

Innovative Use of What if Analysis for Visualization

Amitha P L, Geethu Joy, Geethu S Pillai, Tharakrishnan L, Soman K P

Abstract—The novelty of this paper is aimed at simplifying the tasks in Spreadsheet using a feature called what if analysis, which fires up the performance of the task we are working on. The use of spreadsheet helped us save time, perform many operations such as sorting, searching, classifying and comparing easily and to solve a problem without any programming knowledge. What if or sensitivity analysis is one of the most powerful and valuable concepts in Spreadsheet, the potential of which is not well exploited. The advantage of what if analysis is that if we show one computation in excel, the remaining part of the process will be computed by its own for a given range of variable values. It can be used to solve many problems other than the conventional managerial applications. Two Dimensional function evaluation and graphing, creating Pascal triangles, enumerating Pythagorean triplets in a given range, error function evaluation are some of the real applications. These types of applications can be exploited to enhance computational thinking of children in high schools. Computational thinking brings about a neoteric approach in problem-solving and model simulation.

Index Terms— Computational Thinking, Excel Computation, What if Analysis.

I. INTRODUCTION

Computational Thinking (CT) is a unique combination of thinking skills that, when used together, provide the basis of a new and powerful form of problem solving and model simulation by drawing on the concepts fundamental to computer science. It is necessary for the success of 21st century, a mandatory fundamental skill set. In the context of computational thinking it is necessary that we use computer as an extension of human brain that is, our thinking, to simulate models or to find solutions. It is the method of thinking that helps an individual to participate in the faster growth of society through better utilization of raw materials using the most optimized method. To achieve this it is necessary to encourage our students to think computationally at a very early stage thereby which they will exhibit creative thinking, construct knowledge, and develop innovative products and processes using technology. It should be our aim to develop computational thinkers than software users. Computational thinking applied by each individual will be different in their own ways. An in depth knowledge about the technology is not necessary to solve the problems but the knowledge of how and where it can be applied is needed. Let us now consider a simple example regarding the literacy rate in different parts of

the world. The researchers from different parts discuss together to give a report on the literacy rate. After the discussions, they find that certain parts of the world lack in educational rate. So in order to enhance the literacy rate, they think of different ways. This is known as computational thinking. Computational thinking can be defined as development of one's knowledge rather than utilization of information. One of the innovative tools that can help students to learn uses of CT to represent and help solve problems within the various disciplines is Microsoft Excel, whose power has not been properly utilized so far. What-if analysis can help students to solve models and create simulations for a number of applications in all disciplines of science ranging from management to a number of day-to-day scenarios. It helps students to move beyond technological literacy. By using what-if analysis to solve problems explained below, it will be much easier to convince the students the benefits of using this tool. Also it makes integration of CT into curriculum easier.

Spreadsheet is an innovative method which helps in thinking computationally as it supports problem-solving activities. Given a problem situation with complex quantitative relationships, spreadsheets can be used to represent those relationships which enable user's higher order thinking. In order to come to a better conclusion to the problem, it is necessary to analyze the data logically, to represent them as a model, to obtain the solutions through a sequence of procedures and to debug the errors which can be done effectively with excel. This is because spreadsheets may be used as a cognitive tool for amplifying and reorganizing mental functioning. The users are the rule makers in using spreadsheets which help in simulating any model. In excel, calculating values requires that the user identify relationships and patterns among the data that he or she wants to represent. Next, those relationships must be modeled mathematically, using rules to describe the relationships in the model.

This paper deals with the use of an innovative, powerful tool-what-if analysis. The advantages of using what-if analysis in the context of computational thinking is high as it helps to address problems in all disciplines of science as well as management. It motivates teachers to utilize it in various subjects which encourage students to address any problem at hand computationally. Through this paper we would like to emphasize that the usage of spreadsheets can only be of real value as long as they are put into an educational context and are integrated with the major objectives of the curriculum.

II. WHAT-IF

Spreadsheet gives entrepreneurs an opportunity to plan by simulating any hypothetical situation that can arise in their work of budgeting through the use of What-if analysis.

