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Optimal Ordering Policy for Stock-Dependent Demand Inventory Model with Non-Instantaneous Deteriorating Items

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***Abstract:-** In this paper we have discussed optimal ordering policy for inventory model with non-instantaneous deteriorating items and stock-dependent demand. Here shortage is not allowed. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution have been shown. Sensitivity analysis of the optimal solution with respect to major parameters is carried out. A numerical example is presented to demonstrate the developed model and the solution procedure.*

***Keywords:** Non-instantaneous deterioration, Inventory, purchasing cost, Sales revenue cost, Stock-dependent demand.*

I. INTRODUCTION

In real life situations maintenance of quality and originality of products like as fruit, fish, meat, vegetables, and so on is naturally maintained for some time and no deterioration occurs. We term this phenomenon as ‘‘non-instantaneous deterioration’’. Now a days it is observed in the supermarket that display of the consumer goods in large quantities attracts more customers and generates higher demand. **Gupta and Vrat (1986)** were the first to develop models for stock-dependent consumption rate. **Baker and Urban (1988)** established an economic order quantity model for a power-form inventory-level-dependent demand pattern. **Mandal and Phaujdar (1989)** developed an economic production quantity model for deteriorating items with constant production rate and linearly stock-dependent demand. Other researchers related to this area such as **Pal et al. (1993), Giri et al. (1996), Ray et al. (1998), etc. Soni and Shah (2008)** discussed the optimal ordering policy for an inventory model with stock-dependent demand. **Chang et al. (2010)** developed an optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand. **Sana, S.S., (2012)** developed an EOQ model for perishable items with stock-dependent demand.

This paper deals with an inventory model with optimal replenishment policy for non-instantaneous deteriorating items and stock-dependent demand. The total profit of the whole system is maximized by a proposed solution algorithm. A numerical example, graphical illustration and sensitivity analysis are used to illustrate the model.

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II. NOTATION AND ASSUMPTIONS

The following assumptions and notations are used in this paper:

1) The demand rate D(t) at time t is $D(t) = a + bI(t)$. Where a, b are positive constants and I(t) is the inventory level at time t.
2) t_1 is the length of time in which the product has no deterioration (*i.e.*, fresh product time).
3) α is the deterioration rate.
4) A, C_h, C_p, C_d, C_s Denote the ordering cost per order, inventory holding cost per unit time, purchasing cost per unit, deteriorating cost per unit and sales revenue cost per unit respectively.
5) I_1, the inventory level at time $[0, t_1]$ in which the product has no deterioration. I_2, the inventory level at time $[t_1, t_2]$ in which the product has deterioration.
6) TP (t_1, t_2) is the total present value of profit per unit time of inventory system.

III. MATHEMATICAL MODEL

The inventory levels are governed by the following differential equations:

$$\frac{dI_1(t)}{dt} = -[a + b\,I_1(t)] \qquad 0 \le t \le t_1 \qquad \ldots (1)$$

$$\frac{dI_2(t)}{dt} + \alpha\, I_2(t) = -[a + b\,I_1(t)] \qquad t_1 \le t \le T \qquad \ldots (2)$$

with the boundary conditions $I_1(0) = L$, $I_2(T) = 0$ respectively. By solving these differential equations, we get the inventory level as follows:

$$I_1(t) = \frac{a}{b}(e^{-bt} - 1) + Le^{-bt}, \qquad 0 \le t \le t_1 \qquad \ldots (3)$$

$$I_2(t) = \frac{a}{b+\alpha}(e^{(b+\alpha)(T-t)} - 1), \qquad t_1 \le t \le T \qquad \ldots (4)$$

Considering continuity of I (t) at t=t_1, it follows from Equations (3) and (4) that $I_1(t_1) = I_2(t_1)$

$$\Rightarrow \quad L = \frac{a}{b+\alpha}(e^{(b+\alpha)t_2} - 1)e^{bt_1} - \frac{a}{b}(1 - e^{bt_1}) \qquad \ldots (5)$$

Now the total present value of profit per cycle consists of the following elements:

1) Ordering cost per cycle is OC= A. …(6)

2) Holding cost per cycle is given by

$$HC = C_h \left(\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right)$$

$$= C_h\left[\frac{a+Lb}{b^2}(1 - e^{-bt_1}) - \frac{a}{b}t_1 + \frac{a}{(b+\alpha)^2}(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1)\right]$$

….(7)

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3) Deterioration cost per cycle is given by

$$DC = C_d \int_{t_1}^{t_2} \alpha\, I_2(t)dt$$

$$= \alpha\, C_d \left[\frac{a}{(b+\alpha)^2}\left(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1\right)\right]$$

…(8)

4) Purchasing cost per cycle is given by

$$PC = C_p \times L = C_p \left[\frac{a}{b+\alpha}\left(e^{(b+\alpha)t_2} - 1\right)e^{bt_1} - \frac{a}{b}\left(1 - e^{bt_1}\right)\right] \quad \ldots(9)$$

5) Sales Revenue cost per cycle is given by

$$SRC = C_s \int_0^T D(t)dt$$

$$= C_s \left[a t_2 + \frac{a+Lb}{b}\left(1-e^{-bt_1}\right) + \frac{ab}{(b+\alpha)^2}\left(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1\right)\right]$$

…(10)

