distributed Estimation," IEEE Transactions on Signal Processing, vol. 58, pp. 3777-3787, 2010.
18. C. Hao and P. K. Varshney, "Performance Limit for Distributed Estimation Systems With Identical One-Bit Quantizers," IEEE Transactions on Signal Processing, vol. 58, pp. 466-471, 2010.
19. G. Liu, B. Xu, M. Zeng, and H. Chen, "Distributed estimation over binary symmetric channels in wireless sensor networks," IET Wireless Sensor Systems, vol. 1, pp. 105-109, 2011.