Manuscript received on January, 2013.

Amitha P L, Centre for Excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India.

Geethu Joy, Centre for Excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India.

Geethu S Pillai, Centre for Excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India.

Tharakrishnan L, Centre for Excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India.

K P Soman, Centre for Excellence in Computational Engineering and Networking, Amrita Vishwa Vidyapeetham, Coimbatore, India.

The extensive usage of microcomputers supplemented with the innovative power of excel data sheet lead to the development of VisiCalc in 1978. But what if analysis that has not been much in use till now, especially in the field of education. It has the power of increasing the performance of the program we are working on. The most vibrant and extraordinary use of what if analysis is that it helps in the execution of the same operation for different set of data inputs. Then it records the output of these given input sets. This makes the process easier with less time consumption rather than performing the operations manually. This feature of what-if analysis should be utilized at the educational level or in any other occasion for that reason. It is not necessary for a student to have any programming knowledge to implement a problem but a complicated problem can be broken down in a simple and easy manner by a student with basic mathematical knowledge.

The three methods that enable what- if analysis to be powerful for visualization is scenarios, data tables, and goal seek. Data table helps us to calculate the values for different equations by inserting a row input cell and column input cell value. This is done by selecting the range of cells. The data table shows the change in the values in the selected range by changing the equation. We can decide the number of variables according to our problem. In a two variable data table, one formula is used with two input values.

In this paper, some experiments are done with this feature which may become tedious to visualize with other programming languages. So let us see how they are implemented via what-if analysis.

III. EXCEL IMPLEMENTATION USING WHAT-IF

A. Pascal's Triangle

With the help of spreadsheet in Microsoft Excel we can easily generate Pascal triangle, which is a number triangle with binomial coefficients. Aim of this experiment is to make students in exploring Pascal triangle and its interesting properties with a simpler tool named what if.

$$a_{n,r} = \frac{n!}{r!(n-r)!} \tag{1}$$

Steps:

1. Enter value 1 in cell B1 and string 'n' in A1.
2. Enter value 1 in cell B2 and string 'r' in A2.
3. Type expression =IF (\$B2\leq\$B1,COMBIN(B1,B2),"") in cell B3 and enter string 'a' in cell A3 as shown in Fig.1.
4. Create two-variable data table. Enter n values as along a row and r values as along a column. Enter the reference value in cell D1 as =B3.
5. Select the range of cells and now go to the data tab and click what-if analysis from which we have to select the data table. In the row input cell box, select cell B2 and in the column input cell box select the cell B1 and then click ok which is shown in Fig.2.

In addition we can infer so many interesting properties from the implementation of Pascal's triangle that it shows Hockey stick property (Fig.3) and each row is a power of 11 (Fig.4) and so on.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	n	1		1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	r	1		0													
3	a	1		1													
4				2													
5				3													
6				4													
7				5													
8				6													
9				7													
10				8													
11				9													
12				10													
13				11													
14				12													

Fig.1: Initial data table for creating Pascal's Triangle

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	n	1		1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	r	1		0	1												
3	a	1		1	1												
4				2	1	2	1										
5				3	1	3	3	1									
6				4	1	4	6	4	1								
7				5	1	5	10	10	5	1							
8				6	1	6	15	20	15	6	1						
9				7	1	7	21	35	35	21	7	1					
10				8	1	8	28	56	70	56	28	8	1				
11				9	1	9	36	84	126	126	84	36	9	1			
12				10	1	10	45	120	210	252	210	120	45	10	1		
13				11	1	11	55	165	330	462	462	330	165	55	11	1	
14				12	1	12	66	220	495	792	924	792	495	220	66	12	1

Fig.2: Pascal's Triangle

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	n	1		1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	r	1		0	1												
3	a	1		1	1												
4				2	1	2	1										
5				3	1	3	3	1									
6				4	1	4	6	4	1								
7				5	1	5	10	10	5	1							
8				6	1	6	15	20	15	6	1						
9				7	1	7	21	35	35	21	7	1					
10				8	1	8	28	56	70	56	28	8	1				
11				9	1	9	36	84	126	126	84	36	9	1			
12				10	1	10	45	120	210	252	210	120	45	10	1		
13				11	1	11	55	165	330	462	462	330	165	55	11	1	
14				12	1	12	66	220	495	792	924	792	495	220	66	12	1