Thus the total present value of profit per cycle per unit time is given by

$$TP = \frac{1}{T}\left[SRC - OC - HC - DC - PC\right] \quad \ldots(11)$$

Substituting equations (6–10) in the above equation (11), we get

$$TP = \frac{1}{T}\Bigg[C_s\left\{ a t_2 + \frac{a+Lb}{b}\left(1-e^{-bt_1}\right) + \frac{ab}{(b+\alpha)^2}\left(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1\right)\right\}$$

$$- A - C_h\left\{ \frac{a+Lb}{b^2}\left(1-e^{-bt_1}\right) - \frac{a}{b}t_1 + \frac{a}{(b+\alpha)^2}\left(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1\right)\right\}$$

$$- \alpha\, C_d\left\{ \frac{a}{(b+\alpha)^2}\left(e^{(b+\alpha)t_2} - (b+\alpha)t_2 - 1\right)\right\}$$

$$- C_P\left\{ \frac{a}{b+\alpha}\left\{e^{(b+\alpha)t_2} - 1\right\}e^{bt_1} - \frac{a}{b}\left(1-e^{bt_1}\right)\right\}\Bigg] \quad \ldots.\ (12)$$

The total present value of profit per unit time is maximum if

$$\frac{dTP}{dt_2} = 0 \quad \ldots.\ (13)$$

and

$$\frac{d^2TP}{dt_2^2} < 0 \quad \ldots.\ (14)$$

IV. SOLUTION ALGORITHM FOR PROPOSED MODEL

Step 1. Input A, C_h, C_p, C_s, C_d, α, a, b, t_1;

Step 2. From equation (13) compute t_2 and from Relation (12) compute TP;

Step 3. Put the value of t_2 in equation (14) to check the optimal solution. If satisfied then go to stop otherwise go to step 1 for changing the parameters values.

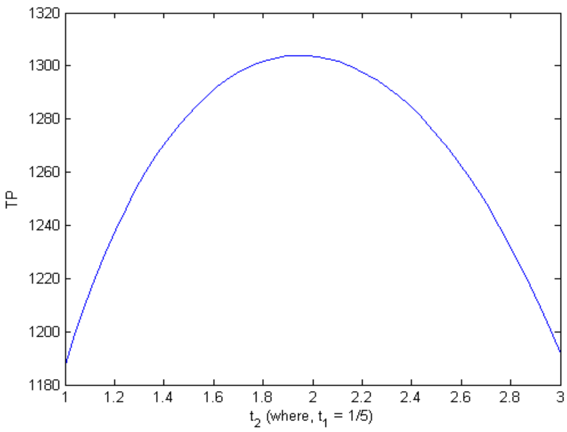
V. NUMERICAL EXAMPLE

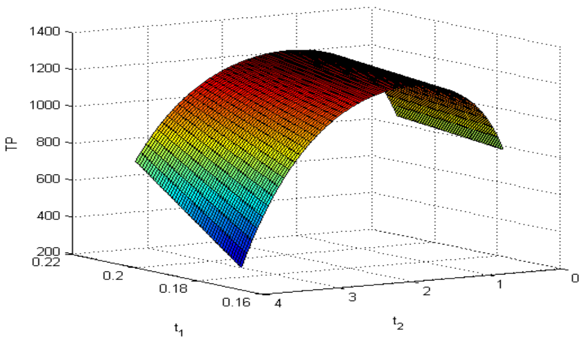
To illustrate the above results, we consider the following example: A=500, C_s = 30 per unit, Cp = 15 per unit, C_h = 0.50 per unit, C_d = 0.2 per unit, α=0.60 and D(t) = 100 + 0.5I(t) units. From Table 1, we observe that the system cost (TP) is Maximum when t_1=1/5 and t_2=1.9505941 (month).

Table 1

Change in		t_2	L	TP
a	100	1.951	779.34	1303.91
	110	1.899	805.31	1457.83
	120	1.853	830.17	1612.26
b	0.50	1.951	779.34	1303.91
	0.49	1.773	618.39	1264.23
	0.48	1.646	521.78	1230.71
α	0.60	1.951	779.34	1303.91
	0.61	1.810	663.56	1276.52
	0.62	1.699	584.19	1252.37
C_s	28	1.311	345.68	917.50
	29	1.524	457.66	1095.79
	30	1.951	779.34	1303.91
C_p	15	1.951	779.34	1303.91
	16	1.317	348.11	1028.85
	17	1.080	250.02	818.75
C_h	0.40	2.112	945.86	1333.67
	0.50	1.951	779.34	1303.91
	0.60	1.829	671.74	1278.03
A	500	1.951	779.34	1303.91
	600	2.051	879.66	1258.49
	700	2.138	975.89	1214.92

The following graphs show the relation between total profit and time period t_1 and t_2.





VI. SENSITIVITY ANALYSIS

As can be observed in the above study the sensitivity analysis of the parameters present in this model, the TP changes significantly with changes in the different parameters values. Furthermore the sensitivity analysis represents the following results:

1. If demand rate (*a*) increases, then the lower time t_2 is, the longer order quantity and total profit increases but if demand rate (*b*) increases, then the longer time t_2 is, the longer order quantity and total profit increases.

2. If deterioration rate (α) increases, then t_2, order quantity and total profit decreases.

3. If the sales revenue cost (C_s) increases, then t_2, order quantity and total profit increases.

4. If purchasing cost (C_p) and holding cost (C_h) increase, then t_2, order quantity and total profit decreases.

5. If ordering cost (A) of the produced items increases then it is quite natural that the profit for this purpose decreases.

VII. CONCLUSION

In this article, a deterministic inventory model has been proposed for non-instantaneous deteriorating items with stock-dependent demand. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. Here in this paper we have used a numerical example to demostrate sensivity analysis and to get the optimal solution. A possible future research direction is the study of a multi-item inventory model for production rate, inflation, shortages, partial backlogging, two warehouses and permissible delay in payments.

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