Fig.3: Pascal's Triangle -Hockey stick property

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1	n	1		1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	r	1		0	1												
3	a	1		1	1												
4				2	1	2	1										
5				3	1	3	3	1									
6				4	1	4	6	4	1								
7				5	1	5	10	10	5	1							
8				6	1	6	15	20	15	6	1						
9				7	1	7	21	35	35	21	7	1					
10				8	1	8	28	56	70	56	28	8	1				
11				9	1	9	36	84	126	126	84	36	9	1			
12				10	1	10	45	120	210	252	210	120	45	10	1		
13				11	1	11	55	165	330	462	462	330	165	55	11	1	
14				12	1	12	66	220	495	792	924	792	495	220	66	12	1

Fig.4: Property of Pascal's Triangle

B. Use of What-If in Managerial Application

To enhance business performance as well as manage inherent business risks, it is necessary to have an integrated planning approach. What-if analysis is a cornerstone of such an approach since it can predict outcome based on combination of several parameters.



It helps in visualizing hypothetical situations that can occur in course time of a business firm, for example, In Customer Relationship Management (CRM), you generally analyze clients by looking at different sub-groups (or 'target groups') and seeing which are most lucrative.

Consider the example where you want to buy a car, and want to compare six price points and nine loan terms to calculate the monthly payment amount. Enter the initial data as given in the Fig.5.

	A	B
1	PRICE	450,000
2	TERM OF PAYMENT	40
3	INTEREST	890%
4	PAYMENT	333,750

Fig.5: Input Data

Here interest is taken as 890%. The Payment is calculated with the PMT formula.

PMT is a built in function provided in excel. Now we need is a table of various prices and terms of payment. Upon applying what-if analysis we get the output table as in Fig.6 for different values of price and term of payment.

	333,750.00	400,000	425,000	450,000	475,000	500,000	525,000
15	296738.8	315284.9	333831.1	352377.3	370923.45	389469.6	
20	296671.2	315213.1	333755.1	352297	370838.96	389380.9	
25	296666.9	315208.6	333750.3	352292	370833.68	389375.4	
30	296666.7	315208.4	333750	352291.7	370833.36	389375	
35	296666.7	315208.3	333750	352291.7	370833.33	389375	
40	296666.7	315208.3	333750	352291.7	370833.33	389375	
45	296666.7	315208.3	333750	352291.7	370833.33	389375	
50	296666.7	315208.3	333750	352291.7	370833.33	389375	
55	296666.7	315208.3	333750	352291.7	370833.33	389375	

Fig.6: Data table depicting the use of what-if analysis in management

Now we need to change the interest only, the computed values in the table changes according to the new interest rate.

C. Plotting a Two Dimensional Function via Automatic Range Changing

By default, Microsoft Excel follows the relative addressing. Let us consider a simple example.

$$f(x, y) = x^2 + y^2 \tag{2}$$

For the above example, we enter the x values as a column and the y values as a row. We now enter a single formula (as shown in blue cell) in the upper-left-hand corner as shown in the Fig.7. This formula should be dragged each time to form a new matrix, once the formula is changed. If \$A3 is copied or moved into a different column, the column designation A will not change, but if copied to a different row, the row designation 3 would change appropriately. This task is made easier in the case of what-if analysis, where we just need to enter the row input cell and the column input cell in the data table. Even after varying the equations, we get the corresponding values in the matrix automatically.

	A	B	C	D	E	F	G
1		x					
2	y	1	2	3	4	5	6
3		1	2	5	10	17	26
4		2	5				
5		3	10				
6		4	17				
7		5	26				
8		6	37				
9		7	50				
10		8	65				
11		9	82				
12		10	101				

Fig.7: Relative addressing for $f(x, y) = x^2 + y^2$

In order to plot a 2 dimensional function in what-if, we have to create a two-variable data table. So we needed two lists of input variables. So, here we plotted two simple functions (refer to "(2),") which is defined above and one more which is defined below (refer to "(3),").

$$f(x, y) = x * \cos(y) \tag{3}$$

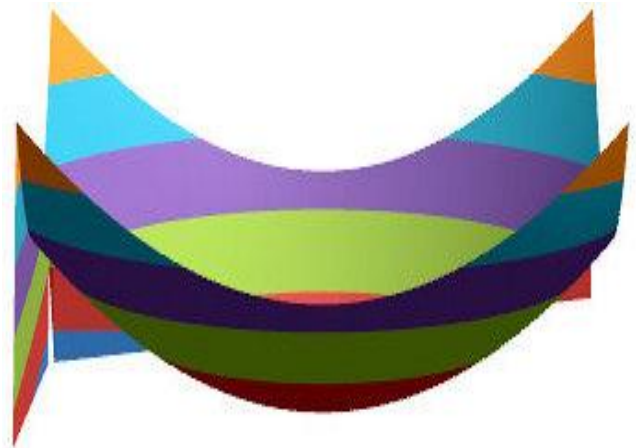


Fig.8: Surface plot for $f(x, y) = x^2 + y^2$

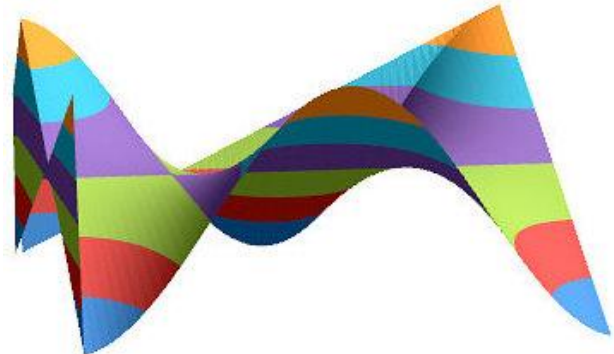


Fig.9: Surface plot for $f(x, y) = x * \cos(y)$

In all these examples (Fig.8, Fig.9) we can see that the value of a function depends entirely on the values of x and y and that we are plotted using 3D surface plot. So, it is just need to change the value of a function alone, to visualize how they get affected for different combination of values.

We can automatically change the two lists of input values by simply changing two parameters. For that, let us initially declare two variables like start and end with range values and range as (end-start). Instead of dragging the cells, we could just change the values of these two parameters and could get all the values lying within that range. This method is called automatic range changing. This makes the work easier and less time consuming rather than changing the range values each time by dragging the cells.

D. Plotting 2D Trigonometric Function

We are familiar with one dimensional trigonometric functions like sine and cosine. There is another set of functions called two dimensional trigonometric functions, which most of us are not familiar with. This experiment helps to create such functions and to visualize them. A big difference from the one dimensional trigonometric function is that, it has two frequencies, one along the X-axis and the other along the Y-axis. In addition we can have sine and cosine waves in the same 2D trigonometric function, i.e. in one axis we may see 3 sine waves and 2 cosine waves along the other.



We are creating such functions by using the concept of outer product. Here we multiply two vectors of size $m \times 1$ and $1 \times n$ then we get a matrix of size $m \times n$. This matrix produces the required trigonometric function. Let us now see the excel implementation of this function.

Steps:

1. Click on cell A5 and I1 and type =COS (RADIANS ((ROW ()-5)*n1)).

2. Click on cell B4 and I2 and type =COS (RADIANS ((COLUMN ()-2)*n2)).

In the above expressions, it returns cosine values for zero degree angles. Here n1 and n2 refers to the frequencies along the x and y directions. As we keep on changing the frequency values, the number of cycles also changes accordingly. For instance if our frequency is 20, it produces two cycles. Next step is to create our row vector and column vector for the creation of our data table.

3. Click on cell A5 and drag down till A41 to produce one cycle of cosine values.

This can be taken as the column vector of size 37×1 . Next let's create a row vector of size 1×37 .

4. Click on the cell B4 and drag right till AL4. This produces the second set of values.

5. Click on cell A4 and type the reference value =I3 which the outer product of the expression =I1*I2.

6. After creating the vectors, our next aim is to create a matrix of size 37×37 . This can be done using what-if analysis. For this, select the range of cells and go to the data tools group and click what-if analysis from which we have to select the data table. The row input cell box, select as the cell I1 and in the column input cell box select the cell I2 and then click ok. The resulted data table for different combinations is shown in Fig.10.

7. Next go to the insert tab and select other charts from which we selected a 3D surface plot in order to get a complete visualization of the trigonometric function (Fig.11, Fig.12). You can repeat this for different combinations of sine and cosines and different number of cycles also.

E	F	G	H	I	J	K	L	M	N	O
		x	0.766							
		y	-0.866		n1	10				
		z	-0.6634		n2	30				
0.866	0.766	0.6428	0.5	0.34202	0.1736	6.1E-17	-0.1736	-0.342	-0.5	-0.6428
0.866	0.766	0.6428	0.5	0.34202	0.1736	6.1E-17	-0.1736	-0.342	-0.5	-0.6428
0.75	0.6634	0.5567	0.433	0.2962	0.1504	5.3E-17	-0.1504	-0.2962	-0.433	-0.5567
0.433	0.383	0.3214	0.25	0.17101	0.0868	3.1E-17	-0.0868	-0.171	-0.25	-0.3214
5E-17	5E-17	4E-17	3E-17	2.1E-17	1E-17	3.8E-33	-1E-17	-2E-17	-3E-17	-4E-17
-0.43	-0.383	-0.321	-0.25	-0.171	-0.0868	-3E-17	0.0868	0.171	0.25	0.32139
-0.75	-0.663	-0.557	-0.433	-0.2962	-0.1504	-5E-17	0.1504	0.2962	0.433	0.55667
-0.87	-0.766	-0.643	-0.5	-0.342	-0.1736	-6E-17	0.1736	0.342	0.5	0.64279
-0.75	-0.663	-0.557	-0.433	-0.2962	-0.1504	-5E-17	0.1504	0.2962	0.433	0.55667
-0.43	-0.383	-0.321	-0.25	-0.171	-0.0868	-3E-17	0.0868	0.171	0.25	0.32139

Fig.10: Data table for two variable cosine waves

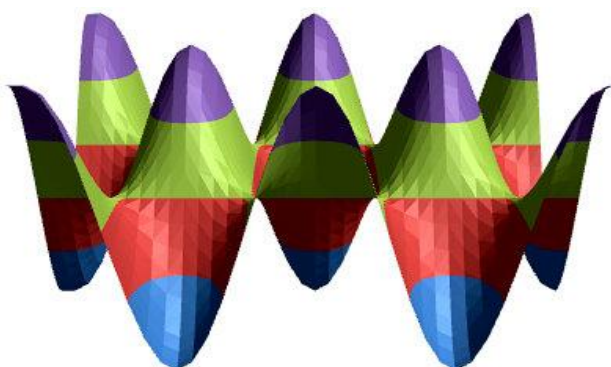


Fig.11: Two variable cosine waves



Fig.12: Two variable sine waves

E. Sum of Squared Error of a Mathematical Model

Here we aimed at helping the students to visualize a line or curve of best fit or regression analysis in spreadsheet using what-if analysis. In the mathematical modeling of data we are trying to fit an equation to the data. Regression techniques minimize the sum of the squared errors (SSE) by adjusting equation parameters. As we all aware that, error or residual can be calculated as the difference between the measured values of data point and the predicted values of the data point. The predicted value is calculated from the regression equation. The difference is referred to as the error or the residual.

$$e = (y_{\text{measured}} - y_{\text{predicted}}) \tag{4}$$

$$\text{squared error, } e^2 = (y_{\text{measured}} - y_{\text{predicted}})^2 \tag{5}$$

Fig.13 shows the column set-up for the data and its calculations. To start with this in spreadsheet, we are giving parameter values m and c in the equation manually (as shown in yellow cells). So they can visualize how these parameters influence the line or curve.

Let us consider fitting a regression equation $y = mx + c$ for the data values shown in Fig.13.

	A	B	C	D	E	F	G
1			m	4			
2			c	5			
3							
4	x	y predicted	y=y measured	Squared error			$y' = mx + c$ $\text{error} = (y - y')^2$
5	0	2.9	5	4.41			
6	1	4	9	25			
7	2	6.5	13	42.25			
8	3	8.5	17	72.25			
9	4	10.4	21	112.36			
10	5	12.4	25	158.76			
11	6	14.6	29	207.36			
12	7	17	33	256			
13	8	18.8	37	331.24			
14	9	21.1	41	396.01			
15	10	24	45	441			
16	11	24.6	49	595.36			
17	12	26.9	53	681.21			
18	13	29	57	784			
19	14	31	61	900			
20	15	32.5	65	1056.25			
21			SSE=	6063.46			
22							

Fig.13: Data table for $y = mx + c$

Initially generate a set of values for x. Similarly we generate the predicted values of y. In the next column, we must calculate the measured values of y using our regression equation $y = mx + c$. In the last column, using the predicted values of y and the measured values of y, we can calculate the error square. The last column is then summed yielding the sum of the squared errors (SSE). Now we must select the range of cells that contains values we want to substitute and move to the data tab, in the Data Tools group, click What-If Analysis, and then click Data Table. In the row input cell box, give the input as m and in the column input cell box, give the value of c. Select the range of cells that contains values and now go to insert tab, select other charts and choose 3-D Surface plot to get the visualization of this model which is shown in Fig.14.



IV. CONCLUSION

Through this paper we have presented a number of practical applications. Two Dimensional function evaluation and graphing, creating Pascal triangles, enumerating Pythagorean triplets in a given range, error function evaluation are the real applications that has been implemented. Spreadsheet provides a way for What-if thinking to exploit various ideas of powerful mathematics. The paper has also portrayed the exploits of computational thinking using excel as a tool to bestow the novel ideologies using what-if analysis which can be visualized by almost all individual. The main motive of this paper is to inculcate computational thinking by employing the functionality of what if analysis in excel for school going children, so that they can create their own ideas in a systematic way for easy visualization.

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Fig.14: Error surface for $y = mx + c$

F. Pythagorean Triplet

A Pythagorean Triplet is a set of positive integers a, b and c that fits the rule:

$$a^2 + b^2 = c^2 \quad (6)$$

where a and b are the legs of the triangle and c is the hypotenuse. This theorem serves as the basis for the distance between two points. There are many existing proofs for this basic fundamental theorem. One of the most fundamental proofs for this theorem is the Euclid's geometric proofs. The Pythagorean triplet has been implemented in Excel for a single iteration using the command:

=IF(AND(F1-F2>=0,int(sqrt(F1^2+F2^2))=sqrt(F1^2+F2^2)),sqrt(F1^2+F2^2),"")

In this experimentation, a variable count was introduced which is nothing but the frequency count for how many times each number is repeated as a different combination of sum of squares of two integers. We inferred that the frequency of occurrence of integer 65 and 85 was 4 which was new information for us.

Fig. 15 shows the excel implementation of triplets. Euclid's proves the Pythagorean Theorem by a method called the bride's chair which uses Fig. 16.

tripletttable		fx {=TABLE(F1,F2)}													
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1					x	1									
2					y	1									
3					z										
4															
5	count		1	2	3	4	5	6	7	8	9	10	11	12	13
6		0	1												
7		0	2												
8		0	3			5									
9		0	4												
10		1	5											13	
11		0	6							10					
12		0	7												
13		0	8												
14		0	9												
15		1	10											15	

Fig.15: Triplet Implementation in Excel

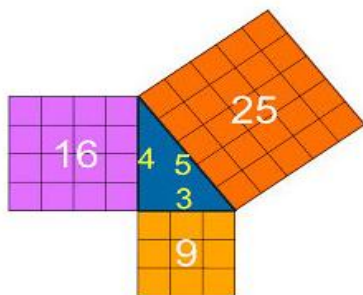


Fig.16: Bride's chair